Lab 3: Introduction to Signal Processing – Transfer Functions, Responses, and Further Transforms

This laboratory requires the following equipment:

- Matlab

The laboratory duration is approximately 3 hours. Although this laboratory is not graded, we encourage you to take your own personal notes as the examinations might leverage results acquired during this laboratory session. For any questions, please contact us at **sis-ta@groupes.epfl.ch**

Information

In the following, text you will find several exercises and questions.

- The notation **S** means that the question can be solved using only additional simulation.
- The notation **Q** means that the question can be answered theoretically, without any simulation.
- The notation **I** means that the problem has to be solved by implementing a piece of code and performing a simulation.
- The notation **B** means that the question is optional and should be answered if you have enough time at your disposal.

Outline

This lab continues about signal processing, and reviews the transfer functions, responses and Laplace and Z-transforms. In Part 1, you will apply different continuous transforms to a first-order continuous system to find the step response and to analyze it. In Part 2, you will be given the discrete version of the system. You will apply discrete transformations to obtain the step response and analyze it. Every part includes both theoretical and Matlab related questions. You can find the Laplace and Z-transform tables in the Appendix B. The table below shows you the main idea behind the questions:

Question No.	Main Idea
1 (Q)	Finding the step response by using the definition of convolution theoretically
2 (S)	Finding the impulse response from the step response programmatically
3 (S)	Finding the step response by using the definition of convolution programmatically
4 (I)	Applying Fourier transform to an arbitrary function to observe the frequency content
5 (B)	Finding the step response by using Laplace transform theoretically
6 (S)	Finding the step response by using Laplace transform programmatically
7 (I)	Finding the step response by using Matlab's built-in functions
8 (B)	Finding the transfer function by using Z-transform theoretically (Discrete time)
9 (I)	Finding the step response by using Matlab's built-in functions (Discrete time)
10 (I)	Finding the step response by using Z-transform programmatically (Discrete time)
11 (B)	Applying DTFT to arbitrary function to observe the freq. content (Discrete time)

Getting Started

To start with this lab, you will need to download the material available on Moodle. Download `lab03.tar.gz` or `lab03.zip` and extract it in your home directory (you can type `tar xvfz lab03.tar.gz`). Now start Matlab and change your "Current directory" to `lab03/part01/`.

Important note: Please install *Control System Toolbox* if you have not installed it already. You can do so by pressing *Add-Ons* button in Matlab main window.

Part 1: System Responses & Continuous-Time Transforms

In this part, you will apply Continuous-Time (CT) transformations (Fourier and Laplace transforms) to a first-order system to obtain and analyze its step response.

In the systems theory, there are three fundamental test signals that can be applied to a system to reveal its characteristic by observing their responses (outputs): impulse function $\delta(t)$, step function $u(t)$ and sinusoidal function $s(t)$. Their definitions are given as follows:

$$\delta(t) = 0 \; if \; t \neq 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

$$u(t) = \begin{cases} 1, & if \; t \geq 0 \\ 0, & otherwise \end{cases}$$

$$s(t) = A sin(\omega t)$$

where A is the amplitude and ω is the angular frequency of the sinusoidal function in radians.

In this lab, we will investigate the responses of impulse and step functions and you will learn the response to sinusoidal function, i.e., frequency response, in the following labs.

Impulse response is defined as the output of the system when the impulse function *δ(t)* (Dirac delta function introduced in Week 2 lecture) is applied to the input. Fig. 1 demonstrates this concept where $h(t)$ is the impulse response. This response contains the "signature" characterizing the system. As you recall from Week 2 lecture, convolving an impulse function centered on 0 (i.e., not shifted in time) with any other signal will deliver the same signal (identity operation), without any distortion. Therefore, perturbing the system with an impulse function at its input, will deliver the non-distorted, characteristic response, containing the system's "signature" at its output. In other words, this operation is equivalent to conduct a convolution operation between the impulse function and the impulse response of the system.

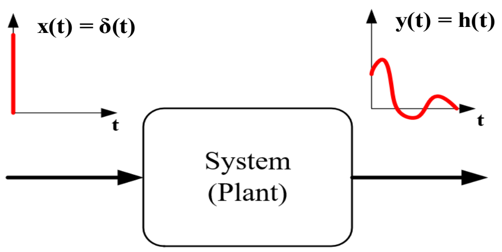

Figure 1. Impulse function and impulse response

Similarly, *the step response* of a system is defined as the time evolution of its output when its input is a step function. It is more commonly used than the impulse response since the realization of the physical step function is easier than the impulse function. Fig. 2 illustrates such concept. Here, $u(t)$ is the step function and $y_u(t)$ is step response.

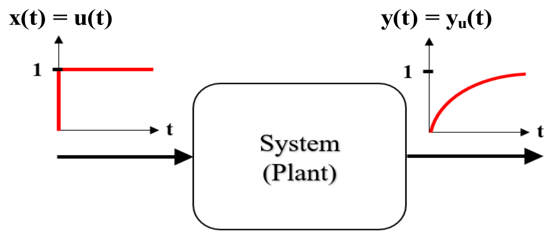

Figure 2. Step input and step response

By observing both responses, we can have an idea about the transient and steady-state characteristics of the system such as how fast it is, how stable it is, how does it respond when time goes to infinity.

Open the script `part_01.m` in the folder `part_01`, read the explanations of this template and check the corresponding questions for step-by-step guidance. You can use *Run Section* button (or *Ctrl* + *Enter* inside the section) to run each section separately in the template. Pay attention to question categorization (i.e., **Q, S, I, B**) to decide whether if an implementation is needed or not.

1. **(Q)** Consider a system whose impulse response (when the impulse is applied at t=0) is given as $h(t) = 3e^{-2t}$ where $t \geq 0$. Calculate the step response of the system $y_u(t)$, by convolving the step function $u(t)$ with the impulse response $h(t)$. Remember that the continuous convolution can be calculated by the following equation:

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)d\tau$$

Now, let us analyze a simple electrical system. Consider the analog RC circuit given in Fig. 3. This circuit consists of an independent voltage source $v_s(t)$, a resistor R, and a capacitor C, whose voltage is shown by $v_c(t)$ (all in SI units).

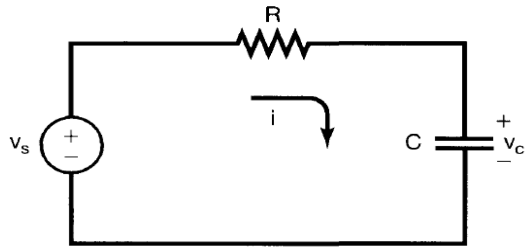

Figure 3. RC circuit [1]

This system can be represented by a continuous first-order differential equation if we select $v_s(t)$ as input and $v_c(t)$ as output. The equation can be obtained as follows by using the Kirchoff's voltage law:

$$v_s(t) = v_r(t) + v_c(t)$$

By knowing that $v_r(t) = i(t)R$ and $i(t) = C\frac{dv_c(t)}{dt}$ for the resistor and the capacitor, we can obtain a relationship between the input and the output as follows:

$$\frac{dv_c(t)}{dt} = \frac{1}{RC}(v_s(t) - v_c(t))$$

If we define $x(t)$ as input, $y(t)$ as output and the multiplication of R and C as Γ, the equation has a new form as follows:

$$\frac{dy(t)}{dt} = \frac{1}{\Gamma}(x(t) - y(t))$$

Here Γ is known as *time constant* of the system and it is an indication of how fast the system responds. It is the duration where the system reaches (approximately) 63% of its steady state (final) value. Pay attention to this definition, you will use this information to extract the time constant of the given system from the plots of the step response. You can add data points on the plot to find the instant where approximately 63% of the final value is reached.

Note: You can find equivalent mechanical and fluid system representations in Appendix A.

To find the step response (i.e., the state of the system when the input is a step function) in time the domain, we should substitute $x(t)$ with the step function $u(t)$. If we solve this first-order differential equation by considering the *complementary* and *particular* solutions, we obtain the following step response:

$$y(t) = y_u(t) = \left(-e^{\left(-\frac{t}{\Gamma}\right)} + 1\right) \cdot u(t)$$

2. **(S):** Given that R = 1 Ω and C = 0.1 F, find the impulse response $h(t)$ of the system from the step response $y_u(t)$ given. Plot both the step response and impulse response using Matlab and observe the figure. Can you identify the time-constant from the figure? Why did we use a "Heaviside" function in the code?

   Hint: Since the system is linear, the impulse response of the system is the derivative of the step response with respect to time.

3. **(S):** Now we will calculate the step response in a different way, i.e., instead of solving a differential equation. Since you know the definition of the step function and found the impulse response in the time domain, you can find the step response by convolving both signals analytically. However, there is no symbolic convolution function in Matlab. Instead, we can use the definition of continuous convolution to calculate it:

$$y_u(t) = u(t) * h(t) = \int_{-\infty}^{\infty} u(\tau) \cdot h(t - \tau)\, d\tau$$

   Obtain the step response, plot, and compare it with the one that is given in Question **1**. Which function did we use to calculate the symbolic integration? Do the overall shape and the time-constant differ?

4. **(I):** Analyze the frequency content of the step response $y_u(t)$ by applying the CT Fourier transform in Matlab. Plot the magnitude of the response and observe it. In which frequency region the frequency content is accumulated? Pay attention to the unit of frequency axis. Is it in Hertz or radians/sec?

   Hint-1: Use `fourier()` *function to calculate the Fourier transform symbolically.*

Another way to obtain system responses (e.g., impulse and step responses) is to leverage the Laplace transform. The associated frequency domain of this transform is also continuous, and the Laplace transform can be considered a more generic version of the CT Fourier transform. Indeed, the convergence of the Laplace transform is broader than that of the CT Fourier transform, i.e., it includes unstable systems. Therefore, while the Laplace transform is mostly used in continuous system analysis, the CT Fourier transform is mostly used in continuous signal processing as you did in Question **4**.

As explained in the lecture, the complex variable s is used instead of the ordinary frequency ξ or the angular frequency ω used for the two variants of the CT Fourier transform. Multiple properties valid for the Fourier transform also apply to the Laplace transform. For instance, a convolution in time domain corresponds to a multiplication in the frequency domain.

$$y_u(t) = u(t) * h(t) \leftrightarrow U(s) \cdot \mathrm{H(s)} = Y_u(s)$$

where $U(s)$ and $H(s)$ are the Laplace transform of $u(t)$ and $h(t)$. Therefore, by taking the inverse Laplace transform of the multiplication of $U(s)$ and $H(s)$, we can obtain the step response in time domain.

$$y_u(t) = \mathcal{L}^{-1}(U(s) \cdot \mathrm{H(s)})$$

where $H(s) = \frac{Y_u(s)}{U(s)}$ or, more generically for an arbitrary input $X(s)$ and a corresponding response $Y(s)$, $H(s) = \frac{Y(s)}{X(s)}$ is also known as the *transfer function* of the CT system. In other words, the *transfer function* represents the relationship between the arbitrary output signal and input signal for the given system in the Laplace domain. Notice that the Laplace transform of the impulse response $h(t)$ is the transfer function $H(s)$.

5. **(B):** Given that impulse response $h(t) = \left(\frac{1}{\Gamma} e^{\left(-\frac{t}{\Gamma}\right)}\right) \cdot u(t)$ from Question **2**, compute $U(s)$ and $H(s)$ by using the definition of Laplace transform or Laplace transform tables in the *Appendix B*. Find the step response $y_u(t)$ in time domain by using the definition of inverse Laplace transform or inverse Laplace transform tables.

6. **(S):** Given that impulse response $h(t) = \left(\frac{1}{\Gamma} e^{\left(-\frac{t}{\Gamma}\right)}\right) \cdot u(t)$ from Question **2,** apply the same procedure explained in question **5** in Matlab symbolically, instead of using tables, and find the step response $y_u(t)$. Which functions did you use to take the Laplace and inverse Laplace transform symbolically? Plot and compare the result with the one you found in Questions **2** and **3**.

7. **(I):** Another way to obtain step response is to use `tf()` function together with the transfer function representation you have found. Observe $H(s)$ and represent it in Matlab using `tf()` function. Use the `step()` function together with the resulting transfer function to obtain the plot of step response. Plot and compare the result with the ones you found in Questions **2, 3,** and **6**. Do the overall shape and time-constant differ?

   Hint-1: After you obtain H(s) symbolically, take note of the numerator and denominator coefficients and enter this into `tf()`*. For example if your H(s)=0.1/(0.4s-0.3) use H_s =* `tf(0.1,[0.4,-0.3])`*.*
   Hint-2: `step(H_s)` *directly gives and plots the step response where* `H_s` *is your transfer function.*

Part 2: Discrete-Time Transforms

In this part, you will apply some Discrete-Time (DT) transformations (Z-transform and DT Fourier Transform) to a first-order system in order to obtain step response of a discrete-time system and analyze the response. Open the script `part_02.m` in the folder `part_02`, read the explanations of this template and check the corresponding questions.

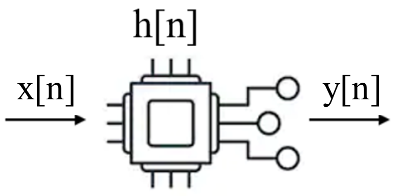

Figure 4. Digital system

Now, we would like to represent our continuous-time electrical system (RC circuit) in an embedded digital computer as in Fig. 4. In this figure, $h[n]$ corresponds to the digital impulse response, i.e., the transfer function that can be programmed in a digital circuit. By using an appropriate discretization method, we can find the difference equation of the system as follows:

$$y[n] = \left(1 - \frac{T}{\Gamma}\right) y[n-1] + \left(\frac{T}{\Gamma}\right) x[n-1]$$

where, T is the sampling period and Γ is defined in the previous questions. Take T as 0.01 seconds.

In order to find the system response $y[n]$ (solution to the difference equation), we first need to find the discrete transfer function $H(z)$ of the system (in the Z continuous frequency domain, or in short Z-domain). We will use the Z-transform (instead of Laplace Transform) to find this discrete transfer function, since the system is represented in discrete time. Note that the Z-transform has a similar purpose for discrete-time functions and systems as the Laplace transform for continuous-time functions and systems.

8. **(B):** Take the Z-transform of the both sides of the difference equation by using Z-transform tables given in the *Appendix B*. Manipulate the equation to obtain the discrete transfer function $H(z) = \frac{Y(z)}{X(z)}$ symbolically.

Hint-1: Use "linearity" and "time shift" properties of the Z-transform while solving the problem analytically.

9. **(I):** By applying the method in Question **8**, it can be found that $H(z) = \frac{0.1}{z-0.9}$, represent it as Matlab transfer function using `tf()`. Use the `step()` function together with the resulting transfer function to obtain the step response. Plot and compare the result with the one you found in Question **7**. Is there any difference between the CT and DT counterparts? Do the overall shape and the time-constant differ?

Hint-1: After you obtain $H(z)$ symbolically, take note of the numerator and denominator coefficients and enter this into `tf()`. For example if your H(z)=0.1/(0.4z-0.3) use H_z = `tf(0.1,[0.4,-0.3],T)` where T is your sample time for discrete system.
Hint-2: `step(H_z)` *directly gives and plots the step response where H_z is your transfer function.*

10. **(I):** Similar to continuous counterpart, take the input $x[n]$ as unit step input $u[n]$ whose definition is given as

$$u[n] = \begin{cases} 1, if\ n \geq 0 \\ 0\ , otherwise \end{cases}$$

Similar to the Laplace transform, a convolution in time domain corresponds to a multiplication in the Z-domain.

$$y_u[n] = u[n] * h[n] \leftrightarrow U(z) \cdot \mathrm{H(z)} = Y(z)$$

where $U(z)$ and $H(z)$ are the Z-transforms of $u[n]$ and $h[n]$. Therefore, by taking the inverse Z-transform of the multiplication, we can obtain the step response in discrete time domain.

$$y_u[n] = Z^{-1}(U(z) \cdot \mathrm{H(z)})$$

Apply this procedure in Matlab to obtain the step response $y_u[n]$. Plot and compare the result with the one you found in question **9**. Do the overall shape and the time-constant differ? For additional step-by-step guidance, please refer to the material.

Hint-1: You already obtained $H(z)$. Find the Z-transform of s[n] by using `ztrans()`*. Multiply this with $H(z)$ and take the inverse transform symbolically by using* `iztrans()`.
Hint-2: You can use `fplot()` *to plot symbolic functions.*

For sake of completeness, we would like you to get a feeling for the Discrete-Time Fourier Transform (DTFT). This transform is associated with a continuous frequency domain and should not be confused with the Discrete Fourier Transform (DFT) that has been exercised in Labs 1 and 2. Moreover, the DT Fourier transform is a special case of the Z-transform. The convergence of the Z-transform is broader than that of the DT Fourier transform, i.e., it includes unstable systems. Therefore, while the Z-transform is mostly used in discrete system analysis, the DT Fourier transform is mostly used in digital signal processing.

11. (B): Define the time series for *n* using the code below:

```
n = 0:1:60;
```

Represent the step response $y_u[n]$ (found in Question **10**) as time-series by using *n,* i.e., $y_u[n] = a - b^n$, where a and b are the real numbers. There is no symbolic function to calculate the DT Fourier transform in Matlab. However, we can use `fft()` to calculate it. Use `calculate_DTFT_and_FFT()` to plot transforms in the folder `part_2`. Input your response series to the functions. Observe how `calculate_DTFT_and_FFT()` is utilizing Matlab's built in function `fft()` to calculate the DT Fourier transform. Observe the differences between FFT and DT Fourier transform plots. Observe the frequency content of the step response and compare it with the one you found in Question **4**.

Appendix A: Mechanical and Fluid systems

Similar to the electrical system (analog RC circuit), let us write the governing differential equation for a mechanical system (mass-damper) shown in Fig. 4. Here, b is the damping coefficient, m is the mass of the object, $v(t)$ is the velocity of the object and $f(t)$ is the force applied to the object (all in SI units).

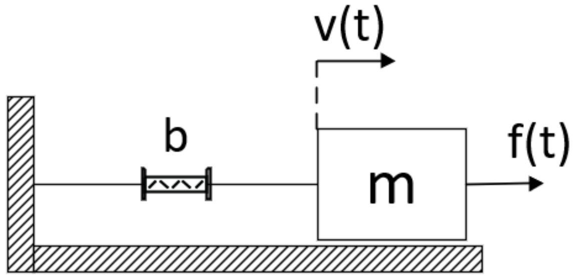

Figure 4. Damper-mass system [2]

By using Newton's second law of motion, the following equation can be written,

$$f(t) - bv(t) = m\frac{dv(t)}{dt}$$

If we rearrange the equation, we obtain,

$$\frac{b}{m}\left[\frac{f(t)}{b} - v(t)\right] = \frac{dv(t)}{dt}$$

Setting $\frac{f(t)}{b}$ as input $x(t)$, v(t) as output $y(t)$ and $\frac{m}{b}$ as time constant Γ gives the following equation,

$$\frac{dy(t)}{dt} = \frac{1}{\Gamma}\,(x(t) - y(t))$$

Finally, let's investigate an equivalent fluid system for a liquid contamination analysis. Here, A is the cross-sectional area of the reservoir, R is the fluid resistance present due to the analysis process, $V(t), Q(t)$, $p(t)$ and $h(t)$ are the time-varying volume of the liquid in the tank, flow rate of the liquid on the pipe, pressure addition due to the pump and height of the liquid, respectively (all in SI units).

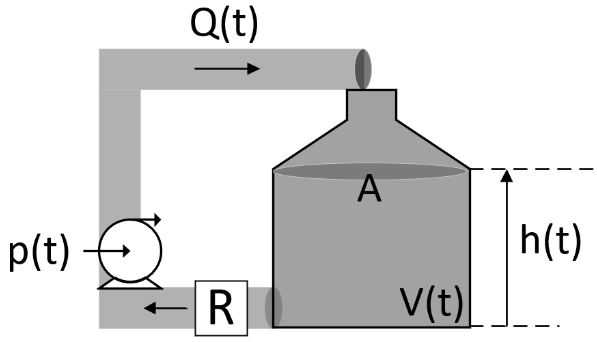

Figure 5. Liquid contamination analysis setup [2]

By using the conservation of mass principle and liquid pressure analysis, we obtain the next governing differential equation,

$$p(t) = Q(t)R + \frac{1}{C}\int Q(t)\,dt$$

where $C = A/\rho g$ and, ρ and g are the density of the fluid and gravitational acceleration in SI units.

If we set Cp(t) as input $x(t)$, $\int Q(t)\,dt$ as output $y(t)$ and $\frac{1}{CR}$ as Γ, we obtain the following equation:

$$\frac{dy(t)}{dt} = \frac{1}{\Gamma}\,(x(t) - y(t))$$

As you can observe, by carefully choosing input and output, the same governing differential equation can be obtained for the different physical systems. In system theory, we name them as equivalent systems. A similar approach can also be applied to thermal systems where thermal resistance and capacitance are present.

Appendix B: Transform tables

Laplace Transform [1]

TABLE 9.1 PROPERTIES OF THE LAPLACE TRANSFORM

Section	Property	Signal	Laplace Transform	ROC
		$x(t)$	$X(s)$	R
		$x_1(t)$	$X_1(s)$	R_1
		$x_2(t)$	$X_2(s)$	R_2
9.5.1	Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
9.5.2	Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
9.5.3	Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
9.5.4	Time scaling	$x(at)$	$\frac{1}{\lvert a \rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
9.5.5	Conjugation	$x^*(t)$	$X^*(s^*)$	R
9.5.6	Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
9.5.7	Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
9.5.8	Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
9.5.9	Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\mathcal{Re}\{s\} > 0\}$

TABLE 9.2 LAPLACE TRANSFORMS OF ELEMENTARY FUNCTIONS

Transform pair	Signal	Transform	ROC
1	$\delta(t)$	1	All s
2	$u(t)$	$\frac{1}{s}$	$\mathcal{Re}\{s\} > 0$
3	$-u(-t)$	$\frac{1}{s}$	$\mathcal{Re}\{s\} < 0$
4	$\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} > 0$
5	$-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} < 0$
6	$e^{-\alpha t}u(t)$	$\frac{1}{s+\alpha}$	$\mathcal{Re}\{s\} > -\alpha$
7	$-e^{-\alpha t}u(-t)$	$\frac{1}{s+\alpha}$	$\mathcal{Re}\{s\} < -\alpha$
8	$\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{Re}\{s\} > -\alpha$
9	$-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s+\alpha)^n}$	$\mathcal{Re}\{s\} < -\alpha$
10	$\delta(t - T)$	e^{-sT}	All s
11	$[\cos \omega_0 t]u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\mathcal{Re}\{s\} > 0$
12	$[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\mathcal{Re}\{s\} > 0$
13	$[e^{-\alpha t}\cos \omega_0 t]u(t)$	$\frac{s+\alpha}{(s+\alpha)^2 + \omega_0^2}$	$\mathcal{Re}\{s\} > -\alpha$
14	$[e^{-\alpha t}\sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s+\alpha)^2 + \omega_0^2}$	$\mathcal{Re}\{s\} > -\alpha$
15	$u_n(t) = \frac{d^n \delta(t)}{dt^n}$	s^n	All s
16	$u_{-n}(t) = \underbrace{u(t) * \cdots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\mathcal{Re}\{s\} > 0$

Z Transform [1]

TABLE 10.1 PROPERTIES OF THE z-TRANSFORM

Section	Property	Signal	z-Transform	ROC
		$x[n]$ $x_1[n]$ $x_2[n]$	$X(z)$ $X_1(z)$ $X_2(z)$	R R_1 R_2
10.5.1	Linearity	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	At least the intersection of R_1 and R_2
10.5.2	Time shifting	$x[n - n_0]$	$z^{-n_0}X(z)$	R, except for the possible addition or deletion of the origin
10.5.3	Scaling in the z-domain	$e^{j\omega_0 n}x[n]$	$X(e^{-j\omega_0}z)$	R
		$z_0^n x[n]$	$X\left(\frac{z}{z_0}\right)$	z_0R
		$a^n x[n]$	$X(a^{-1}z)$	Scaled version of R (i.e., $\|a\|R$ = the set of points $\{\|a\|z\}$ for z in R)
10.5.4	Time reversal	$x[-n]$	$X(z^{-1})$	Inverted R (i.e., R^{-1} = the set of points z^{-1}, where z is in R)
10.5.5	Time expansion	$x_{(k)}[n] = \begin{cases} x[r], & n = rk \\ 0, & n \neq rk \end{cases}$ for some integer r	$X(z^k)$	$R^{1/k}$ (i.e., the set of points $z^{1/k}$, where z is in R)
10.5.6	Conjugation	$x^*[n]$	$X^*(z^*)$	R
10.5.7	Convolution	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	At least the intersection of R_1 and R_2
10.5.7	First difference	$x[n] - x[n-1]$	$(1 - z^{-1})X(z)$	At least the intersection of R and $\|z\| > 0$
10.5.7	Accumulation	$\sum_{k=-\infty}^{n} x[k]$	$\frac{1}{1 - z^{-1}}X(z)$	At least the intersection of R and $\|z\| > 1$
10.5.8	Differentiation in the z-domain	$nx[n]$	$-z\frac{dX(z)}{dz}$	R

TABLE 10.2 SOME COMMON z-TRANSFORM PAIRS

Signal	Transform	ROC
1. $\delta[n]$	1	All z
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$\|z\| > 1$
3. $-u[-n-1]$	$\frac{1}{1 - z^{-1}}$	$\|z\| < 1$
4. $\delta[n - m]$	z^{-m}	All z, except 0 (if $m > 0$) or ∞ (if $m < 0$)
5. $\alpha^n u[n]$	$\frac{1}{1 - \alpha z^{-1}}$	$\|z\| > \|\alpha\|$
6. $-\alpha^n u[-n-1]$	$\frac{1}{1 - \alpha z^{-1}}$	$\|z\| < \|\alpha\|$
7. $n\alpha^n u[n]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\|z\| > \|\alpha\|$
8. $-n\alpha^n u[-n-1]$	$\frac{\alpha z^{-1}}{(1 - \alpha z^{-1})^2}$	$\|z\| < \|\alpha\|$
9. $[\cos\omega_0 n]u[n]$	$\frac{1 - [\cos\omega_0]z^{-1}}{1 - [2\cos\omega_0]z^{-1} + z^{-2}}$	$\|z\| > 1$
10. $[\sin\omega_0 n]u[n]$	$\frac{[\sin\omega_0]z^{-1}}{1 - [2\cos\omega_0]z^{-1} + z^{-2}}$	$\|z\| > 1$
11. $[r^n \cos\omega_0 n]u[n]$	$\frac{1 - [r\cos\omega_0]z^{-1}}{1 - [2r\cos\omega_0]z^{-1} + r^2 z^{-2}}$	$\|z\| > r$
12. $[r^n \sin\omega_0 n]u[n]$	$\frac{[r\sin\omega_0]z^{-1}}{1 - [2r\cos\omega_0]z^{-1} + r^2 z^{-2}}$	$\|z\| > r$

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