Lab 1

School of Architecture, Civil and Environmental Engineering

EPFL, SS 2023-2024

http://disal.epfl.ch/teaching/signals_instruments_systems/

Ubuntu & Matlab Reminder

- Check the terminal commands in the assignment.
- Try to apply them one by one.
- Change key bindings to «Windows Style» in Matlab.

Lab 1 Outline

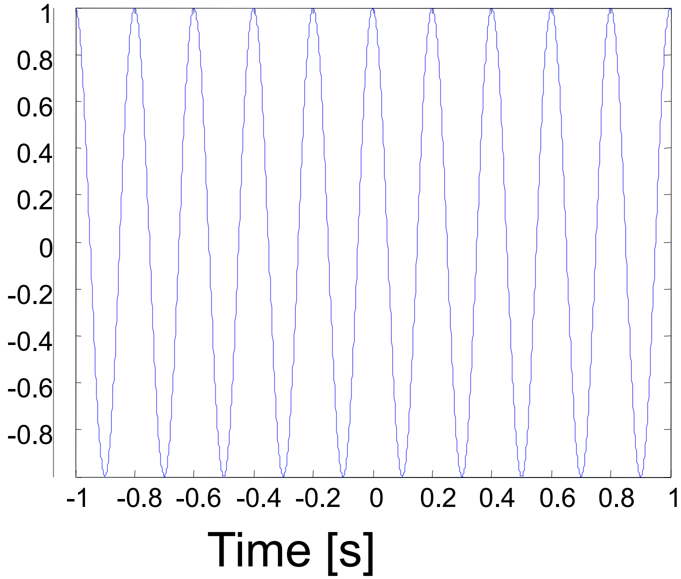
- Concept:
  - Signals (Part 1)
  - Fourier Transforms (Part 2)
  - Fourier Series (Part 3)
  - Convolution (Part 4, 5)
- Tools:
  - Matlab

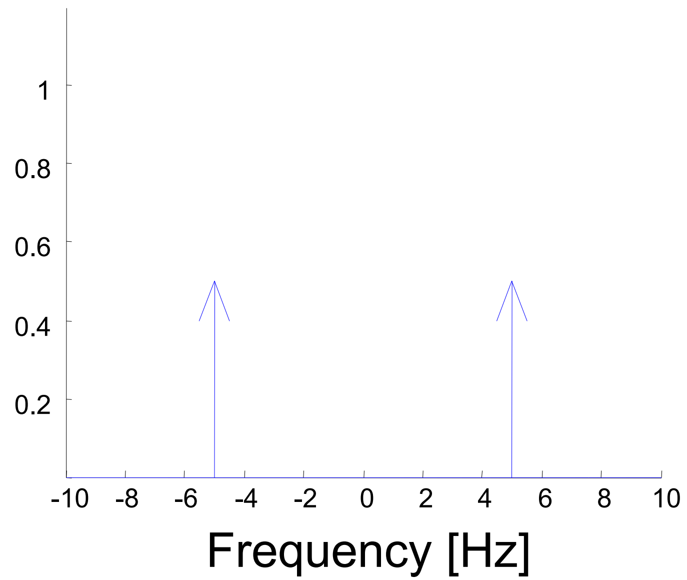
Reminder: Fourier Transform

$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i2\pi\xi t} dt$$

$$f(t) = \int_{-\infty}^{\infty} \hat{f}(\xi) \cdot e^{i2\pi t\xi} d\xi$$

Reminder: Fourier Transform

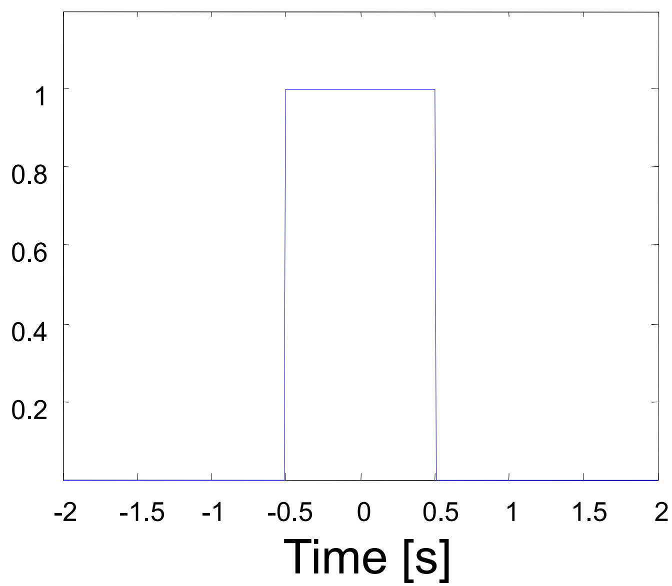




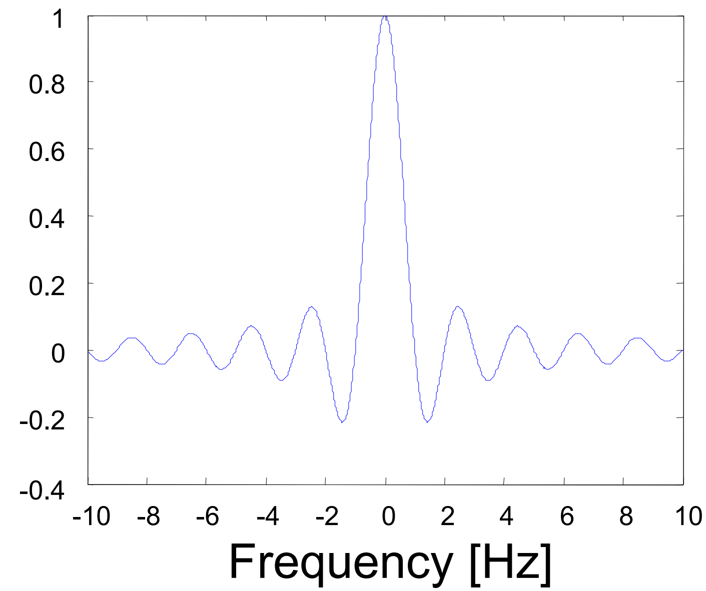
$$f(t) = \cos(2\pi a t)$$

$$\hat{f}(\xi) = \frac{\delta(\xi - a) + \delta(\xi + a)}{2}$$

Reminder: Fourier Transform



$$f(t) = rect(t)$$



$$\hat{f}(\xi) = \operatorname{sinc}(\xi)$$

Fourier Properties Table

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency
	$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi}\, dx$	$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-i\omega x}\, dx$	$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(x) e^{-i\nu x}\, dx$
101	$a \cdot f(x) + b \cdot g(x)$	$a \cdot \hat{f}(\xi) + b \cdot \hat{g}(\xi)$	$a \cdot \hat{f}(\omega) + b \cdot \hat{g}(\omega)$	$a \cdot \hat{f}(\nu) + b \cdot \hat{g}(\nu)$
102	$f(x-a)$	$e^{-2\pi i a \xi} \hat{f}(\xi)$	$e^{-ia\omega} \hat{f}(\omega)$	$e^{-ia\nu} \hat{f}(\nu)$
103	$e^{2\pi i a x} f(x)$	$\hat{f}(\xi - a)$	$\hat{f}(\omega - 2\pi a)$	$\hat{f}(\nu - 2\pi a)$
104	$f(ax)$	$\frac{1}{\|a\|} \hat{f}\left(\frac{\xi}{a}\right)$	$\frac{1}{\|a\|} \hat{f}\left(\frac{\omega}{a}\right)$	$\frac{1}{\|a\|} \hat{f}\left(\frac{\nu}{a}\right)$
105	$\hat{f}(x)$	$f(-\xi)$	$f(-\omega)$	$2\pi f(-\nu)$
106	$\frac{d^n f(x)}{dx^n}$	$(2\pi i \xi)^n \hat{f}(\xi)$	$(i\omega)^n \hat{f}(\omega)$	$(i\nu)^n \hat{f}(\nu)$
107	$x^n f(x)$	$\left(\frac{i}{2\pi}\right)^n \frac{d^n \hat{f}(\xi)}{d\xi^n}$	$i^n \frac{d^n \hat{f}(\omega)}{d\omega^n}$	$i^n \frac{d^n \hat{f}(\nu)}{d\nu^n}$
108	$(f * g)(x)$	$\hat{f}(\xi)\hat{g}(\xi)$	$\sqrt{2\pi} \hat{f}(\omega)\hat{g}(\omega)$	$\hat{f}(\nu)\hat{g}(\nu)$
109	$f(x)g(x)$	$(\hat{f} * \hat{g})(\xi)$	$\frac{(\hat{f} * \hat{g})(\omega)}{\sqrt{2\pi}}$	$\frac{1}{2\pi}(\hat{f} * \hat{g})(\nu)$

(Continuous) Fourier Transform Implementation

- Use of Matlab symbolics tool to compute the Fourier transform analytically.
- The result is true almost everywhere
- To create a symbolic variable, use the command *`syms`*

Reminder: Discrete Fourier Transform

$$\mathrm{X[k]} = \sum_{n=0}^{N-1} x[n] \cdot e^{\frac{-2\pi i}{N} kn}, k = 0, \cdots, N-1$$

$$\mathrm{x[n]} = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{\frac{2\pi i}{N} kn}, k = 0, \cdots, N-1$$

- Input and output sequences are both finite (the integral becomes a finite sum)
- Different from the Discrete-Time Fourier Transform (DTFT)
- Efficient implementation is done using Fast Fourier Algorithms (FFT)

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Continuous Convolution

$$(f*g)(t)=\int_{-\infty}^{\infty} f(\tau)\cdot g(t-\tau)d\tau$$

- For each value of *t*:
  1. Flip (reflect) ***g***
  2. Shift ***g*** by ***t***
  3. Multiply ***f*** and ***g***
  4. Integrate over ***τ***

1) $g(\tau)\rightarrow g(-\tau)$

2) $g(-\tau)\rightarrow g(t-\tau)$

3) $f(\tau)\cdot g(t-\tau)$

4) $\int_{-\infty}^{\infty} f(\tau)\cdot g(t-\tau)d\tau$

- Note that the result does **not** depend on τ!

Discrete Convolution

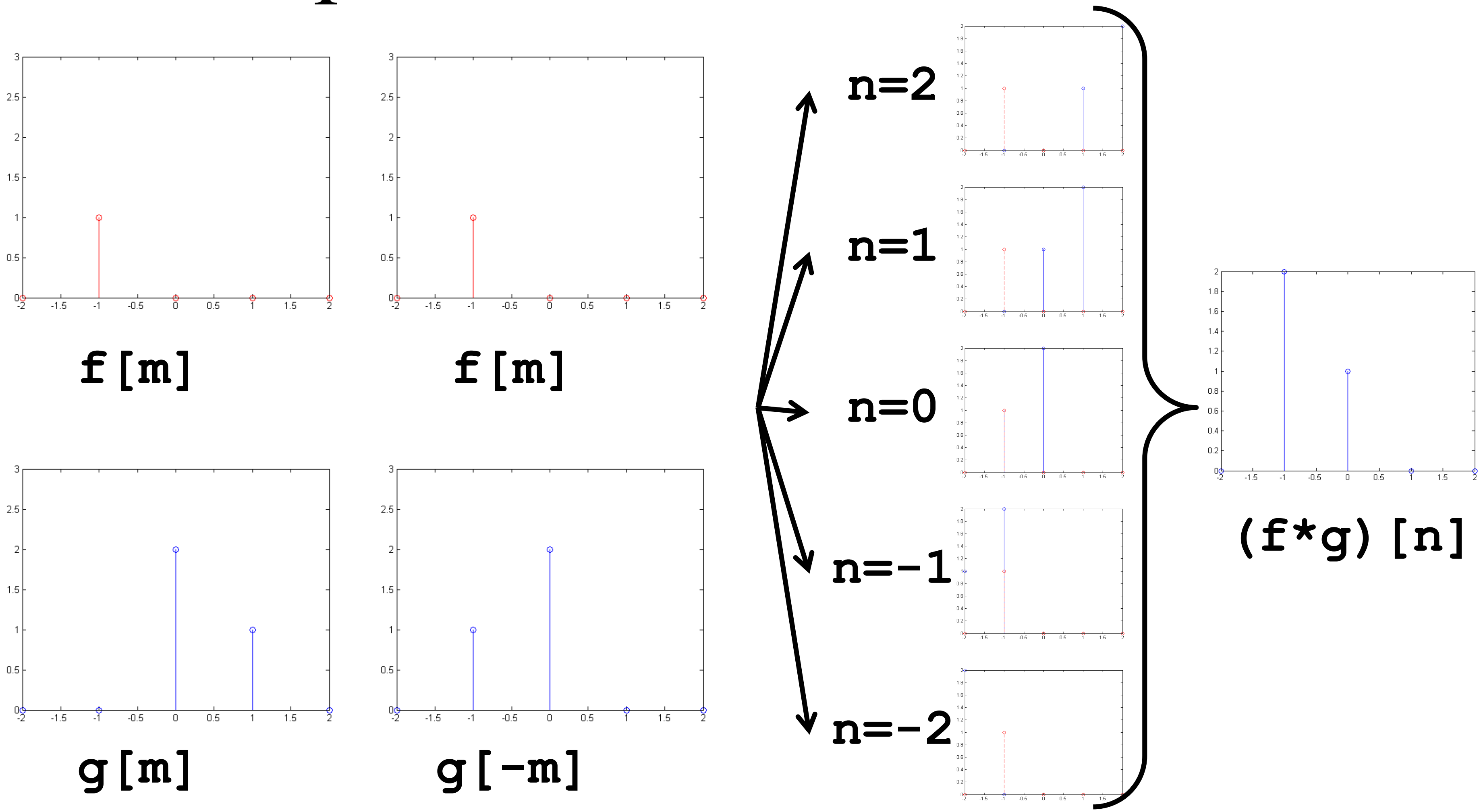
$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t-\tau) d\tau$$

$$\downarrow$$

$$(f * g)(n) = \sum_{m=-\infty}^{\infty} f(m) \cdot g(n-m)$$

- Similar to the continuous version
- The integral becomes an infinite sum
- Matlab, operating on a computer, can only emulate continuity and therefore use the discrete version with an adjustable discretization level in time and amplitude

Example of Discrete Convolution



Feedback form

Please fill the feedback form for Lab 1!