EPFL

Disal

Signals, Instruments, and Systems – W10

Dealing with Non-Deterministic Uncertainties in Localization

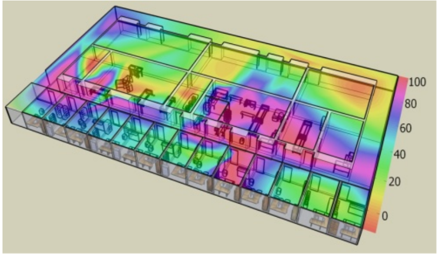
Outline

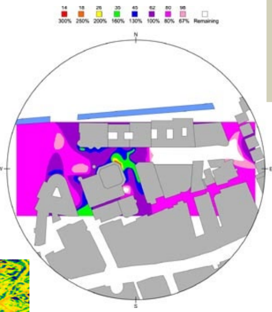
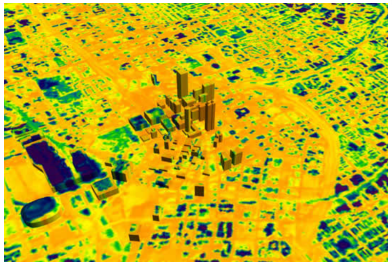
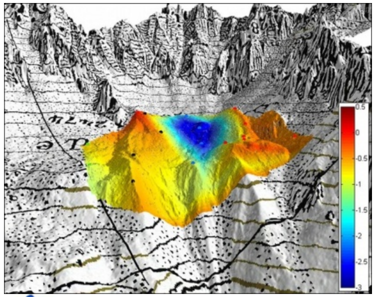
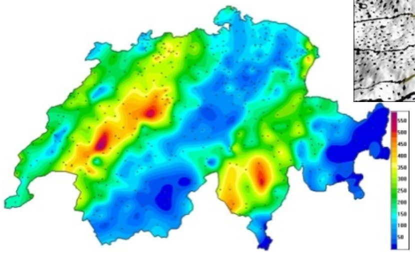
- Motivation for localization and estimation methods
- Fusing information from noisy processes and noisy sensing: the Kalman Filter algorithm in 1D problems
- Localization using proprioceptive sensors with non-deterministic uncertainties
- Feature-based localization
- Kalman filter algorithm for 2D localization
- Particle filters for localization

Motivation for Localization and Sensor Fusion

Motivation for Localization

- Environmental data most often useless without location information
- Localization is required at different scales (1000's km → cm)



Pictures: courtesy of Sensorscope, NASA, NIST

Motivation for Stochastic Models and Estimation Methods

- A mathematical system model is necessarily an approximation of the real system: system structure is often appropriate but parameters are affected by uncertainties
- Non-deterministic sources of noise: we can neither control stochastic disturbances, nor get rid of stochastic sensor noise by calibration, nor model stochastic sources deterministically
- Sensors are noisy and can only partially observe the system state in all its details

Motivation for Stochastic Models and Estimation Methods

- The applicability of estimation methods goes way beyond localization problem
- To this purpose most diverse problems in environmental and civil engineering leverage Kalman filters for handling noisy processes and sensing
- Here are some examples I found in recent literature:
  - Improving accuracy of subsurface contaminant transport model
  - Improving in-situ monitoring of groundwater contamination
  - Identification of civil structural parameters
  - Improving predictive modeling of landslide deformation
  - Improving estimation of environmental performance variables in an acid gas removal process

The Kalman Filter Algorithm in 1D

Two Key Sources of Information

Stochastic models, estimation, and control

VOLUME 1

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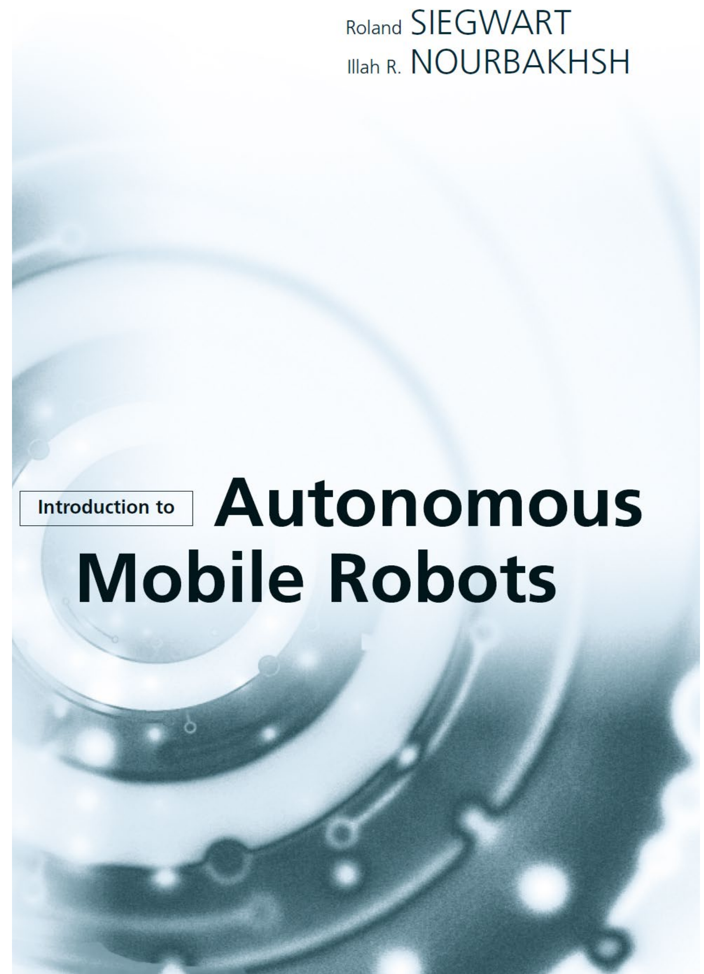
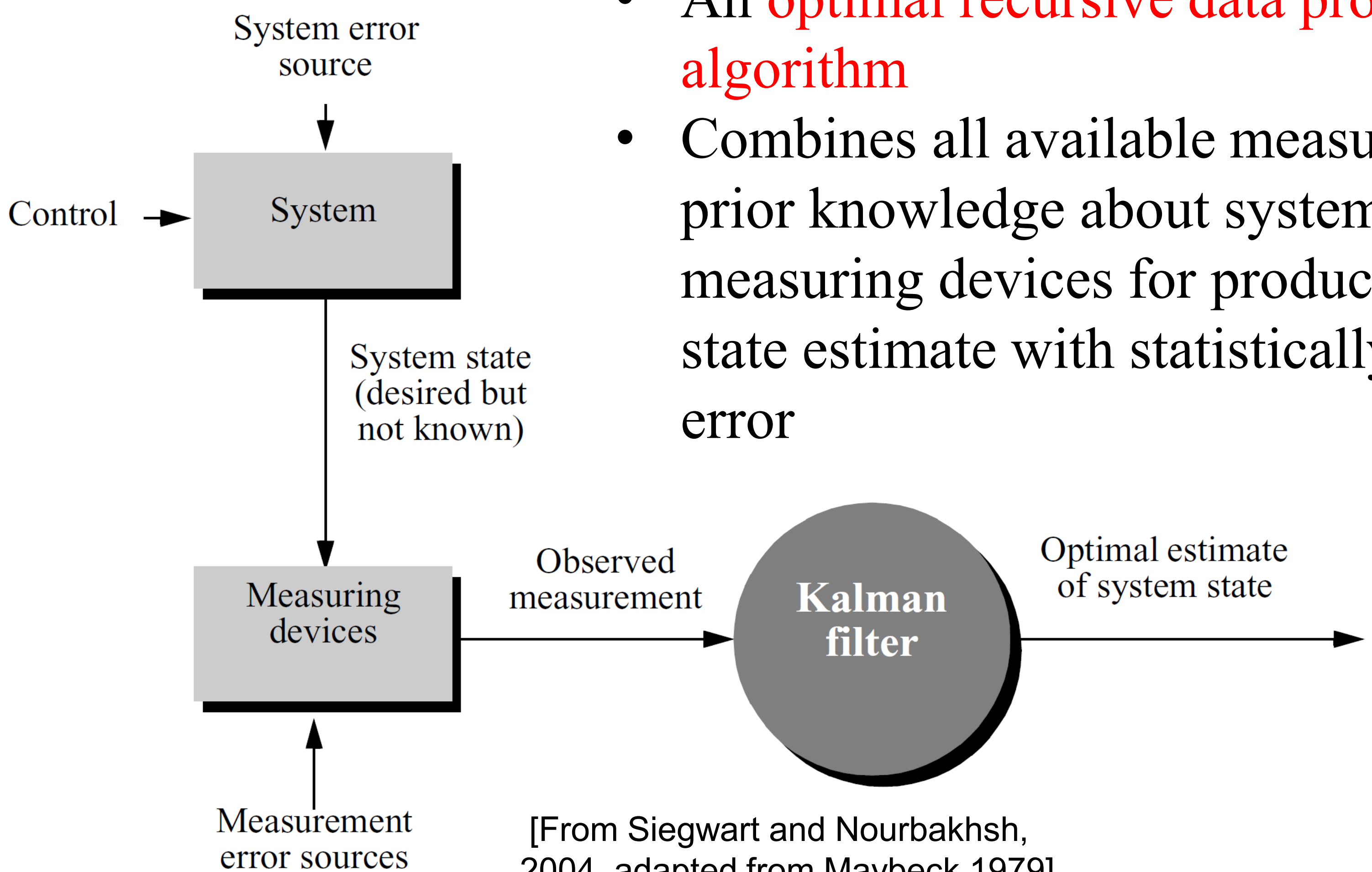


Kalman Filter - Overview

- An optimal recursive data processing algorithm
- Combines all available measurement data, prior knowledge about system and measuring devices for producing a system state estimate with statistically minimized error



[From Siegwart and Nourbakhsh, 2004, adapted from Maybeck 1979]

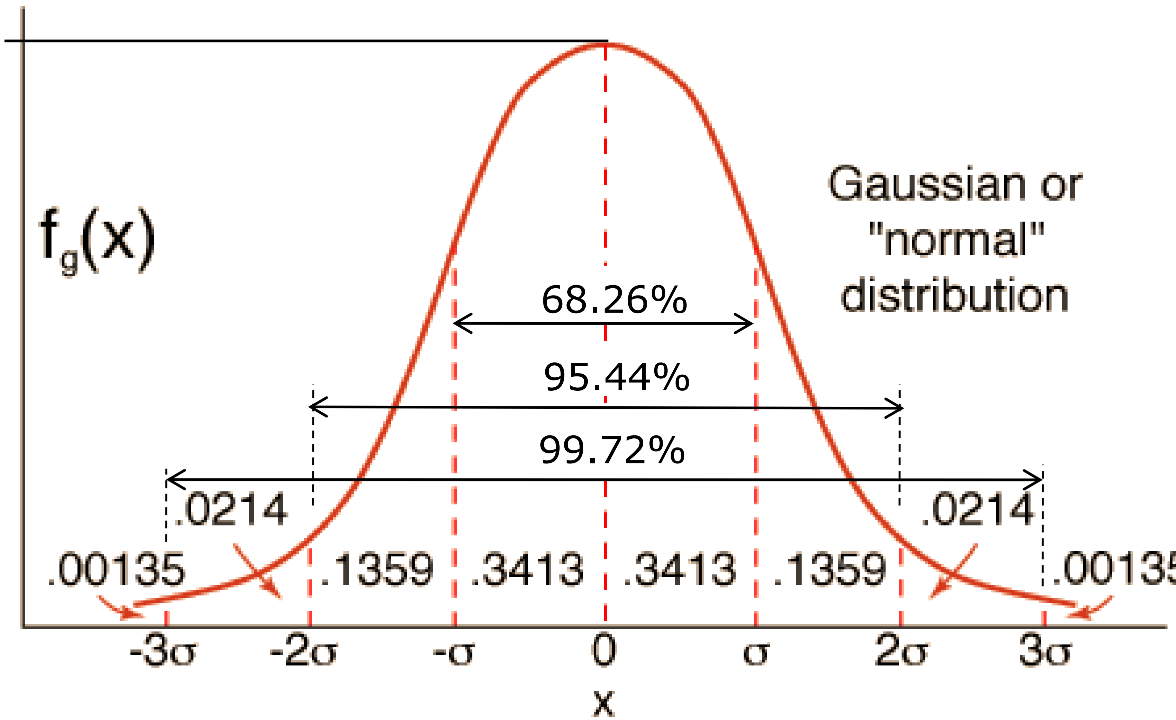
Kalman Filter - Assumptions

- Linear system model
- White Gaussian system and measurement noise

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Notes on noise:

- White: uncorrelated in time
- Gaussian: probability density of amplitude follows bell-shaped curve

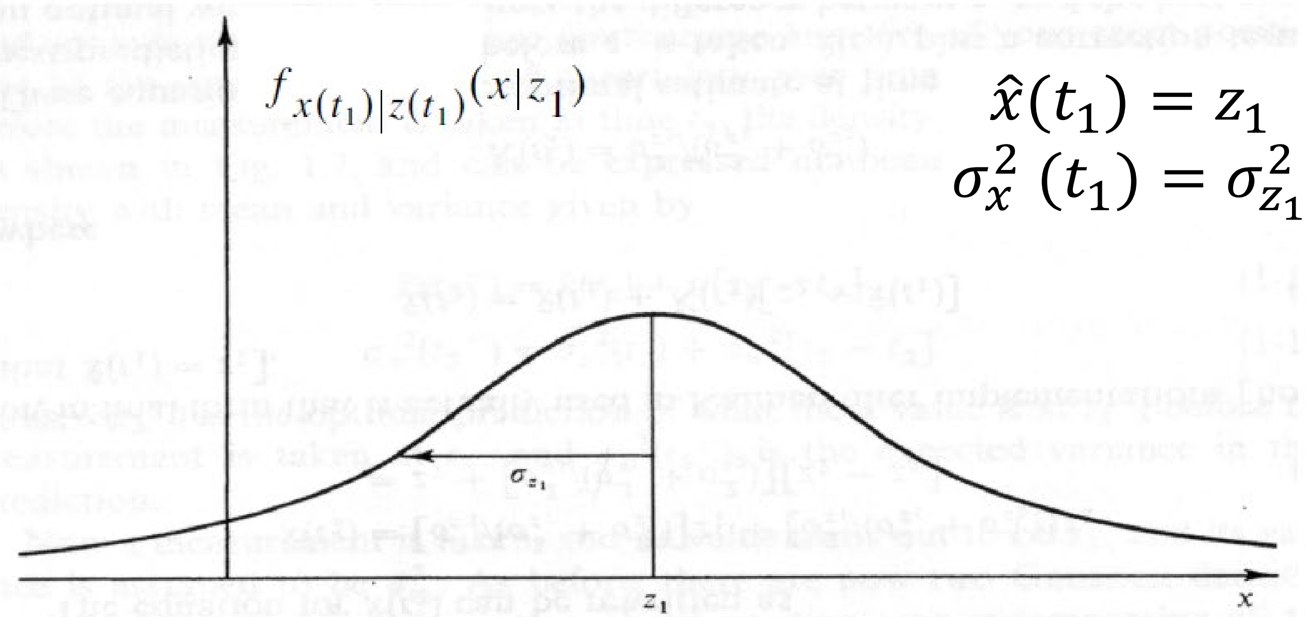


x = μ

A Simple 1D Positioning Example

t = time
x = 1D position
$\hat{x}$ = 1D position estimate
z = measurement or observation

t_i = time instant i
z_i = measurement taken at t_i

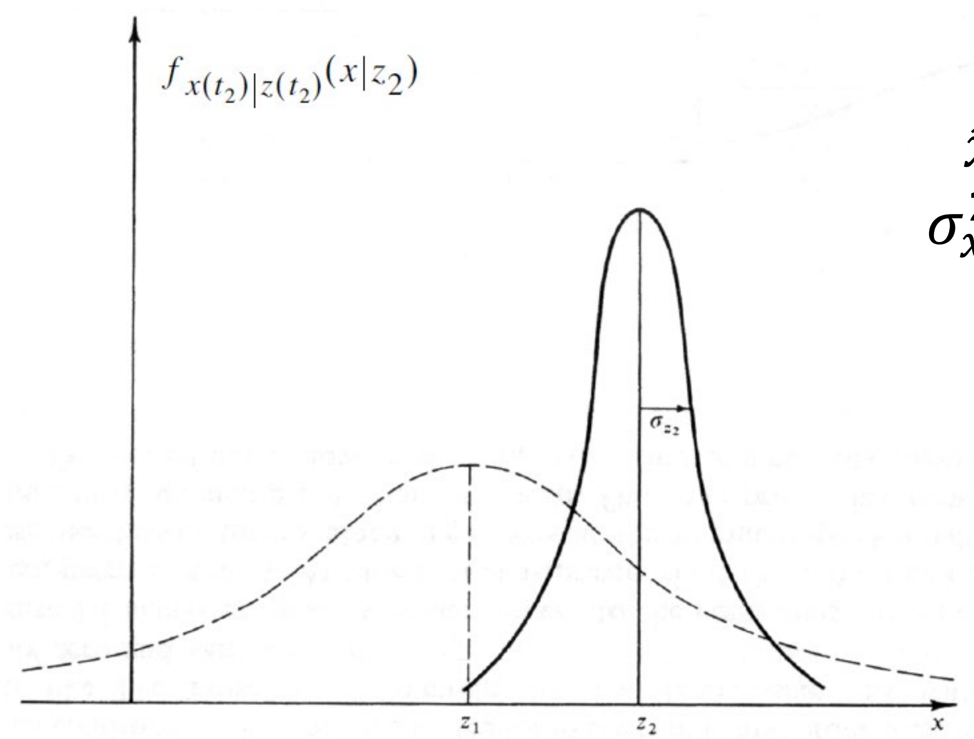


$$\hat{x}(t_1) = z_1$$
$$\sigma_x^2(t_1) = \sigma_{z_1}^2$$

[From Maybeck, 1979]

Estimation Based on Static Measurements

- Second measurement taken at $t_2 \approx t_1$
- The smaller σ, the higher the certitude about the measurement

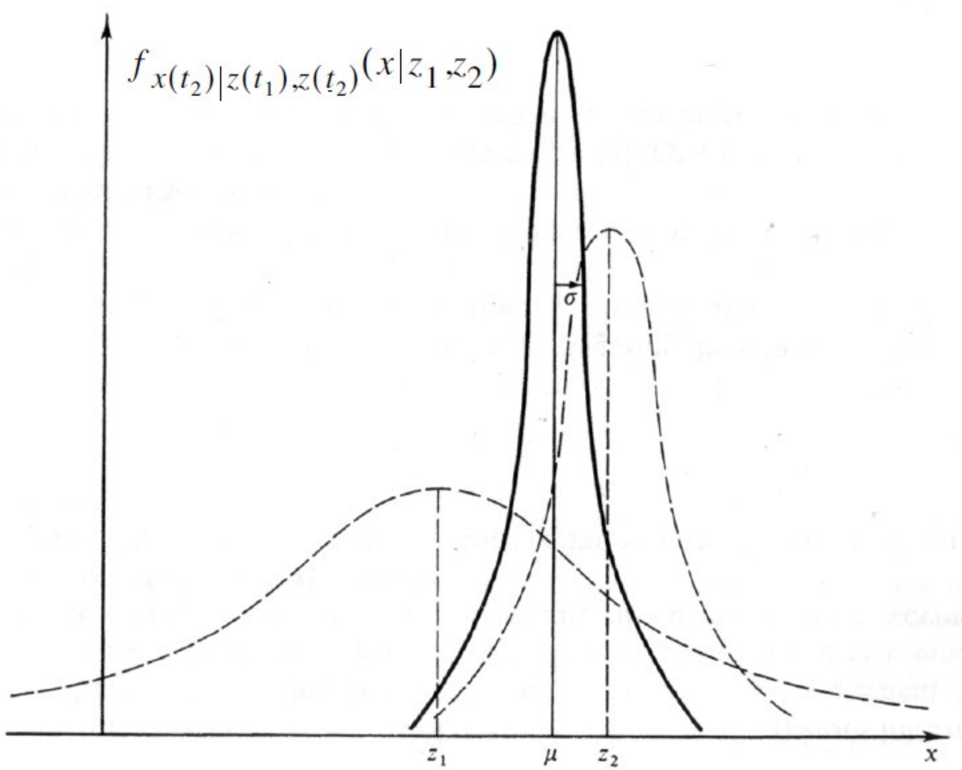


$$\hat{x}(t_2) = z_2$$
$$\sigma_x^2(t_2) = \sigma_{z_2}^2$$

[From Maybeck, 1979]

Improving the Estimate Through Fusion

- Intuition: the smaller σ, and thus σ^2, the higher should be the weight in the fused estimate
- Estimate can be obtained as a weighted average μ of the individual measurement contributions



[From Maybeck, 1979]

$$\mu = \frac{\frac{1}{\sigma_{z_1}^2} z_1 + \frac{1}{\sigma_{z_2}^2} z_2}{\frac{1}{\sigma_{z_1}^2} + \frac{1}{\sigma_{z_2}^2}}$$

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_{z_1}^2} + \frac{1}{\sigma_{z_2}^2}$$

$$\hat{x}(t_2) = \mu$$

$$\sigma_x(t_2) = \sigma$$

$$\sigma < \sigma_{z_1} \text{ and } \sigma < \sigma_{z_2}$$

Mean of the new Estimate

$$\hat{x}(t_2) = \frac{\frac{1}{\sigma_{z_1}^2} z_1 + \frac{1}{\sigma_{z_2}^2} z_2}{\frac{1}{\sigma_{z_1}^2} + \frac{1}{\sigma_{z_2}^2}} = \frac{\sigma_{z_2}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} z_1 + \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} z_2$$

$$= \frac{\sigma_{z_2}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} z_1 \boxed{+ \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} z_1 - \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} z_1} + \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} z_2$$

$$= \frac{\sigma_{z_2}^2 + \sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} z_1 + \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} (z_2 - z_1)$$

$$= z_1 + \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} (z_2 - z_1)$$

Kalman filter formulation

$$\hat{x}(t_2) = \hat{x}(t_1) + K(t_2)[z_2 - \hat{x}(t_1)]$$

with $K(t_2) = \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2}$

$\hat{x}(t_1) = z_1$

Variance of the new Estimate

$$\frac{1}{\sigma_x^2(t_2)} = \frac{1}{\sigma_{z_1}^2} + \frac{1}{\sigma_{z_2}^2}$$

$$\sigma_x^2(t_2) = \frac{\sigma_{z_1}^2 \sigma_{z_2}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2} = \frac{\sigma_{z_2}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2}\,\sigma_{z_1}^2 = \frac{\sigma_{z_1}^2 + \sigma_{z_2}^2 - \sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2}\,\sigma_{z_1}^2 = \left(1 - \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2}\right)\sigma_{z_1}^2$$

$$= \sigma_{z_1}^2 - \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2}\,\sigma_{z_1}^2$$

Kalman filter formulation

$$\sigma_x^2(t_2) = \sigma_x^2\,(t_1) - K(t_2)\sigma_x^2(t_1)$$

with $K(t_2) = \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2}$

$$\sigma_x^2(t_1) = \sigma_{z_1}^2$$

Kalman Filter for Sensor Fusion

$$\hat{x}(t_2) = \hat{x}(t_1) + K(t_2)[z_2 - \hat{x}(t_1)]$$

Mean

Prediction

Correction or Update

$$\sigma_x^2(t_2) = \sigma_x^2(t_1) - K(t_2)\,\sigma_x^2(t_1)$$

Variance

$$K(t_2) = \frac{\sigma_{z_1}^2}{\sigma_{z_1}^2 + \sigma_{z_2}^2}$$

Kalman Gain

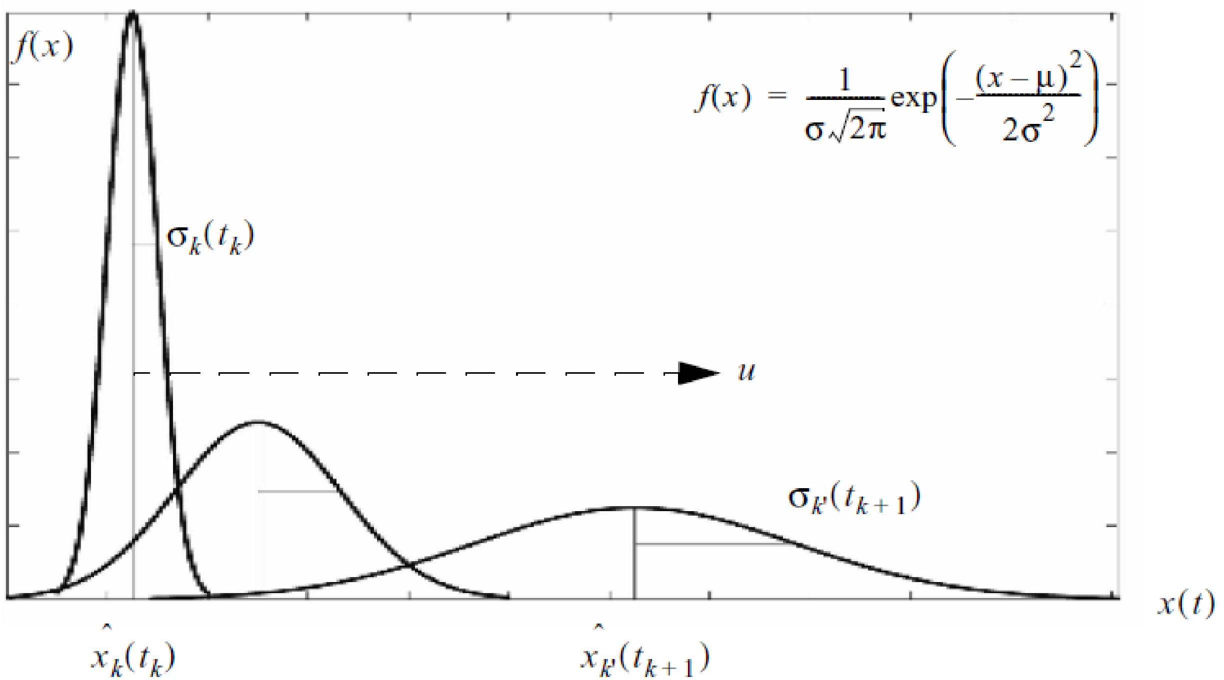
Function of the sensing precision

Estimation Considering Motion Dynamics

Consider a simple but noisy motion model: $\dot{x} = u + w$

u = constant speed (controllable input)
w = Gaussian motion noise

$w = \sigma_w^2$



[From Siegwart and Nourbakhsh, 2004, adapted from Maybeck 1979]

σ_k^2: variance at timestep k (known)

$\sigma_{k'}^2$: variance at timestep k+1

Estimation Based on Motion Model

$$\hat{x}_{k'} = \hat{x}_k + u[t_{k+1} - t_k]$$

- New mean position at timestep t_{k+1}
- Can be estimated with deterministic displacement from motion model

$$\sigma^2_{k'} = \sigma^2_k + \sigma^2_w[t_{k+1} - t_k]$$

- New variance at timestep t_{k+1}
- Variance of noisy motion (constant over time) gets added (cumulated) to previous one

Fusing Motion Model Prediction with New Measurement - Mean

$$\hat{x}_{k+1} = \hat{x}_{k'} + K_{k+1}(z_{k+1} - \hat{x}_{k'}) \qquad \text{with } K_{k+1} = \frac{\sigma_{k'}^2}{\sigma_{k'}^2 + \sigma_z^2}$$

Prediction based on motion model

Correction or update based on observation

z_{k+1} : measurement at timestep k+1

K_{k+1} : Kalman gain at timestep k+1

$\hat{x}_{k+1}$: new estimate at timestep k+ 1 incorporating observation and prediction of motion model

$\hat{x}_{k'}$: estimate just before timestep k+1 based on prediction motion model

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Fusing Motion Model Prediction with New Measurement - Variance

$$\sigma_{k+1}^2 = \sigma_{k'}^2 - K_{k+1}\,\sigma_{k'}^2 \qquad \text{with } K_{k+1} = \frac{\sigma_{k'}^2}{\sigma_{k'}^2 + \sigma_z^2}$$

Prediction based on motion model

Correction or update based on observation

K_{k+1} : Kalman gain at timestep k+1

σ_{k+1}^2 : variance at timestep k+ 1 incorporating correction from observation and prediction of motion model

$\sigma_{k'}^2$: variance just before timestep k+1 based on prediction motion model

σ_z^2 : variance of the sensor meausurement (constant over time)

Kalman Filter for Sensor and Motion Model Fusion

$$\hat{x}_{k+1} = \hat{x}_{k'} + K_{k+1}(z_{k+1} - \hat{x}_{k'})$$

Mean

Prediction   Correction or Update

$$\sigma^2_{k+1} = \sigma^2_{k'} - K_{k+1}\,\sigma^2_{k'}$$

Variance

$$K_{k+1} = \frac{\sigma^2_{k'}}{\sigma^2_{k'} + \sigma^2_z}$$

Kalman Gain

Function of the motion model and sensing precision

Kalman Filter - Some Extreme Cases

$$\hat{x}_{k+1} = \hat{x}_{k'} + K_{k+1}(z_{k+1} - \hat{x}_{k'})$$

$$\sigma_{k+1}^2 = \sigma_{k'}^2 - K_{k+1}\,\sigma_{k'}^2$$

$$K_{k+1} = \frac{\sigma_{k'}^2}{\sigma_{k'}^2 + \sigma_z^2}$$

$\sigma_z^2 \rightarrow \infty$: new measurement extremely noisy, does not add information, then $K_{k+1} \rightarrow 0$ new estimate based exclusively on motion model (both mean and variance)

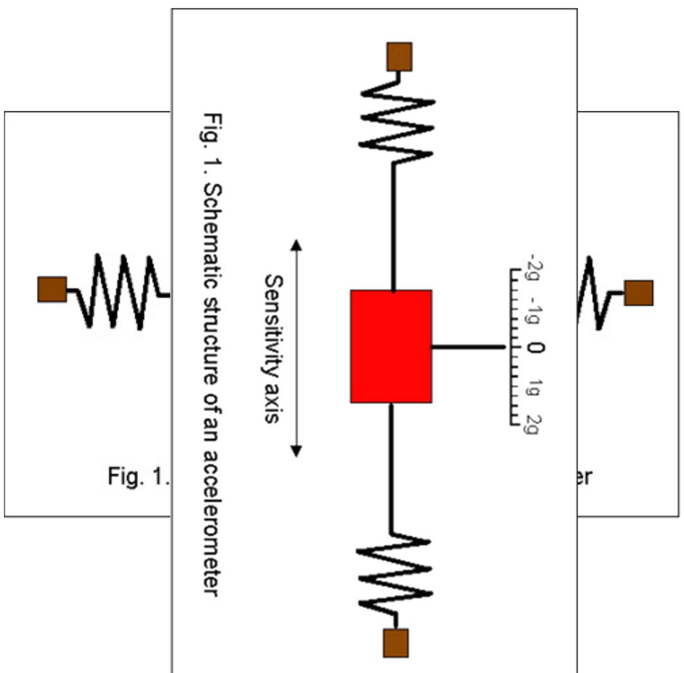
$\sigma_w^2 \rightarrow \infty$, then $\sigma_{k'}^2 \rightarrow \infty$ (see. s. 23): motion model does not add information, then $K_{k+1} \rightarrow 1$ new estimate based exclusively on new observation

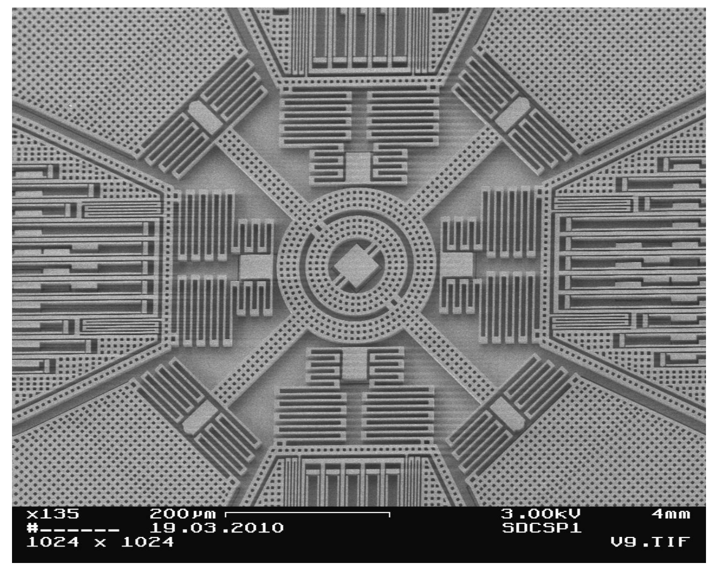
$\sigma_{k'}^2 \rightarrow 0$, motion model is deterministic and perfectly reproducing the reality, then $K_{k+1} \rightarrow 0$, new measurement can be disregarded since model is giving a perfect estimate

Mitigating Localization Uncertainities in Odometry Through Exteroceptive Sensors – The 1D Case

Non-Deterministic Error Sources

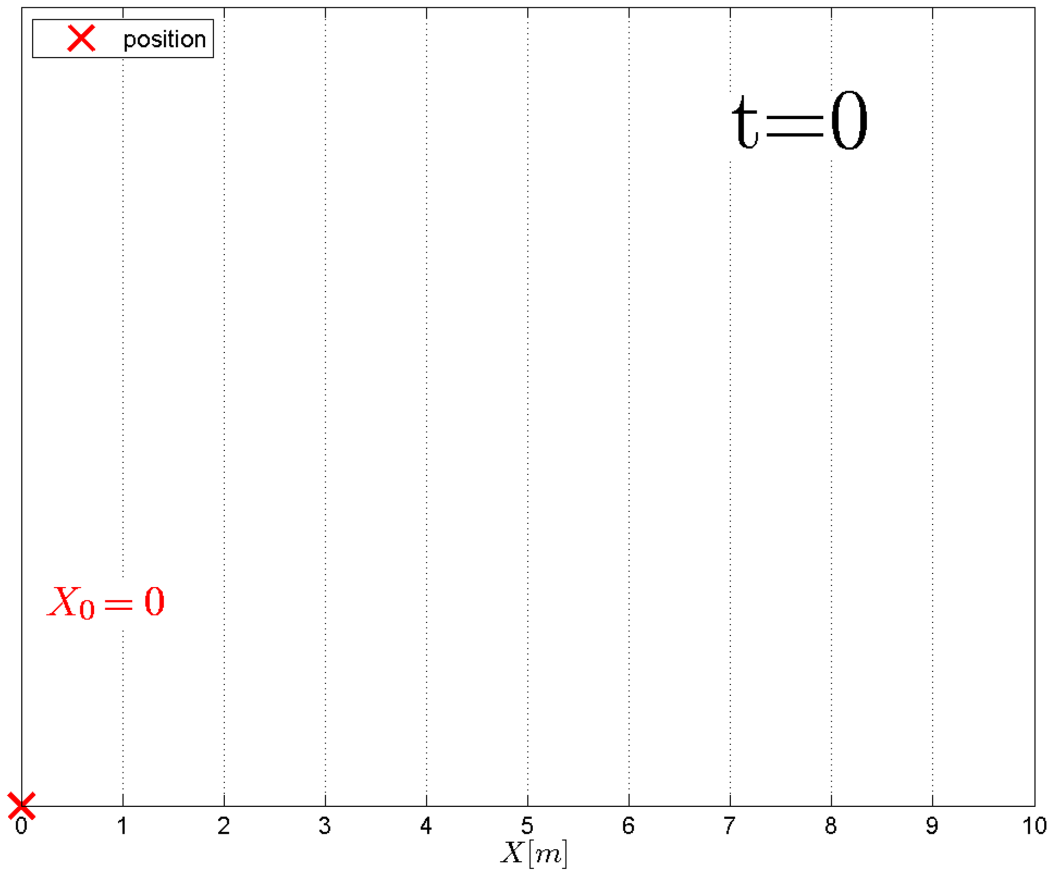
- From Week 9: no deterministic prediction possible
  → we have to describe them probabilistically
- Example: accelerometer-based odometry

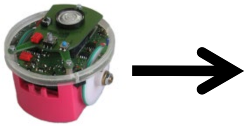




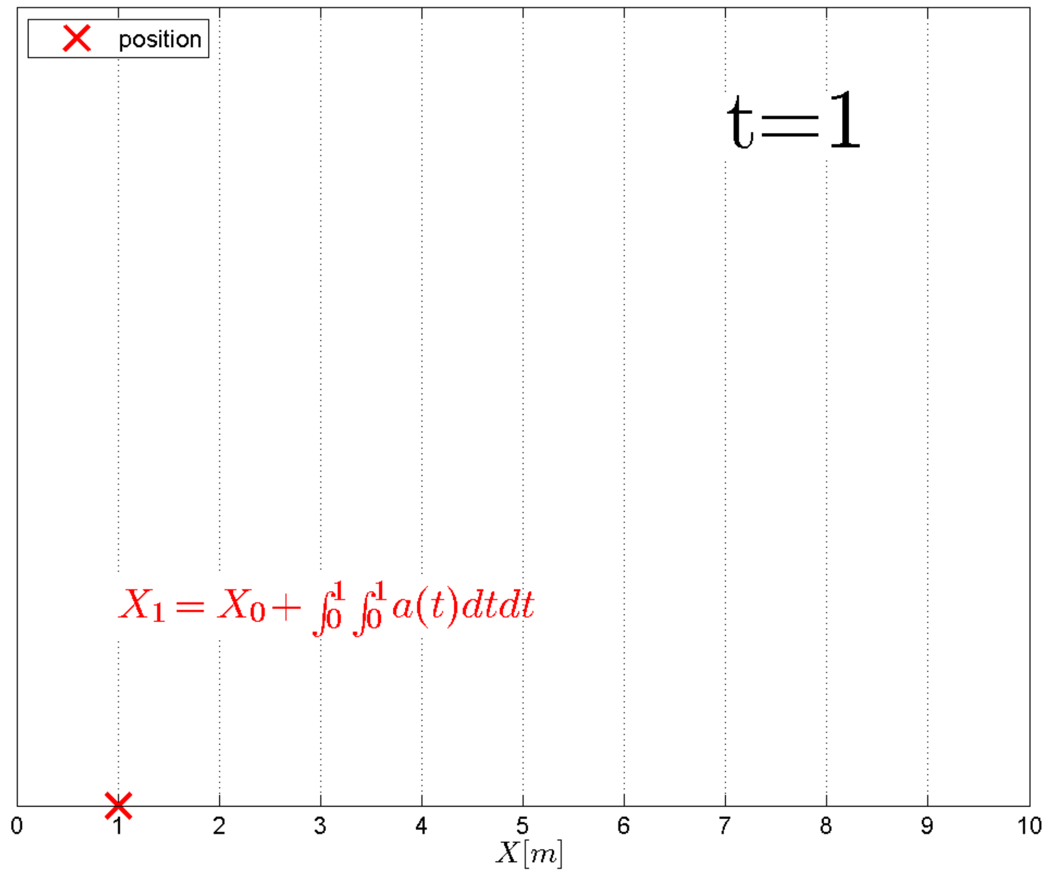
MEMS-Based accelerometer
(e.g., on e-puck)

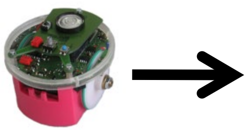
Odometry in 1D





Odometry in 1D





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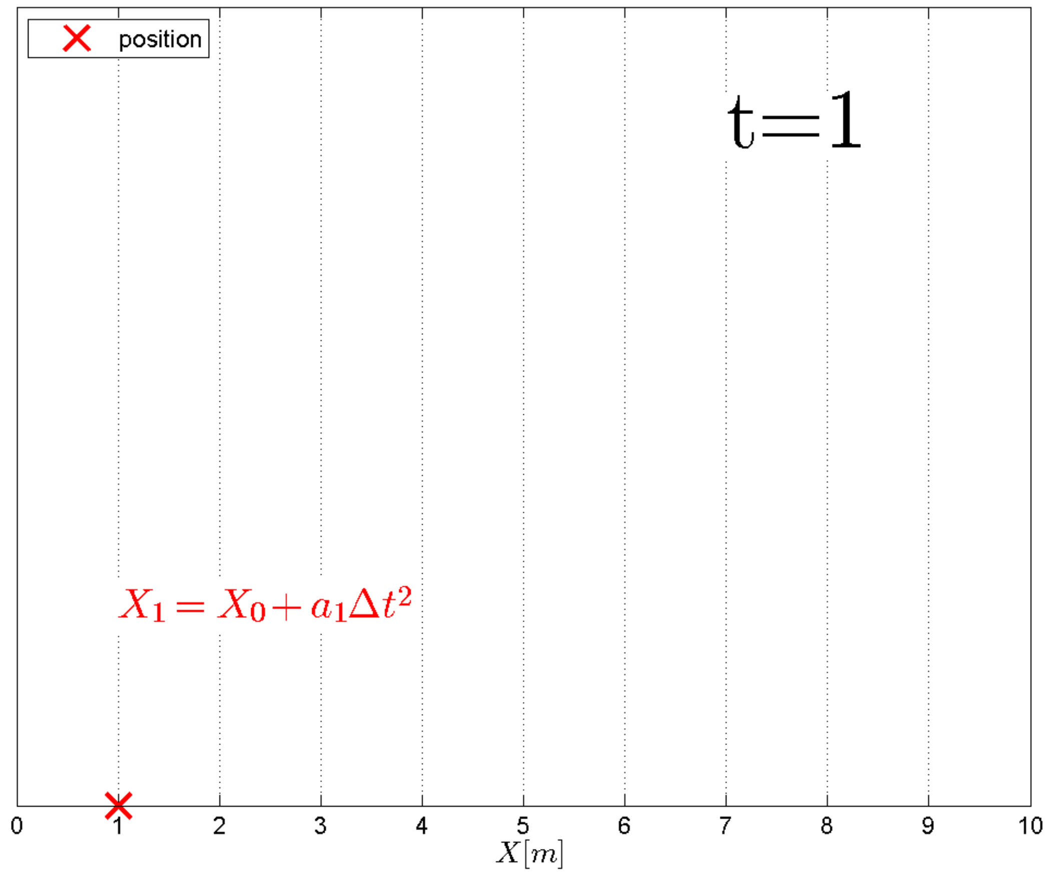

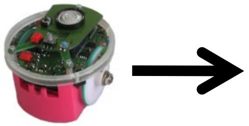

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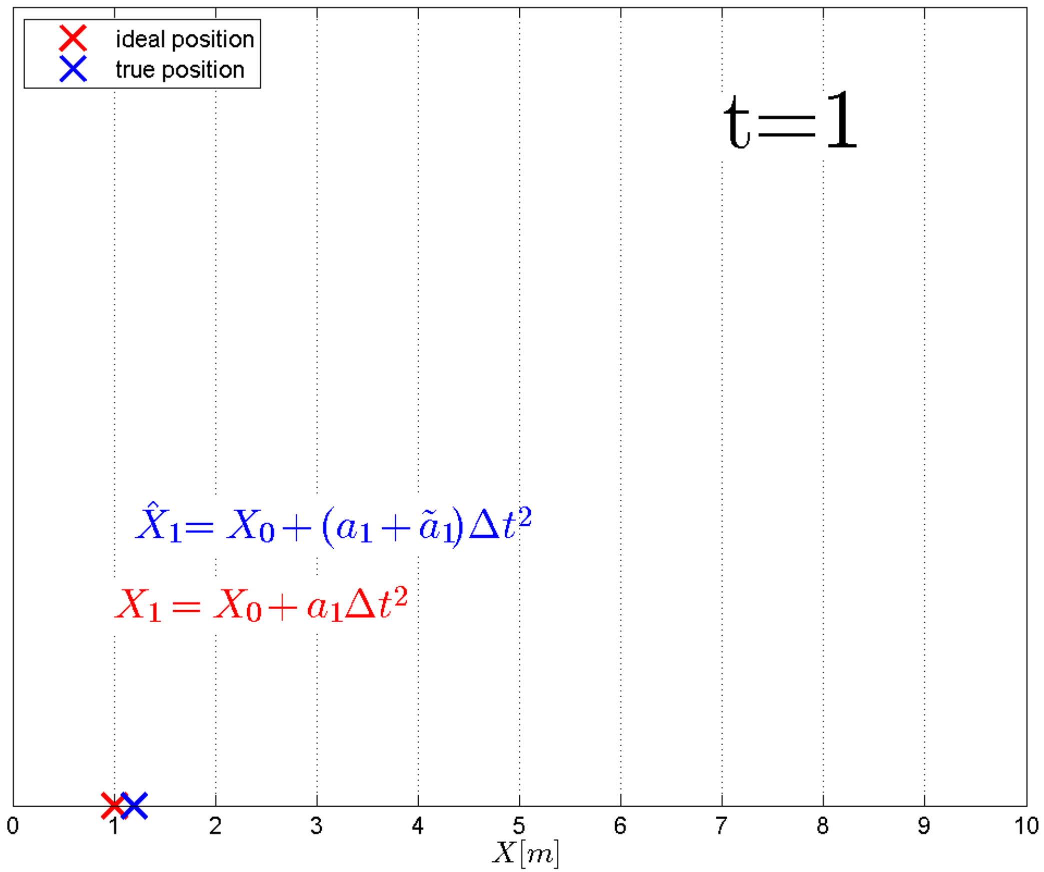

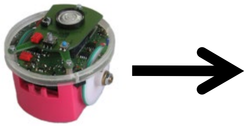

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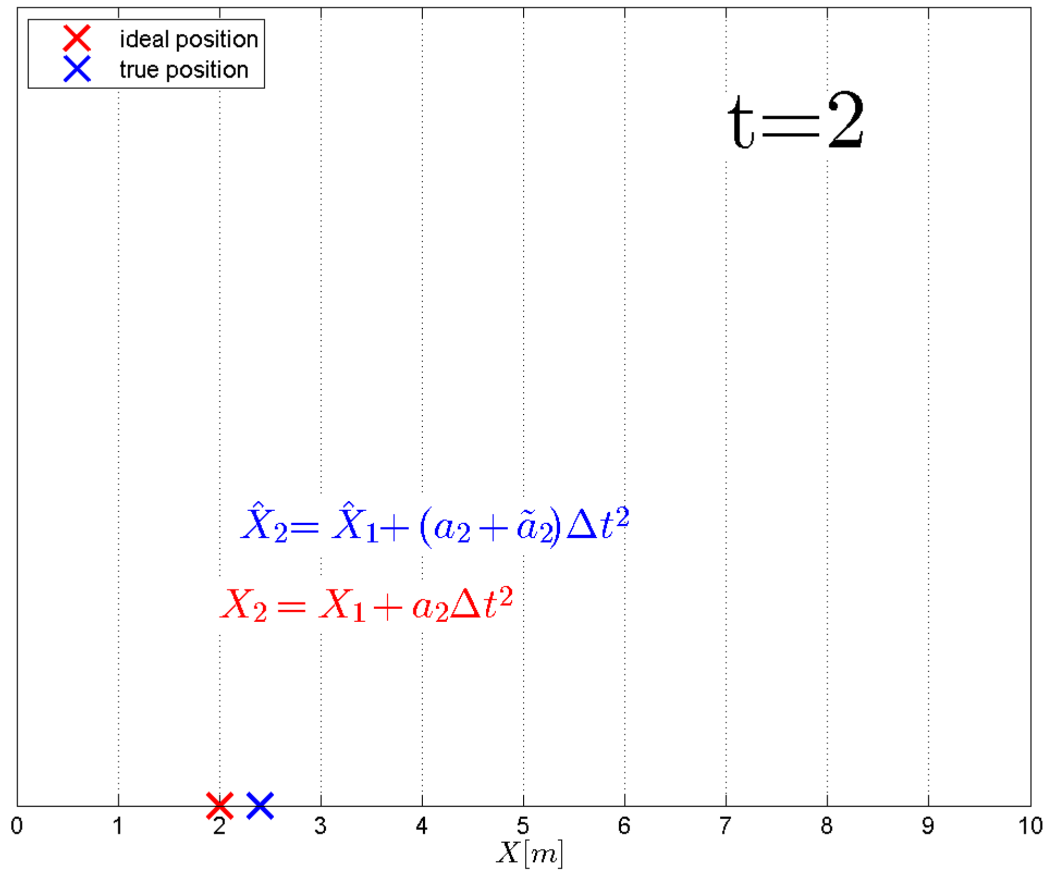

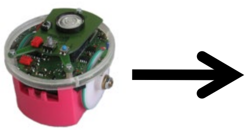

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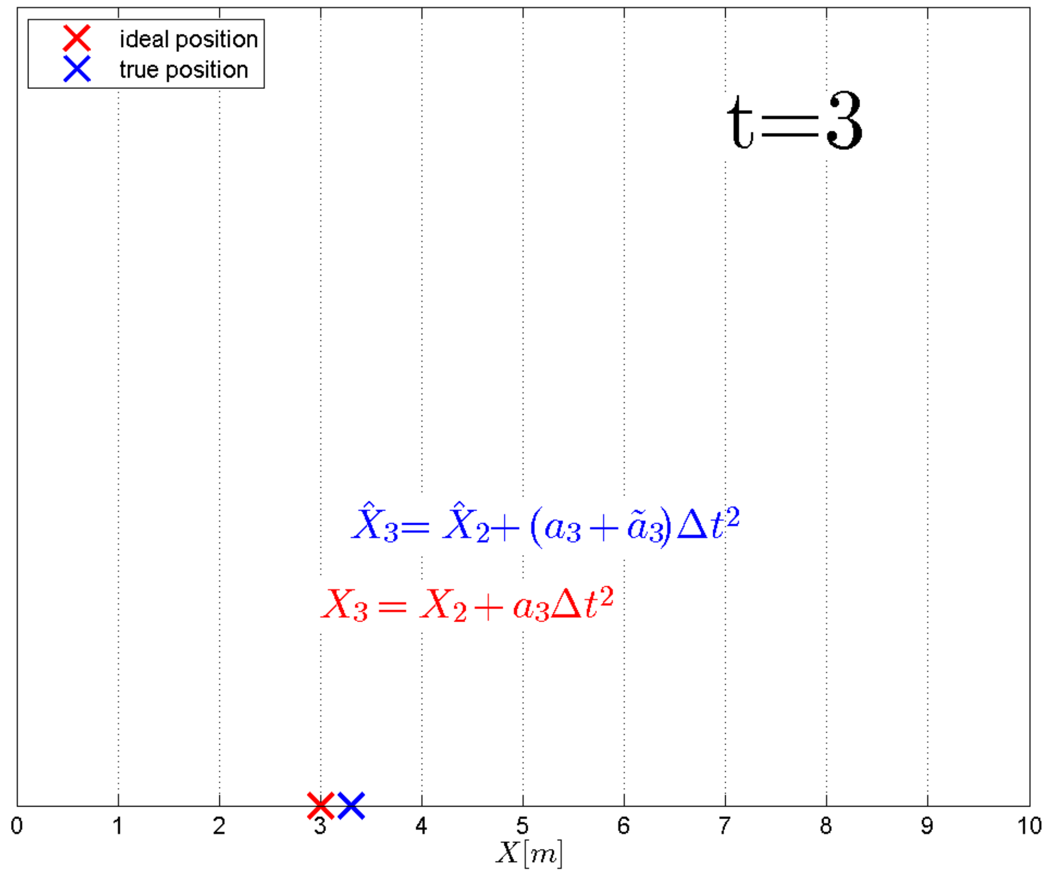

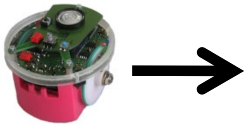

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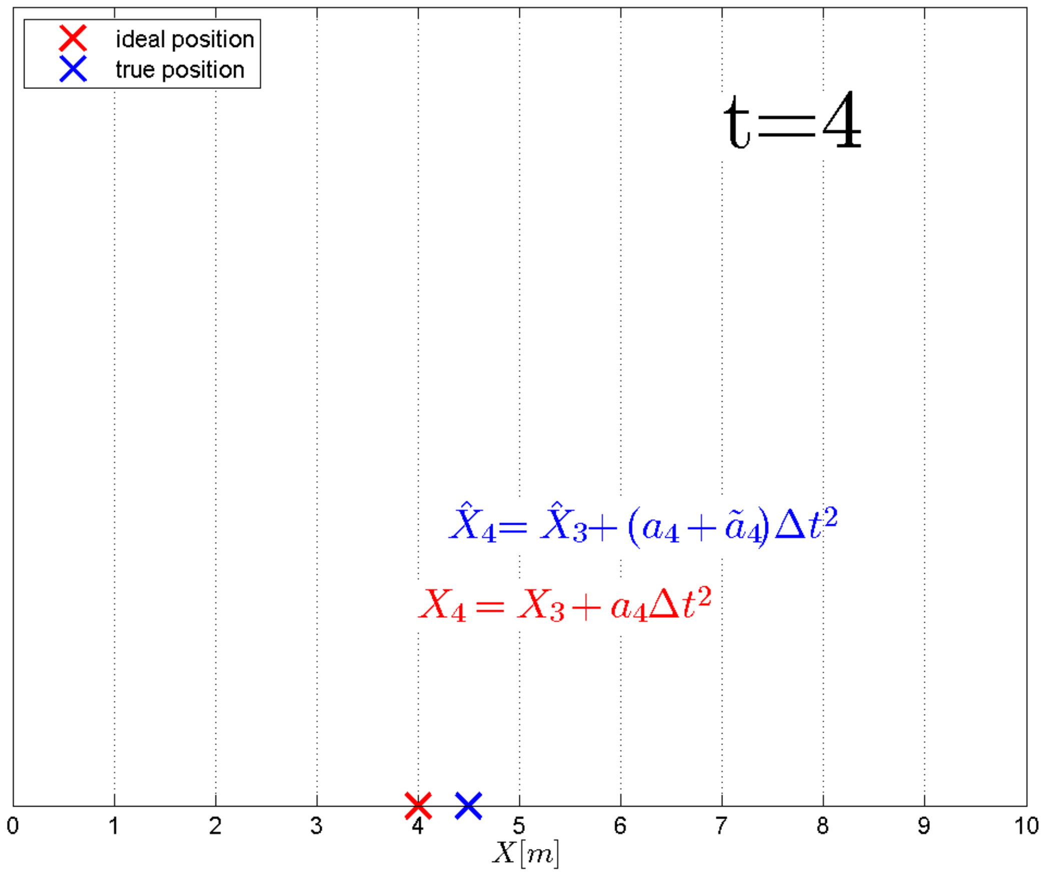

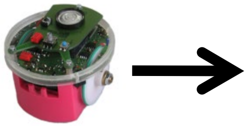

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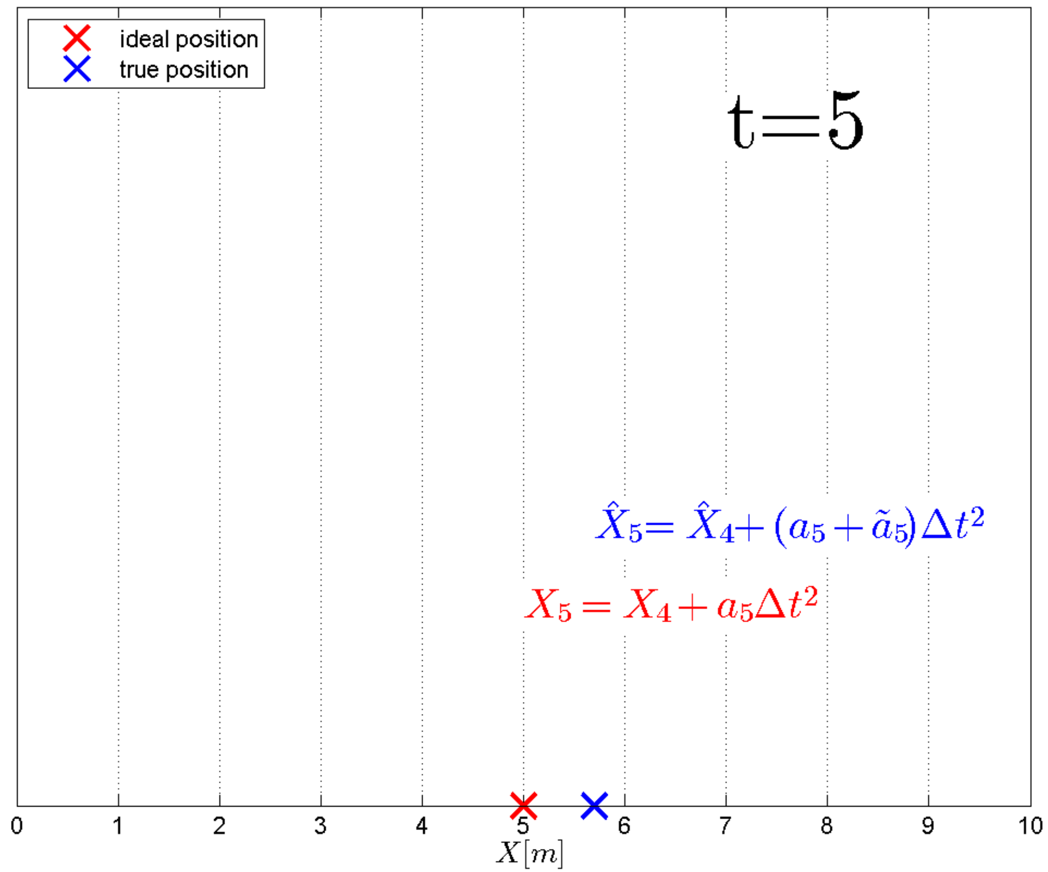
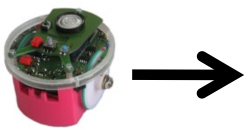
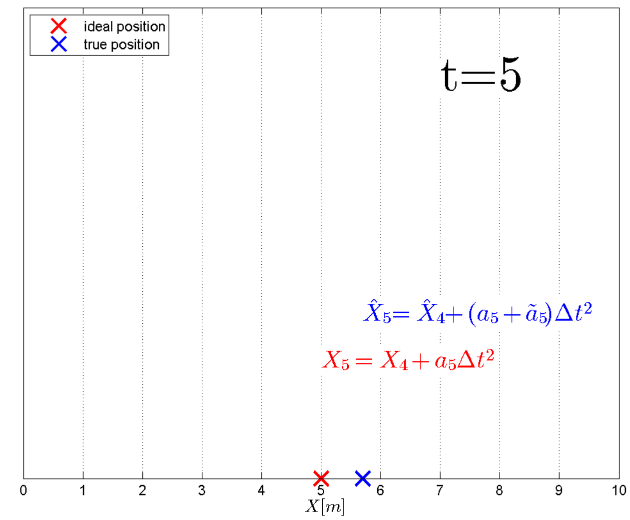
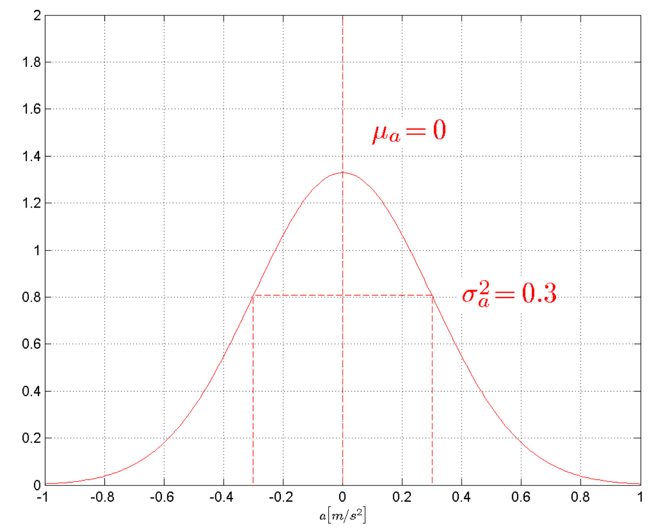




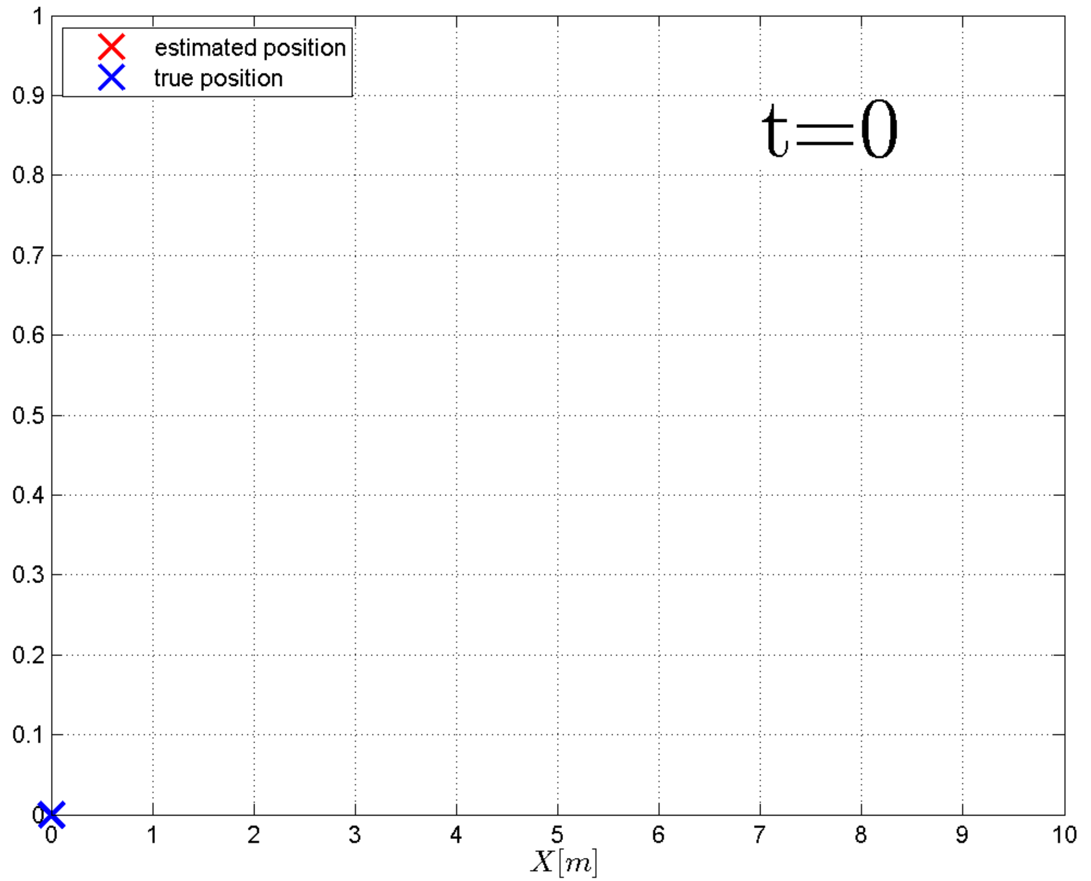
1D Odometry: Error Modeling

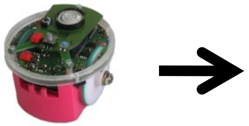
- Error happens!
- Odometry error is cumulative.
  → grows without bound
- We need to be aware of it.
  → We need to model odometry error.
  → We need to model sensor error.
- Multiple independent source of errors with arbitrary distribution combined → Central Limit Theorem → Gaussian assumption reasonable
- Acceleration is random variable A drawn from "mean-free" Gaussian ("Normal") distribution.
  → Position X is random variable with Gaussian distribution.





1D Odometry with Gaussian Uncertainty





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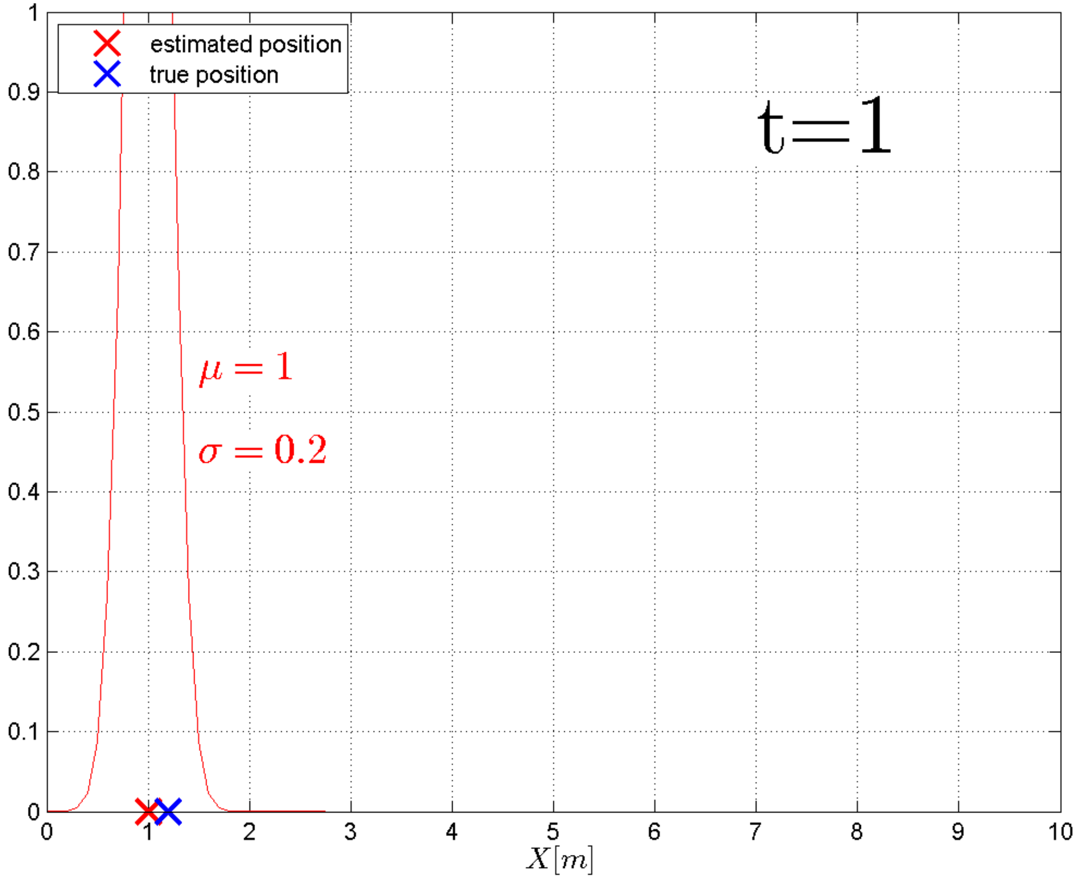
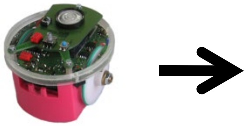
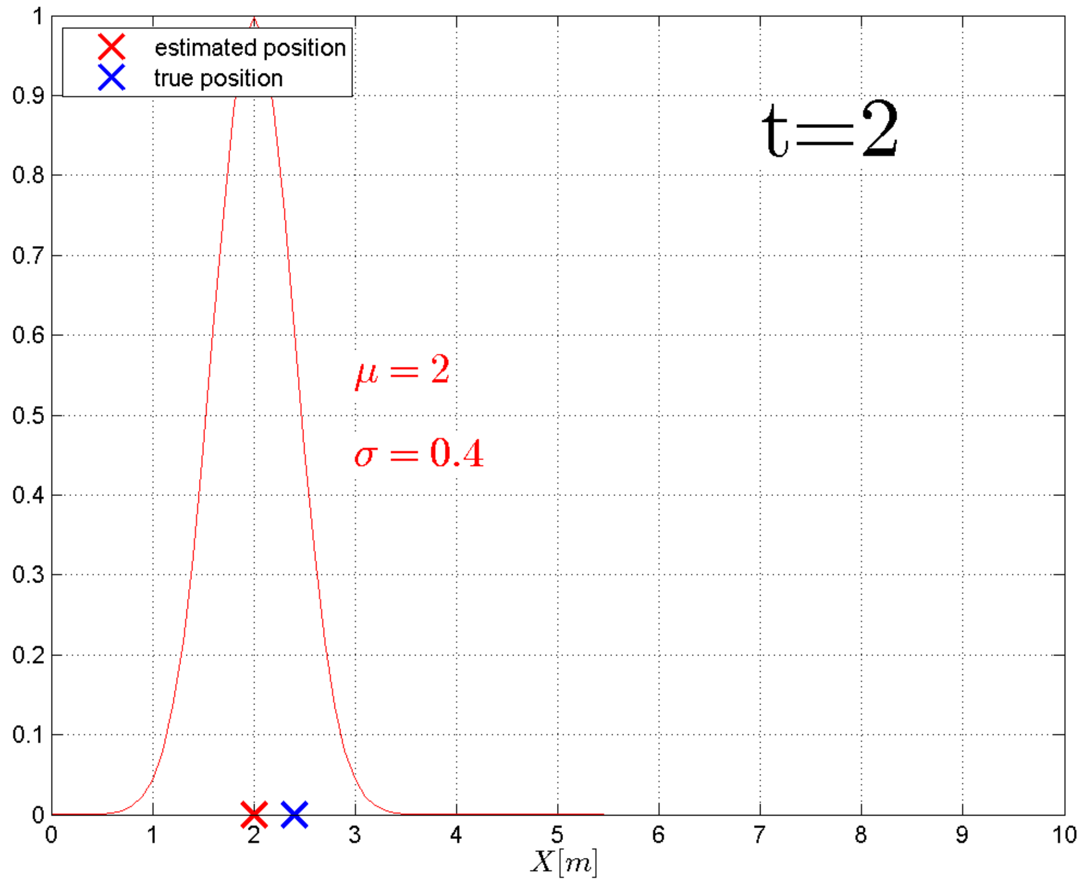
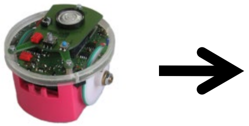


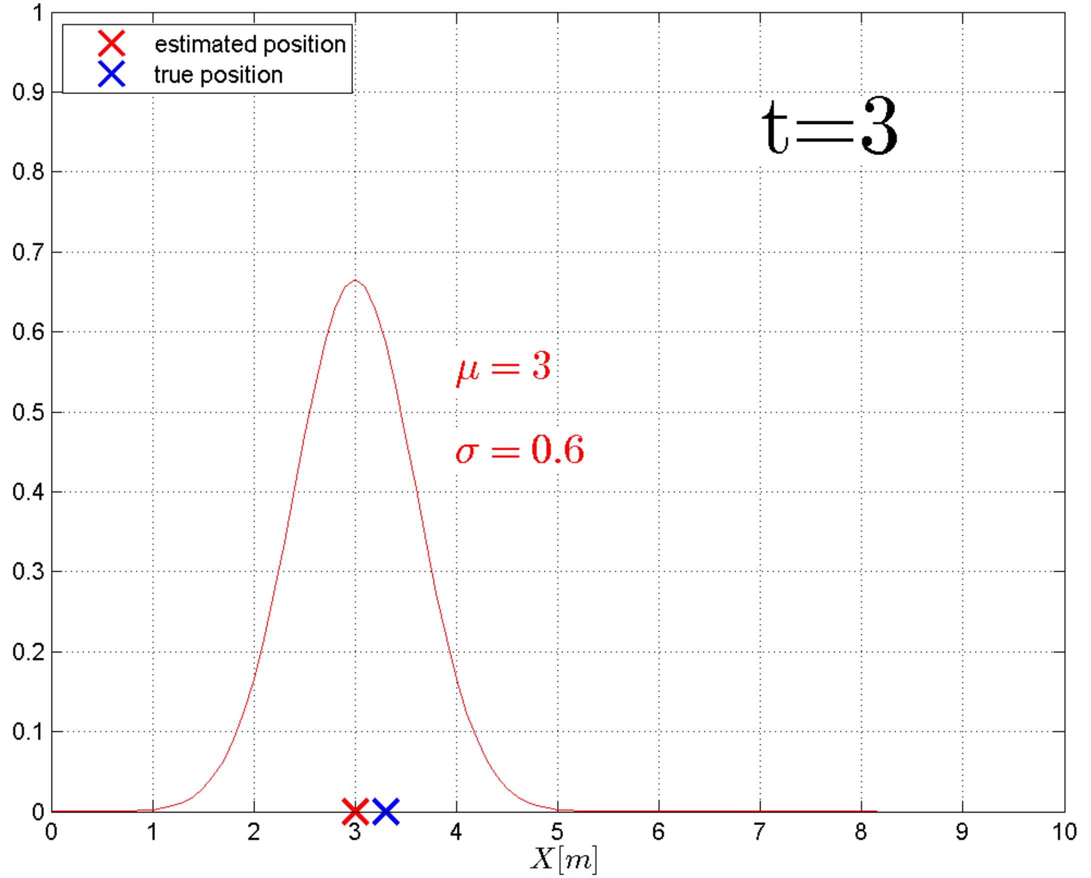


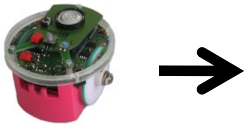
1D Odometry with Gaussian Uncertainty



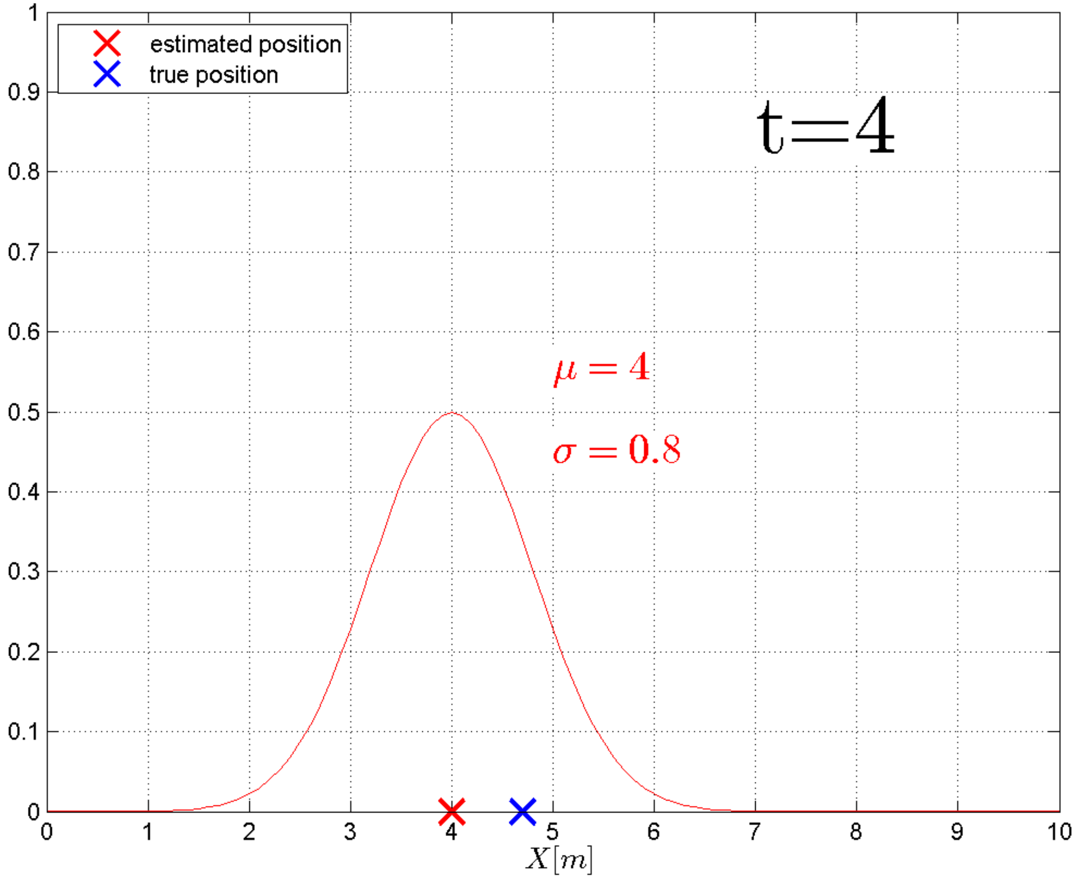


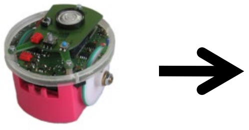
1D Odometry with Gaussian Uncertainty



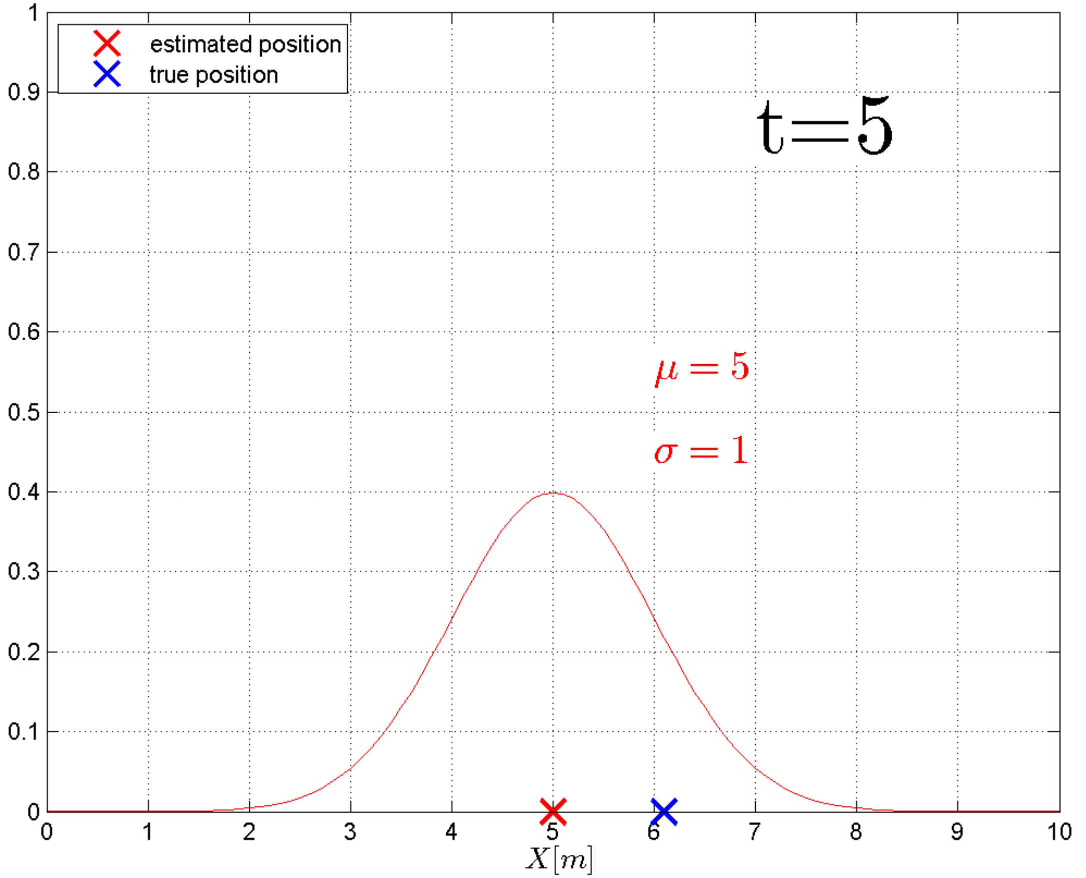


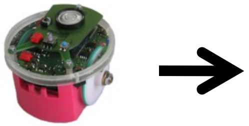
1D Odometry with Gaussian Uncertainty





1D Odometry with Gaussian Uncertainty



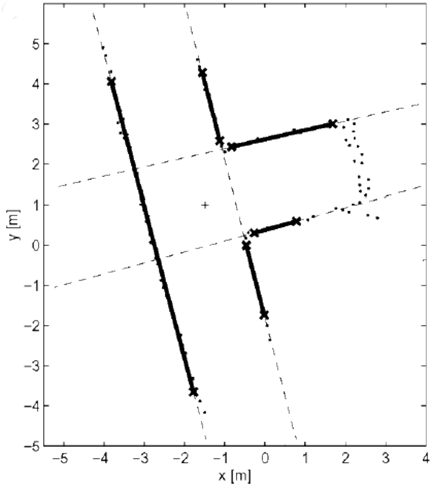


Features

- Odometry based position error grows without bound.
- Use relative measurement to features (“landmarks”) to reduce position uncertainty
- ***Feature***:
  - Uniquely identifiable
  - Position is known
  - We can obtain relative measurements between robot and feature (usually angle or range).
- Examples:
  - Doors, walls, corners, hand rails
  - Buildings, trees, lanes
  - GPS satellites

Automatic Feature Extraction

- High level features:
  - Doors, persons
- Simple visual features:
  - Edges (Canny Edge Detector 1983)
  - Corner (Harris Corner Detector 1988)
  - ***S***cale ***I***nvariant ***F***eature ***T***ransformation (2004)
- Simple geometric features
  - Lines
  - Corners
- "Binary" feature



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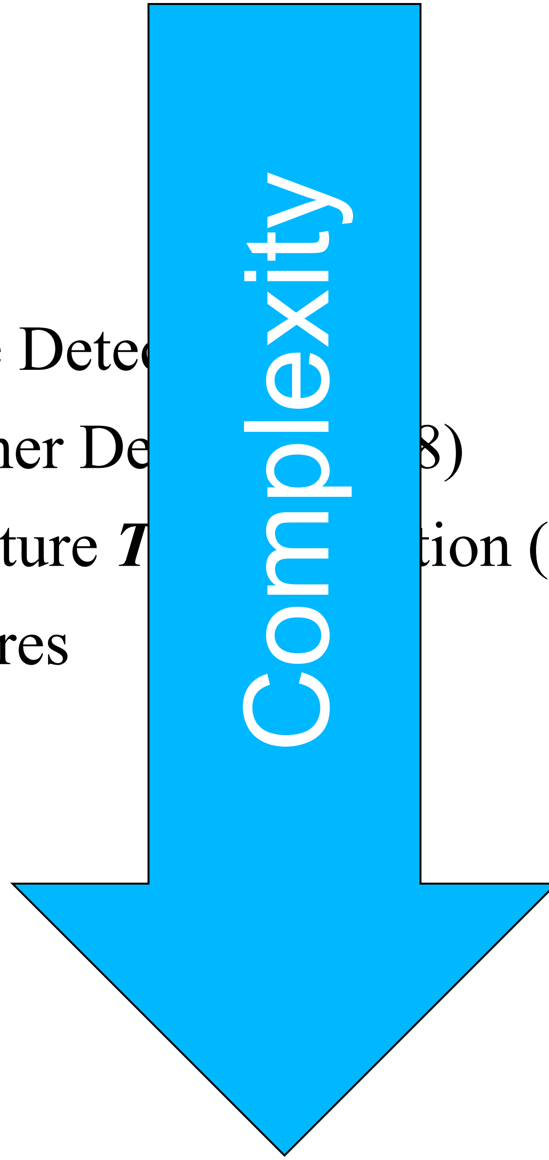
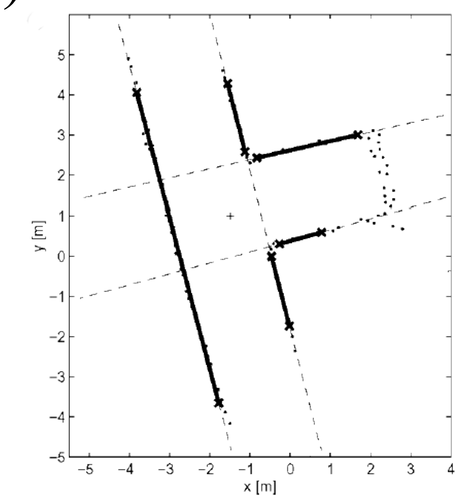
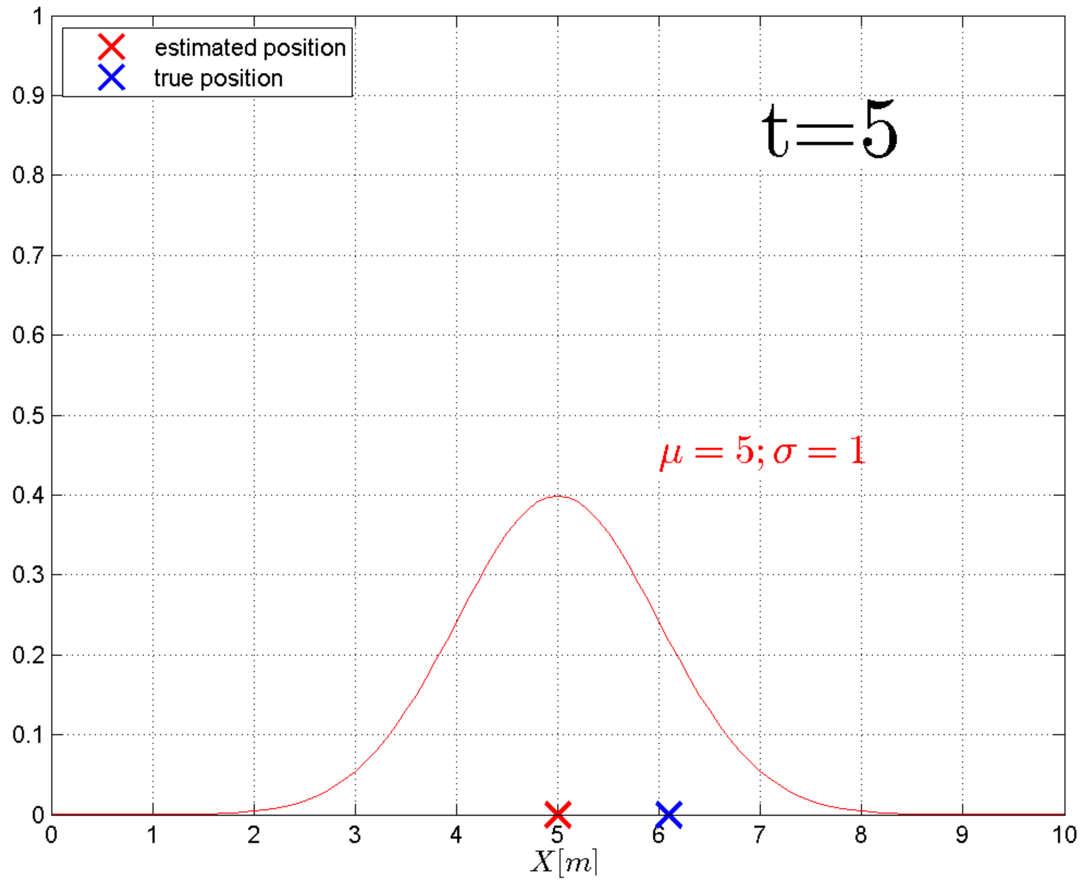




Feature-Based Localization



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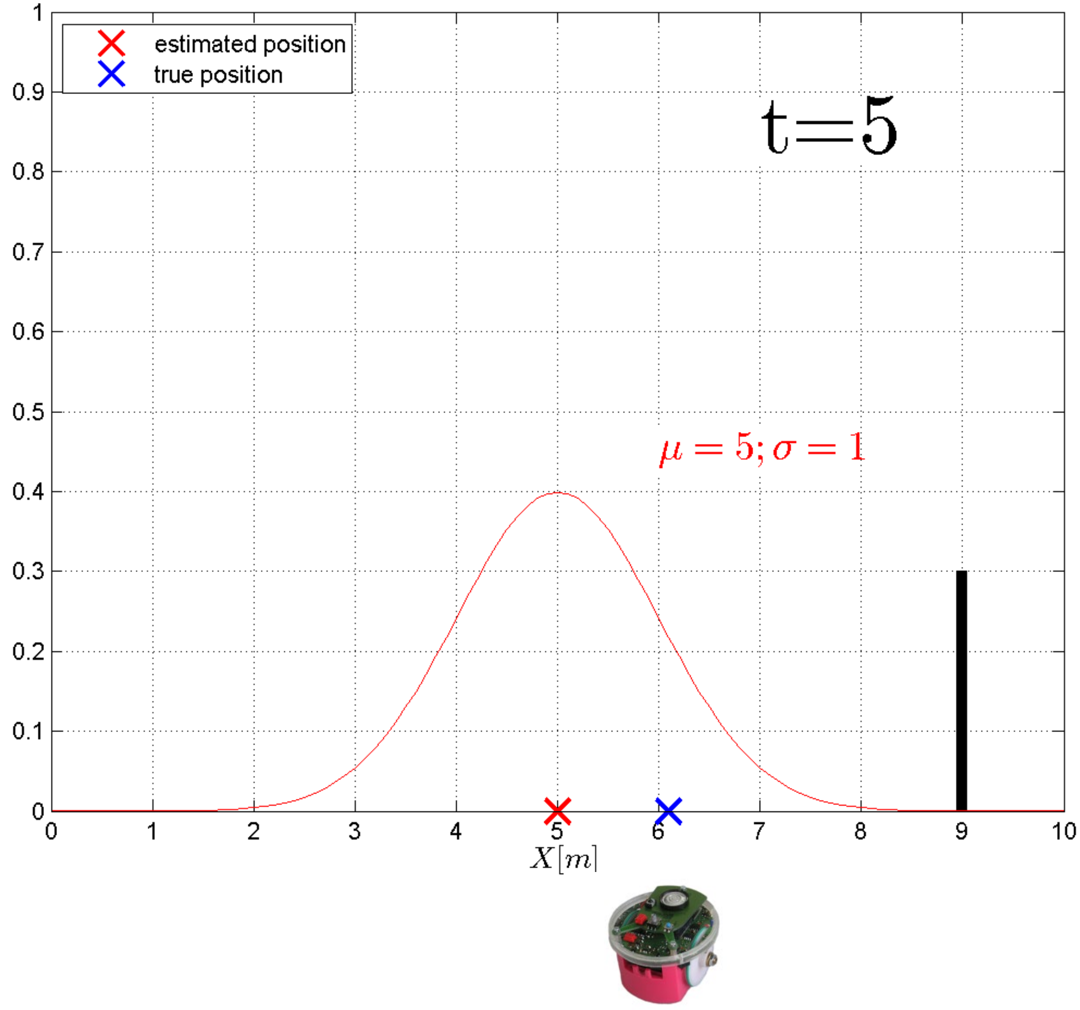

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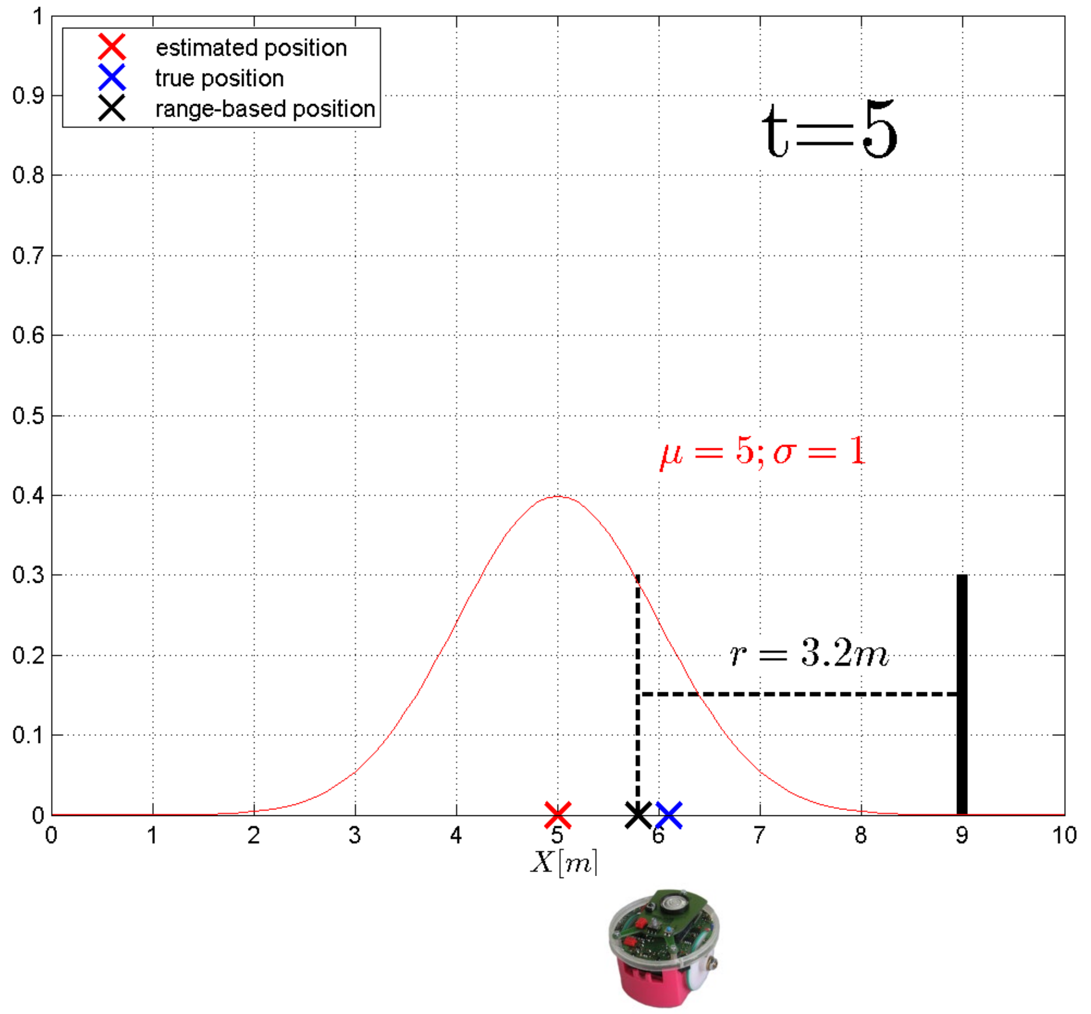

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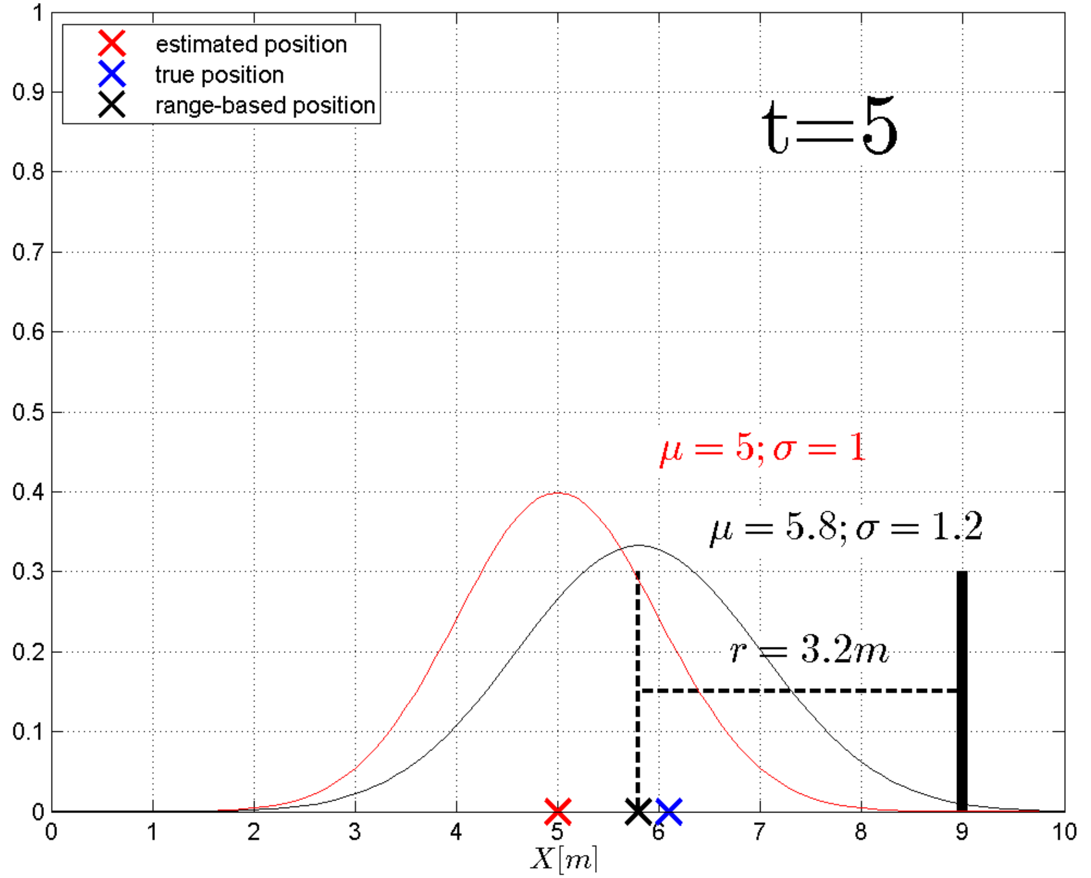
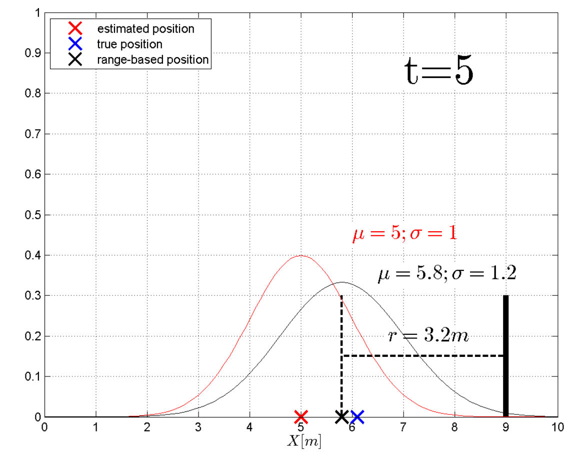


Sensor Fusion

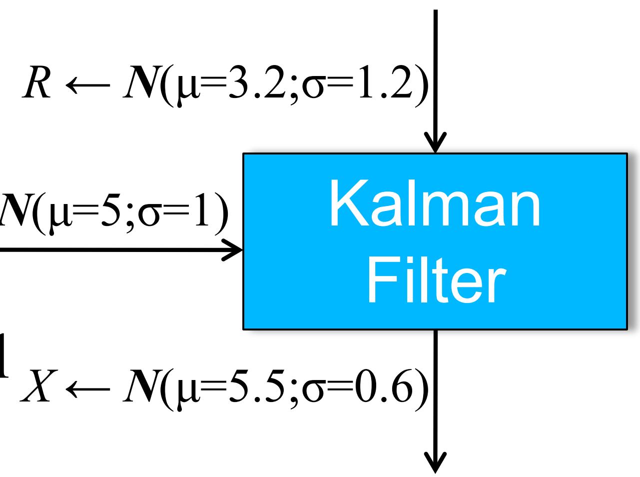
- Given:
  - Position estimate $\underline{X} \leftarrow N(\mu=5;\sigma=1)$
  - Range estimate $R \leftarrow N(\mu=3.2;\sigma=1.2)$

    What is the best estimate AFTER incorporating r ?

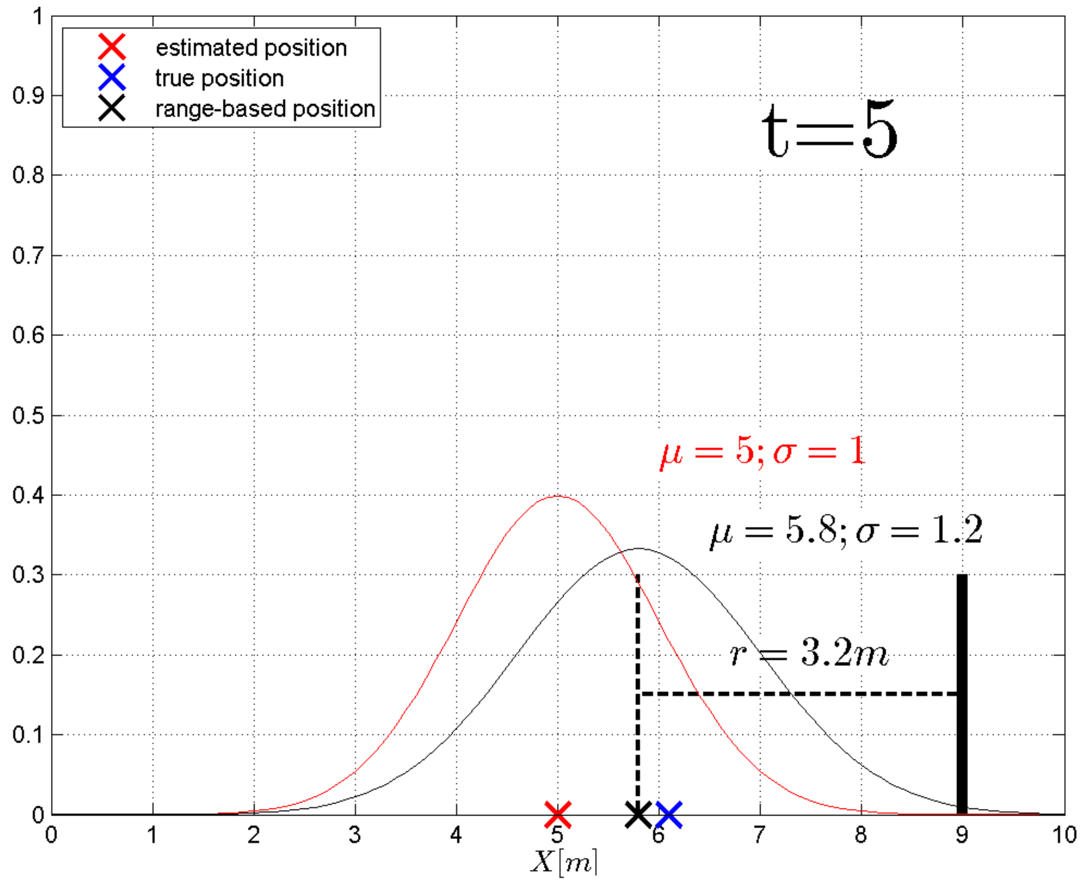


→ ***Kalman Filter***

- Requires:
  - White Gaussian noise distribution for all measurements
  - Linear motion and measurement model
  - …



Feature-Based Localization



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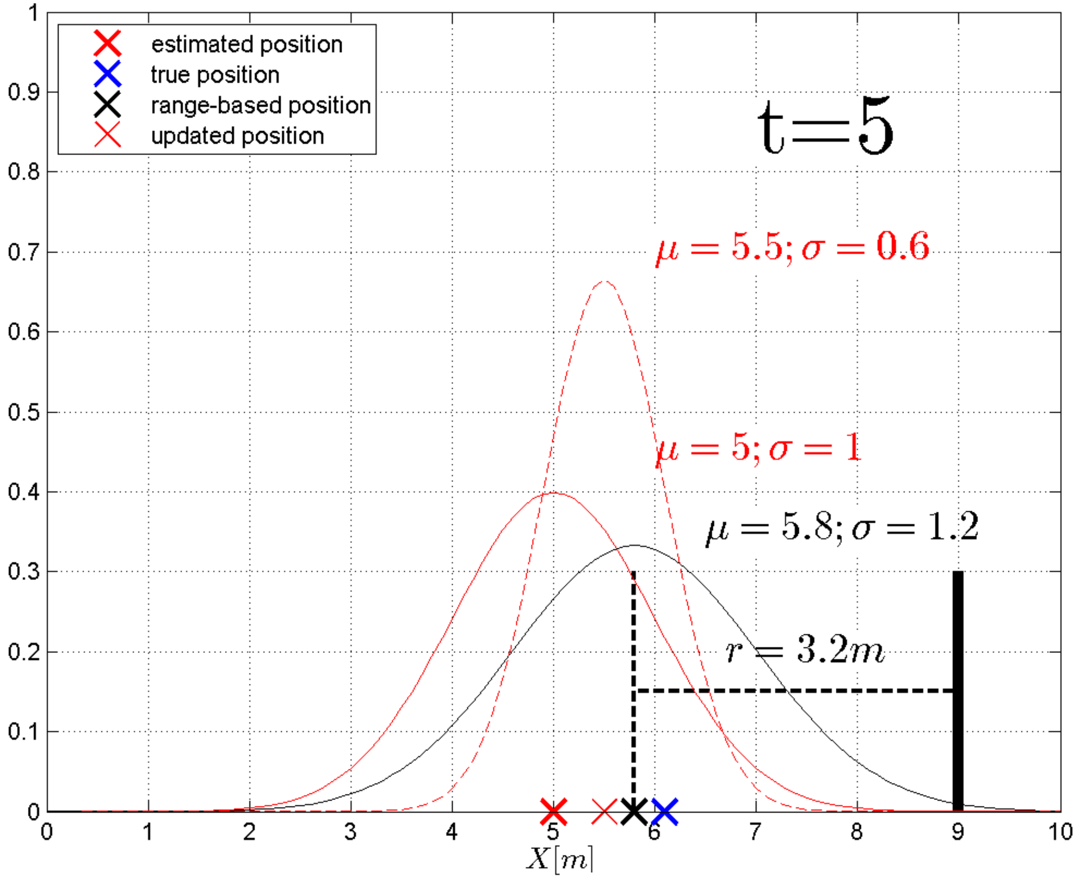

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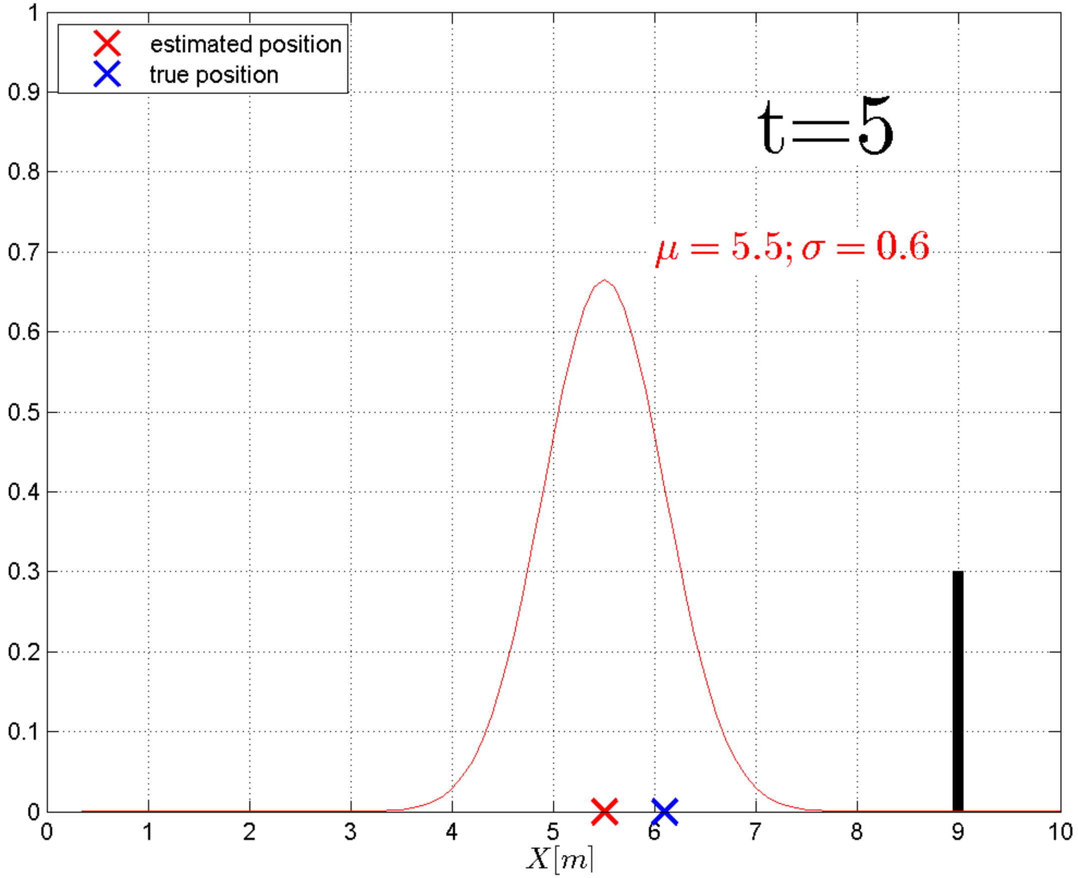

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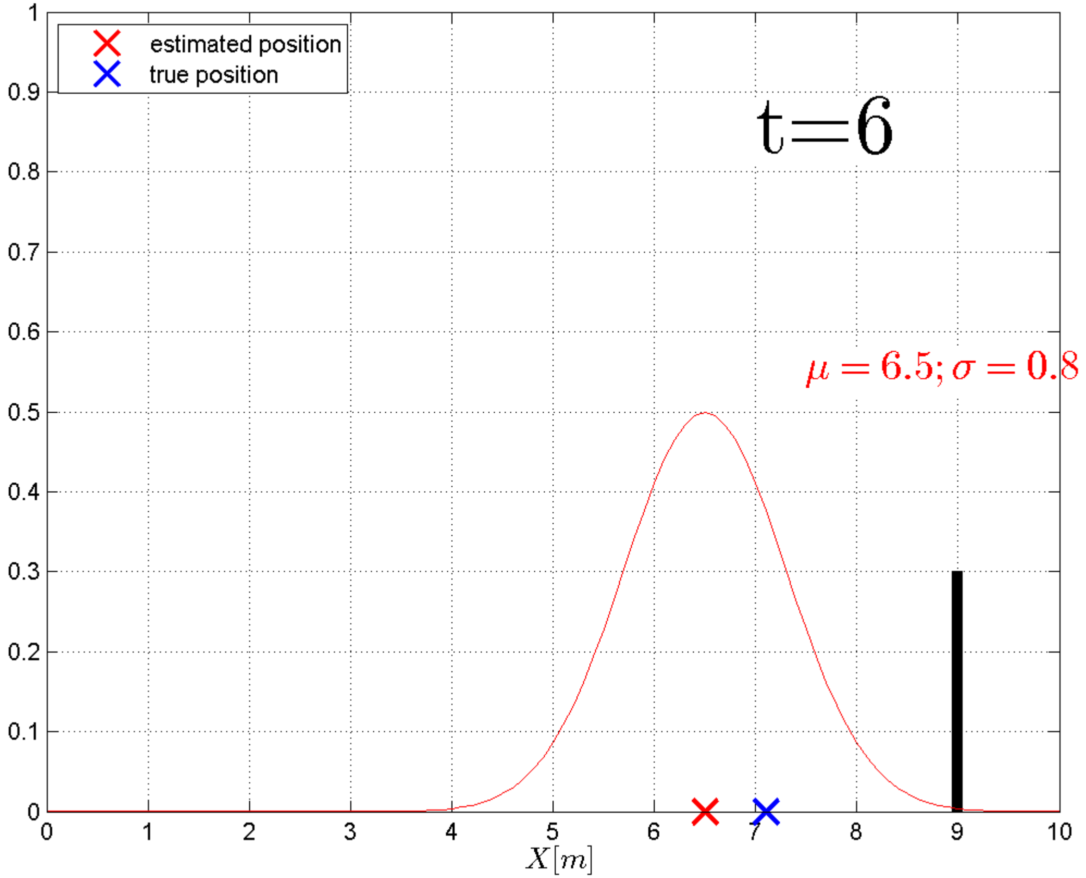
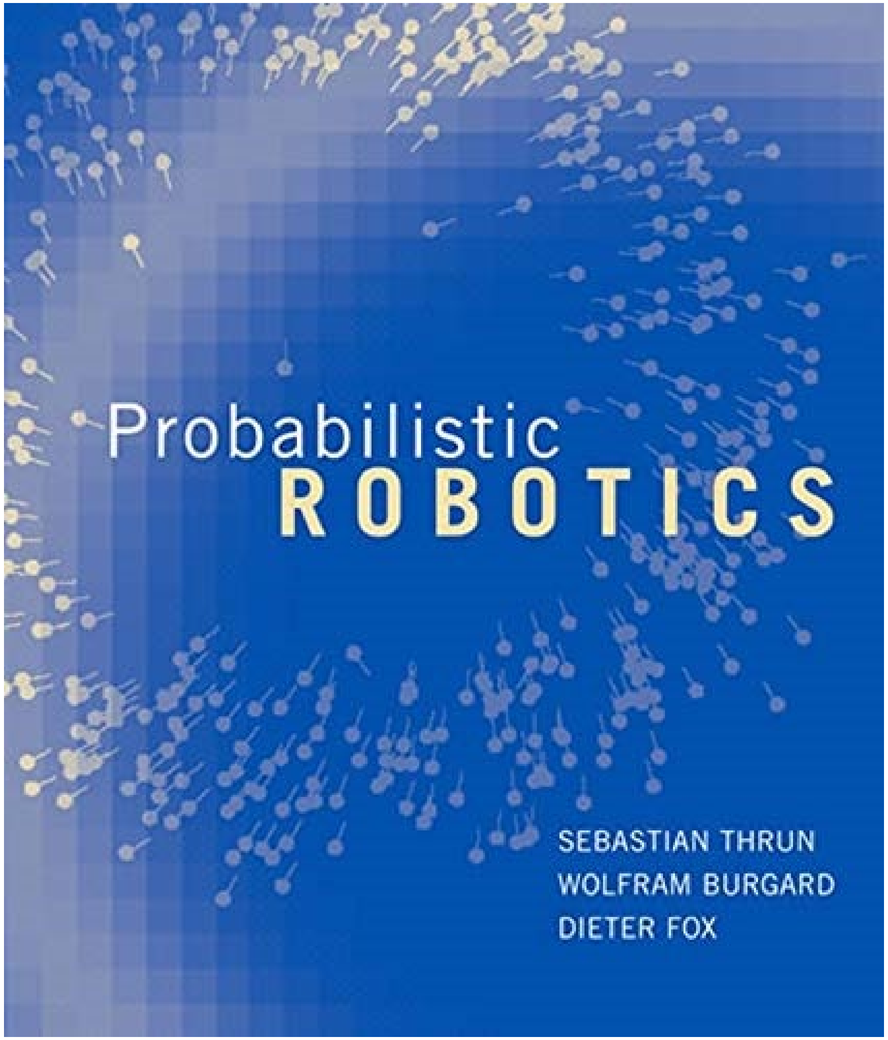


The Multi-Dimensional Kalman Filter Algorithm and its Application to Localization

Two Key Sources of Information



Multi-Dimensional Kalman Filter

Estimates the state x of a discrete-time controlled process that is governed by the linear stochastic difference equation

$$x_t = A_t x_{t-1} + B_t u_t + \varepsilon_t$$

with a measurement

$$z_t = C_t x_t + \delta_t$$

[Adapted from Thrun et al., 2005]

Components of a Kalman Filter

A_t Matrix (nxn) that describes how the state evolves from *t* to *t-1* without controls or noise.

B_t Matrix (nxl) that describes how the control u_t changes the state from *t* to *t-1*.

C_t Matrix (kxn) that describes how to map the state x_t to an observation z_t.

ε_t

δ_t

Random variables representing the process and measurement noise that are assumed to be independent and normally distributed with covariance R_t and Q_t respectively.

[Adapted from Thrun et al., 2005]

Kalman Filter Algorithm

Algorithm **Kalman_filter**(μ_{t-1}, Σ_{t-1}, u_t, z_t)

1. Prediction:
2. $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$
3. $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$
4. Correction or update:
5. $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$
6. $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$
7. $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$
8. Return μ_t, Σ_t

[Adapted from Thrun et al., 2005]

Mitigating Localization Uncertainities in Odometry Through Exteroceptive Sensors – The 2D Case

Nondeterministic Error Sources

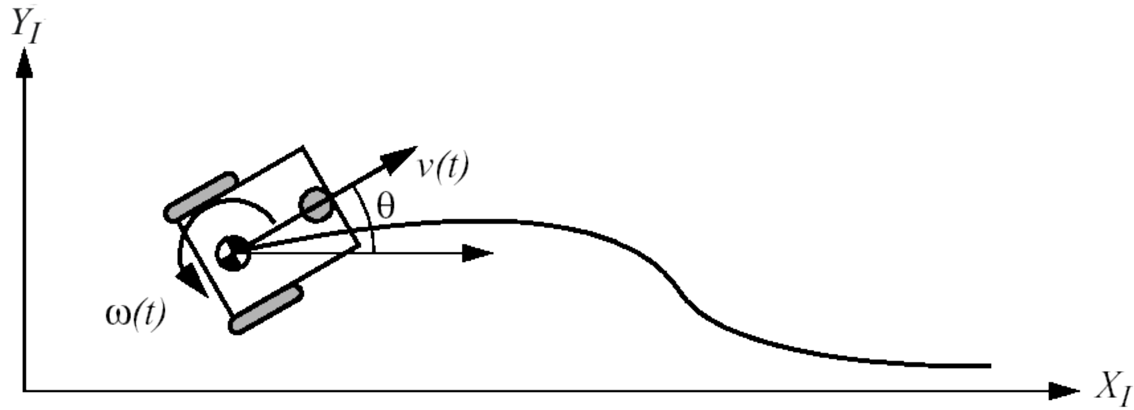
- Variation of the contact point of the wheel
- Unequal floor contact (e.g., wheel slip, nonplanar surface)

- Wheels cannot be assumed to roll perfectly
- Measured encoder values do not perfectly reflect the actual motion
- Pose error is cumulative and incrementally increases
- Probabilistic modeling for assessing quantitatively the error

Pose Variation During Δt

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

$$\begin{cases} \Delta x = \Delta s \cos(\theta + \frac{\Delta\theta}{2}) \\ \Delta y = \Delta s \sin(\theta + \frac{\Delta\theta}{2}) \\ \Delta\theta = \frac{\Delta s_r - \Delta s_l}{b} \end{cases}$$



$$p = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \xrightarrow{t'=t+\Delta t} p' = \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix}$$

b = 2l = inter-wheel distance

Δs_r = traveled distance right wheel

Δs_l = traveled distance left wheel

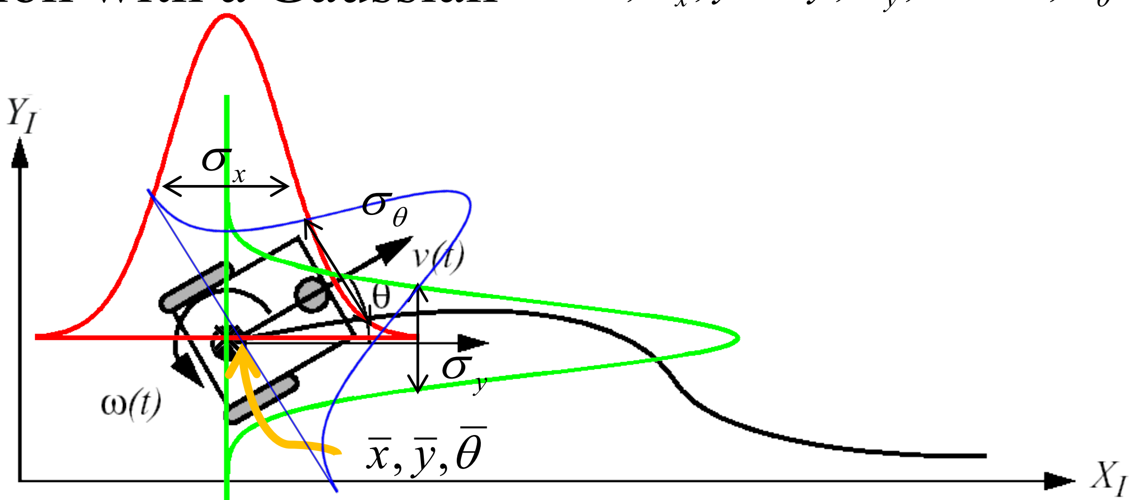
Δθ = orientation change of the vehicle

$$p' = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta s \cos(\theta + \Delta\theta/2) \\ \Delta s \sin(\theta + \Delta\theta/2) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

Noise modeling

Model error in each dimension with a Gaussian $x \rightarrow \bar{x}, \sigma_x; y \rightarrow \bar{y}, \sigma_y; \theta \rightarrow \bar{\theta}, \sigma_\theta$

$$\Sigma_p = \begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{x\theta}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{y\theta}^2 \\ \sigma_{\theta x}^2 & \sigma_{\theta y}^2 & \sigma_{\theta\theta}^2 \end{bmatrix}$$



Assumptions:

- Covariance matrix Σ_p at the beginning is known
- Errors of the two individual wheels are independent
- Errors are independent of direction of motion
- Errors are proportional to the distance traveled (k_r, k_l model parameters)

$$\sum_\Delta = \mathrm{cov}(\Delta s_r, \Delta s_l) = \begin{bmatrix} k_r|\Delta s_r| & 0 \\ 0 & k_l|\Delta s_l| \end{bmatrix} = \begin{bmatrix} \sigma_{s_r}^2 & 0 \\ 0 & \sigma_{s_l}^2 \end{bmatrix}$$

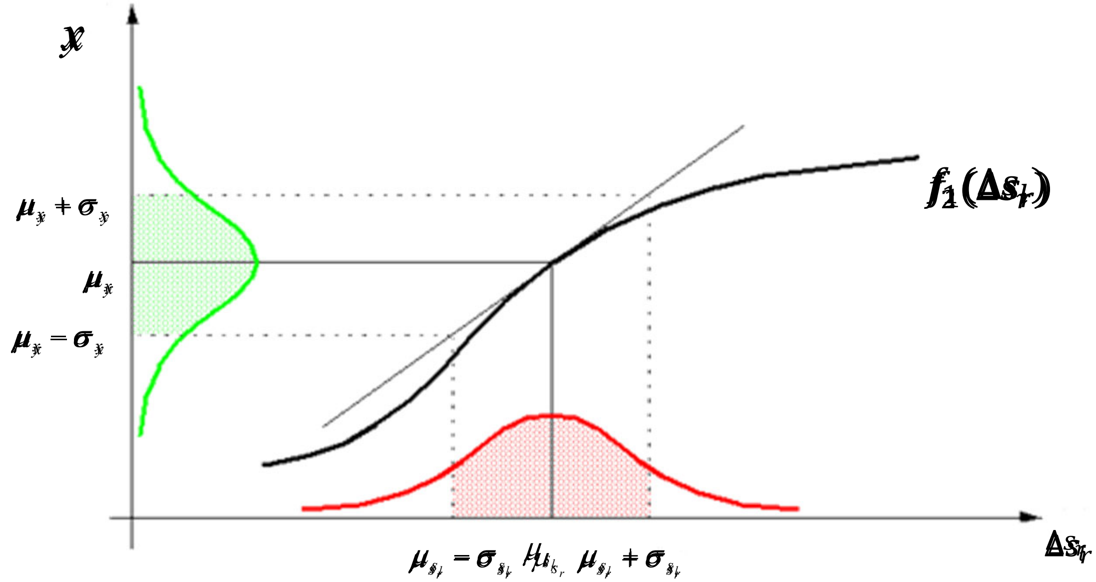
Actuator Noise → Pose Noise

$$x \approx f_1(\Delta s_r)\Big|_{\Delta s_r=\mu_{s_r}} \approx f_1(\Delta s_r) + \frac{\partial f_1}{\partial \Delta s_r}(\Delta s_r - \mu_{s_r})$$

$$x \approx f_1(\Delta s_l)\Big|_{\Delta s_l=\mu_{s_l}} \approx f_1(\Delta s_l) + \frac{\partial f_1}{\partial \Delta s_l}(\Delta s_l - \mu_{s_l})$$

$$y \approx f_2(\Delta s_r)\Big|_{\Delta s_r=\mu_{s_r}} \approx f_2(\Delta s_r) + \frac{\partial f_2}{\partial \Delta s_r}(\Delta s_r - \mu_{s_r})$$

...



$$F_{\Delta rl} = \begin{bmatrix} \frac{\partial f_1}{\partial \Delta s_r} & \frac{\partial f_1}{\partial \Delta s_l} \\ \frac{\partial f_2}{\partial \Delta s_r} & \frac{\partial f_2}{\partial \Delta s_l} \\ \frac{\partial f_3}{\partial \Delta s_r} & \frac{\partial f_3}{\partial \Delta s_l} \end{bmatrix}$$ Jacobian

$$\Sigma_\Delta = \begin{bmatrix} k_r|\Delta s_r| & 0 \\ 0 & k_l|\Delta s_l| \end{bmatrix}$$

- General error propagation law

$$\Sigma_{\Delta rl} = F_{\Delta rl} \Sigma_\Delta F_{\Delta rl}^T$$

Actuator Noise → Pose Noise

Algorithm

Precompute:

- Determine actuator noise Σ_Δ
- Compute mapping actuator-to-pose noise incremental $F_{\Delta rl}$
- Compute mapping pose propagation noise over step F_p

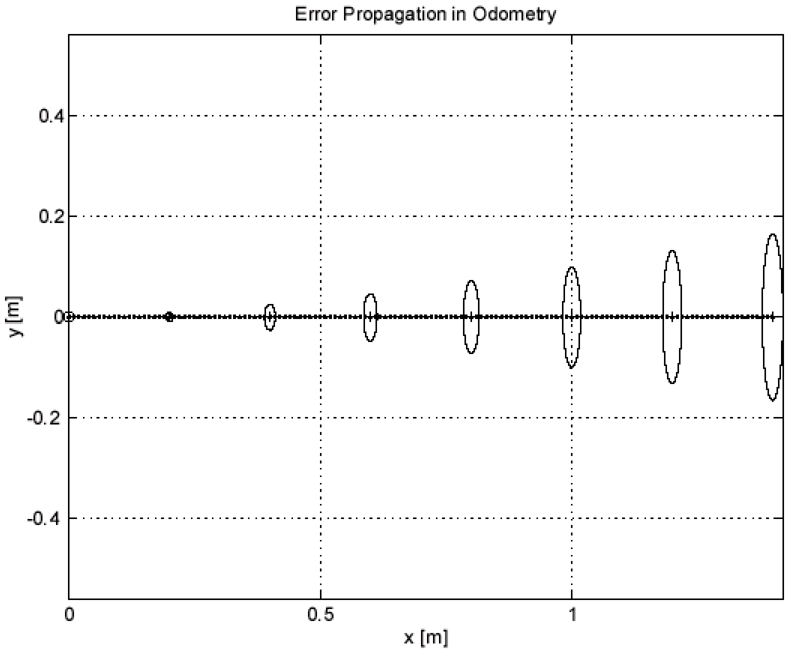
Initialize:

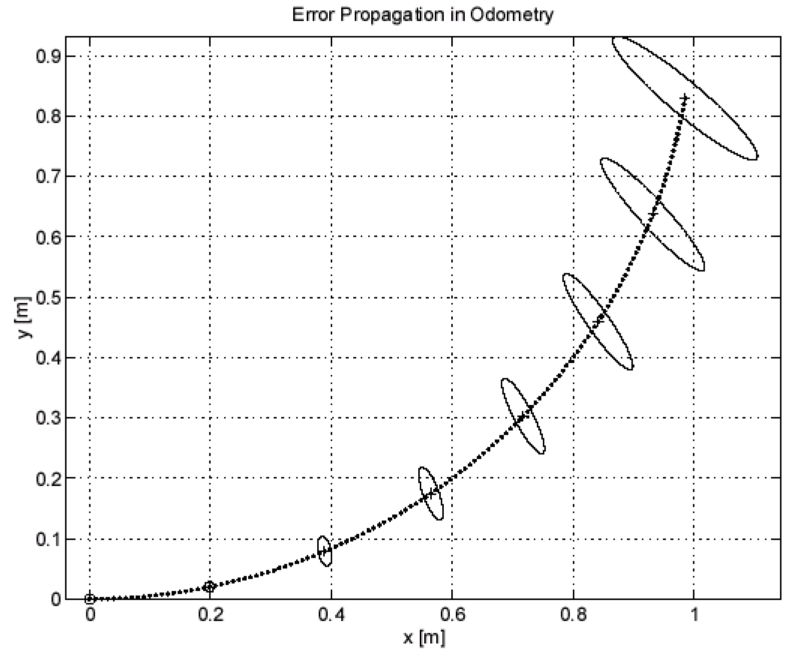
- Initialize $\Sigma_p^{(t=0)} = [0]$

Iterate:

$$\Sigma_p^{(t=(k+1)\Delta t)} = F_p \Sigma_p^{(t=k\Delta t)} F_p^T + F_{\Delta rl} \Sigma_\Delta F_{\Delta rl}^T$$

Classical 2D Representation

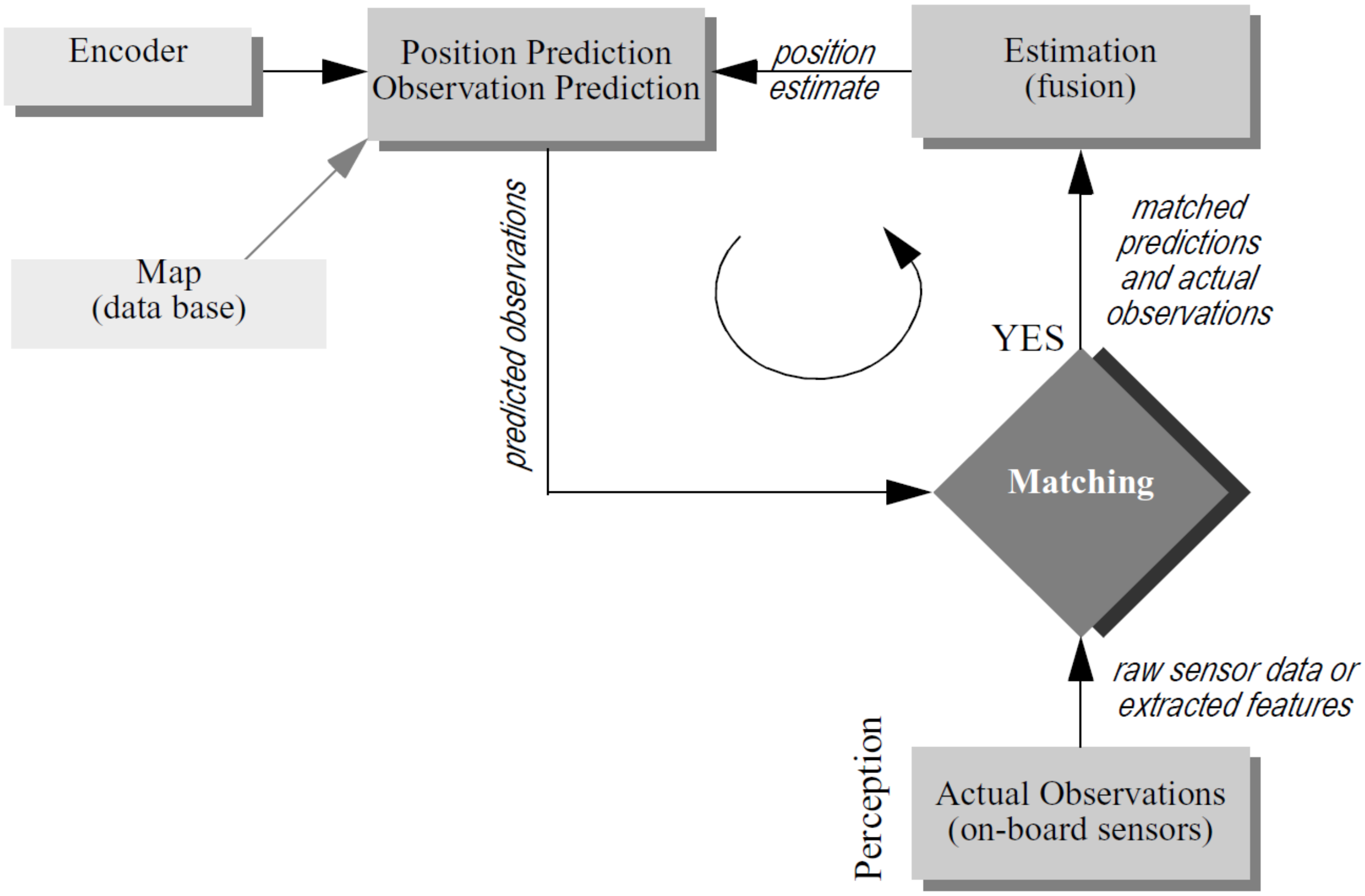




Courtesy of R. Siegwart and R. Nourbakhsh

Ellipses: typical 3σ bounds

2D Localization with Kalman Filter



[From Siegwart and Nourbakhsh, 2004]

A Five-Step Iterative Recipe

1. Robot pose prediction
2. Actual observations
3. Predicted observations
4. Matching
5. Estimation with Kalman filter

Robot Pose Prediction

- Input: motion model of the vehicle, odometry, control actions
- Output: estimation mean and co-variance of the pose and the next timestep (s. 40 and 43)

Actual Observations

- Input: sensor model
- Coordinate transformation needed for getting to the local frame (in this case sensor frame)
- Output: depending on the sensing technology/feature (e.g., distance, line, door)

Predicted Observations

- Input: predicted robot pose and map
- Coordinate transformation needed for getting to the local frame (in this case sensor frame)
- Output: predicted feature observations

Matching

- Input: predicted feature observations and actual observations, all in the sensor frame
- Objective: matching actual observations to predicted features
- Innovation metric: difference predicted and observed measurements (also random variable, with mean and co-variance)
- Validation criterion based on innovation metric
- Output: selection of features worth using in the estimation process

Estimation with Kalman Filter

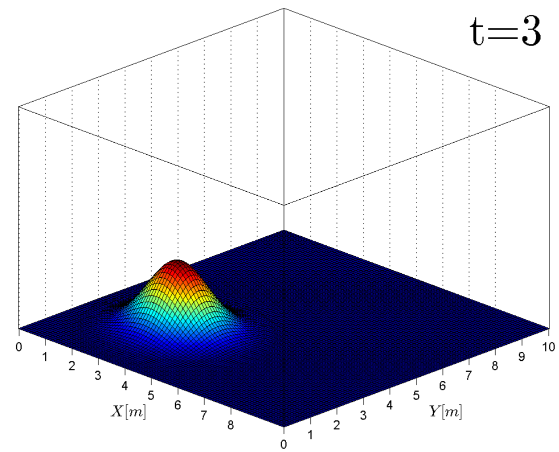
- Input: pose prediction from the motion model and validated observations
- Objective: fuse pose prediction from motion model and all validated observation to create next pose
- Output: next mean and variance of the pose

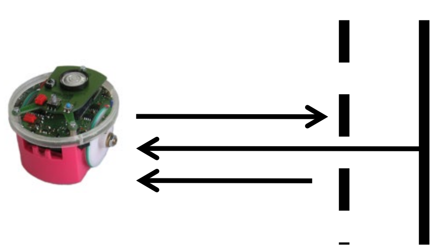
Working Around Kalman Filter Limitations: Particle Filters for Localization

Feature-Based Localization

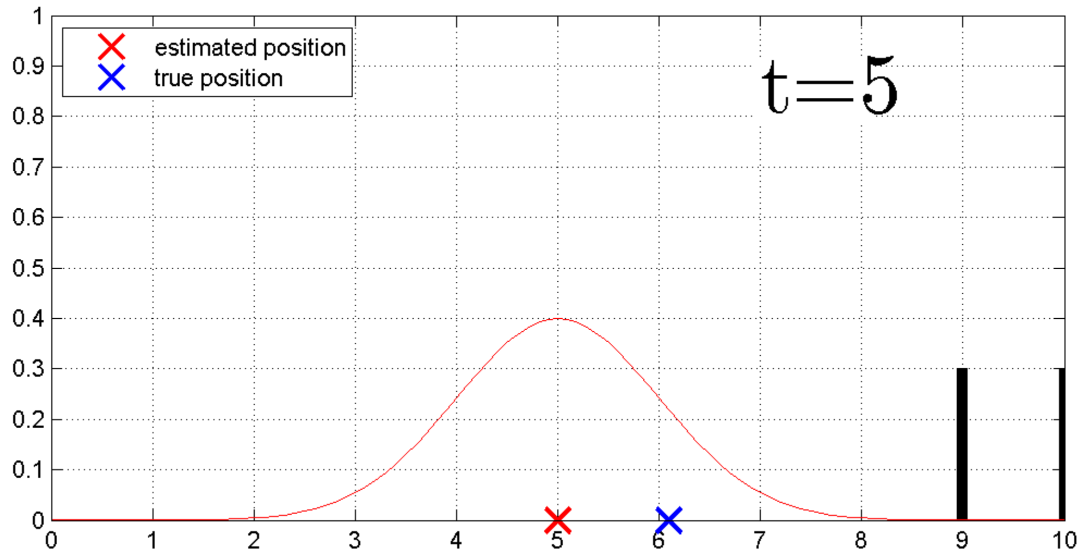
Belief representation through Gaussian distribution

- Advantages:
  - Compact (only mean and variance required)
  - Continuous
  - Powerful tools (Kalman Filter)
- Disadvantages:
  - Requires Gaussian noise assumption
  - Cannot represent ignorance (“kidnapped robot problem”)
  - Uni-modal
- Problematic example:





Particle Filter Localization



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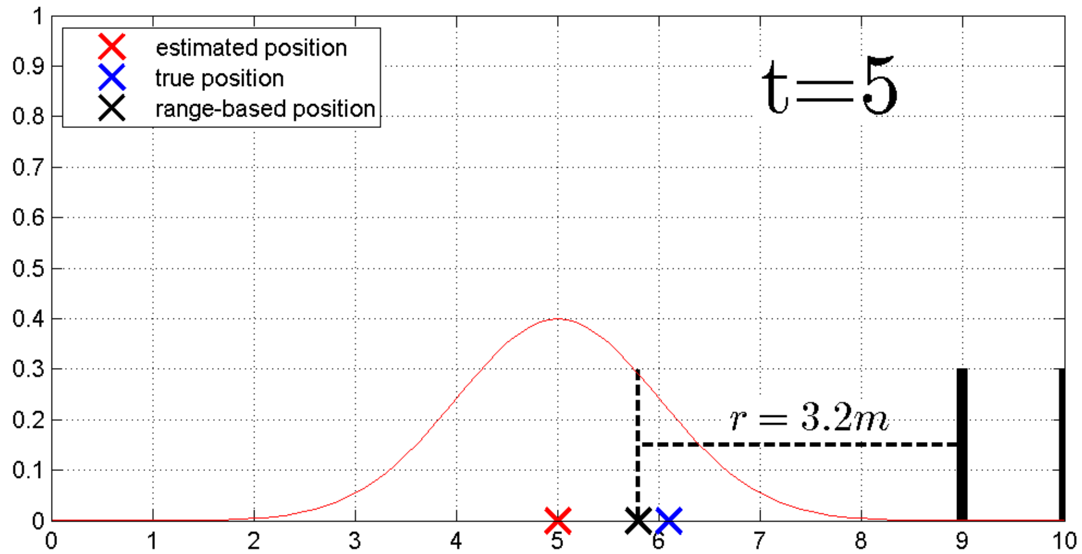

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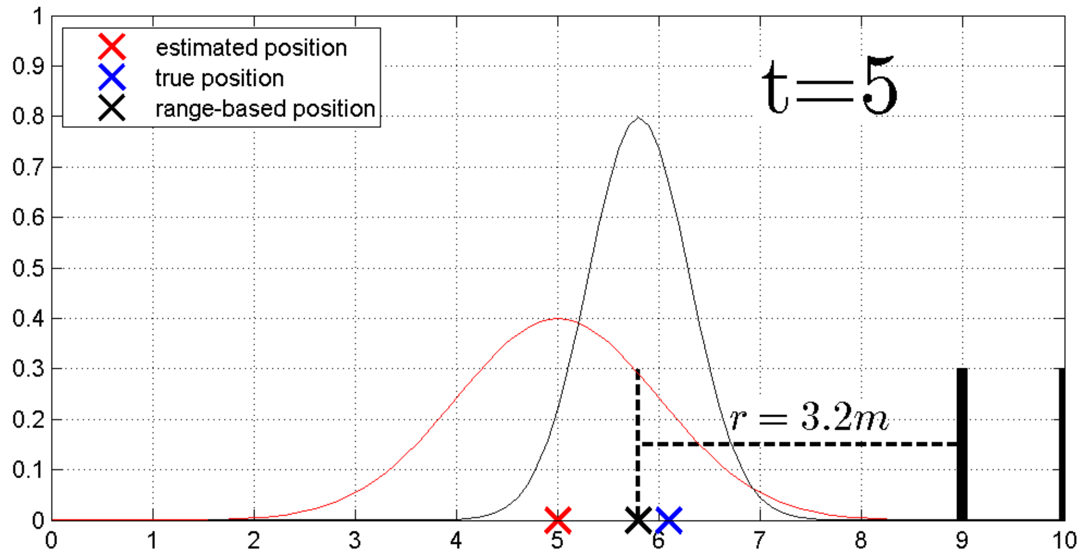

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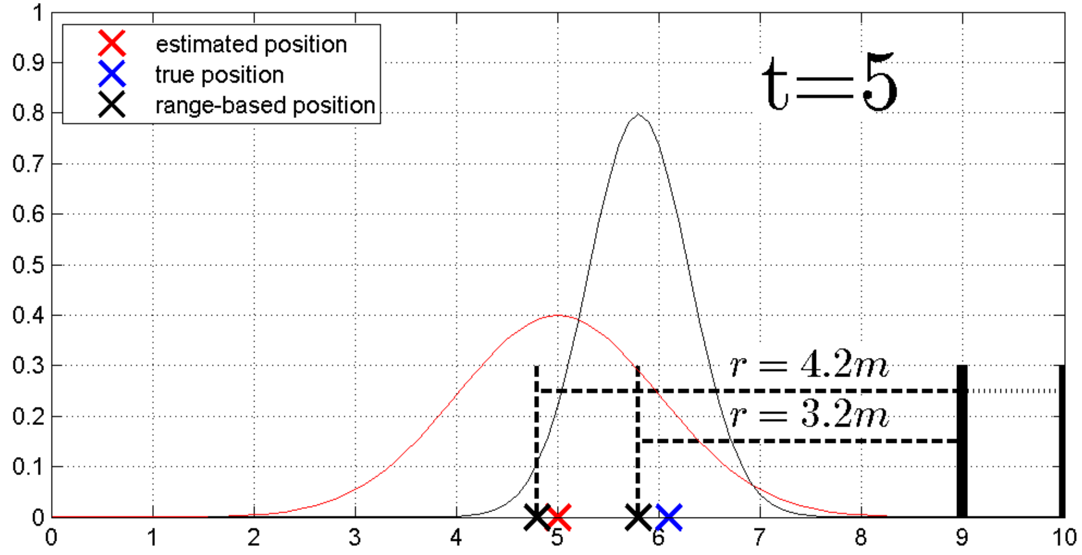

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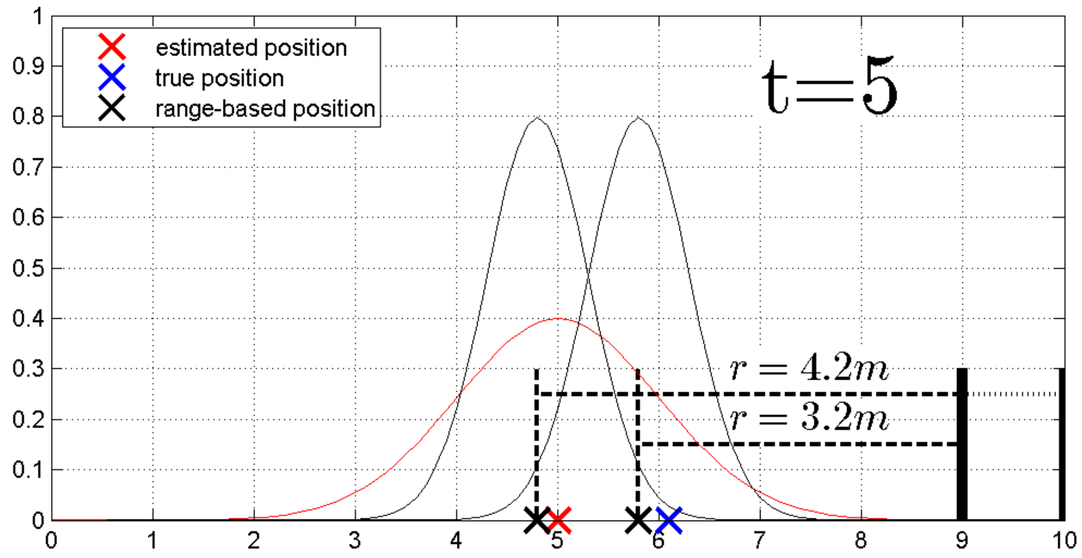

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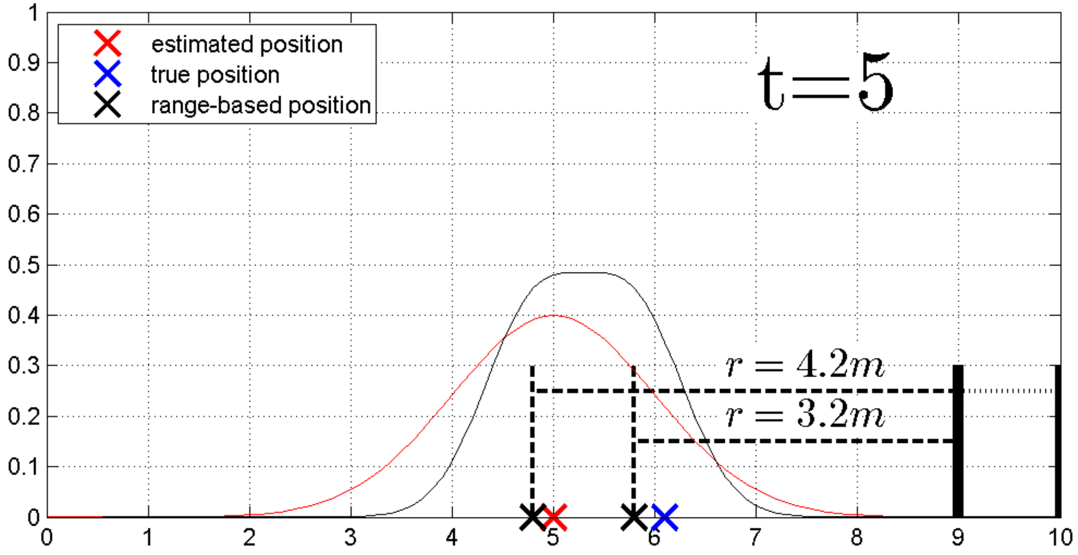

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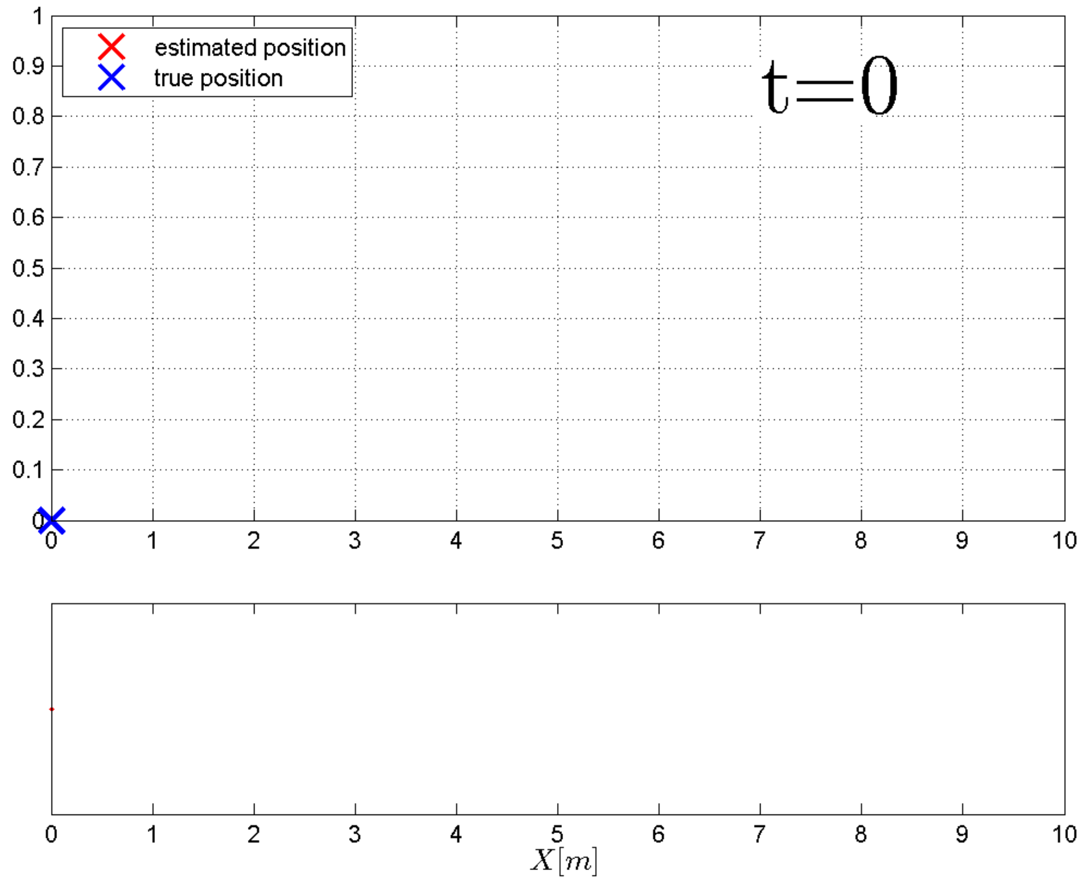

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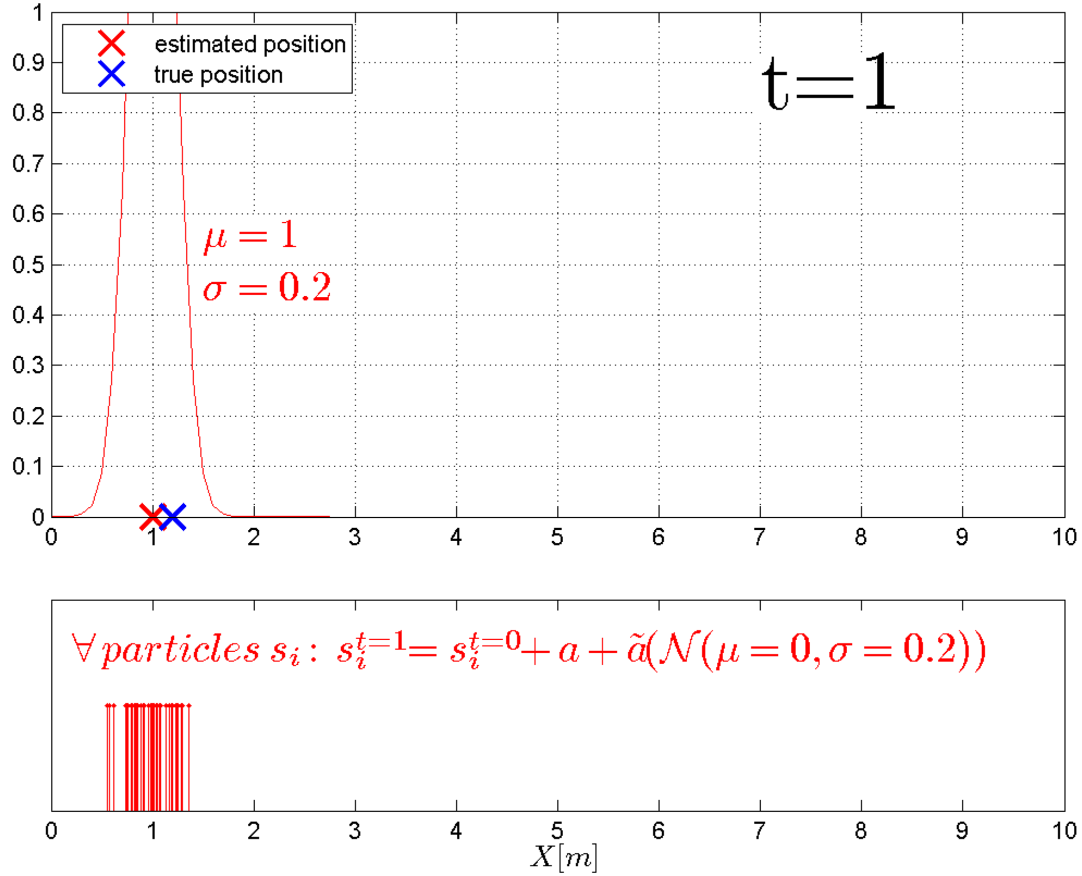

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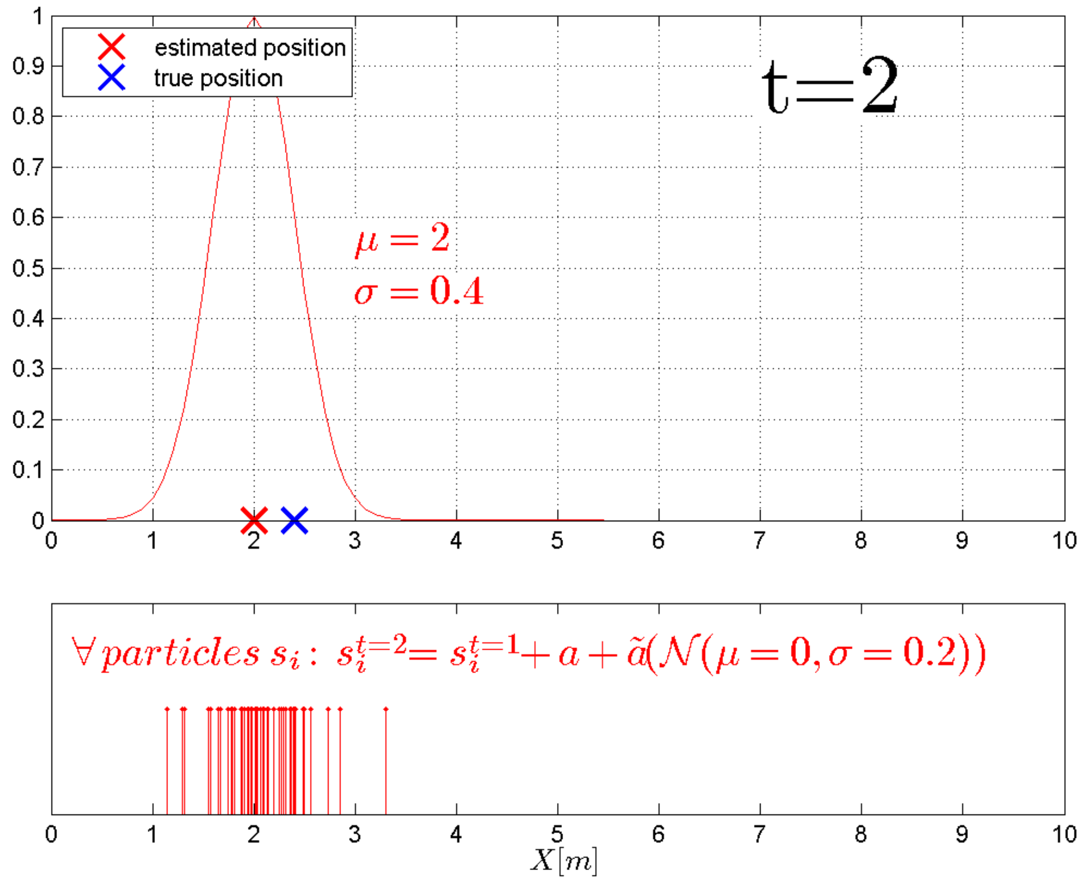

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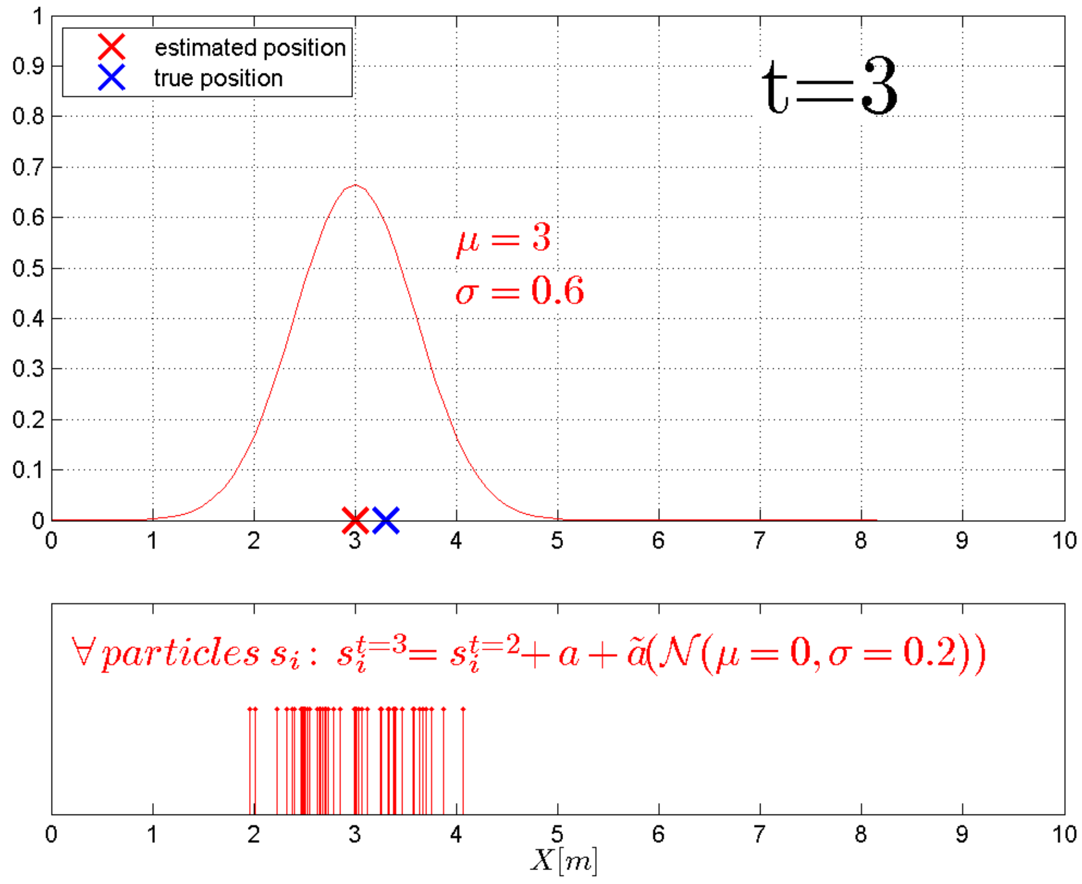

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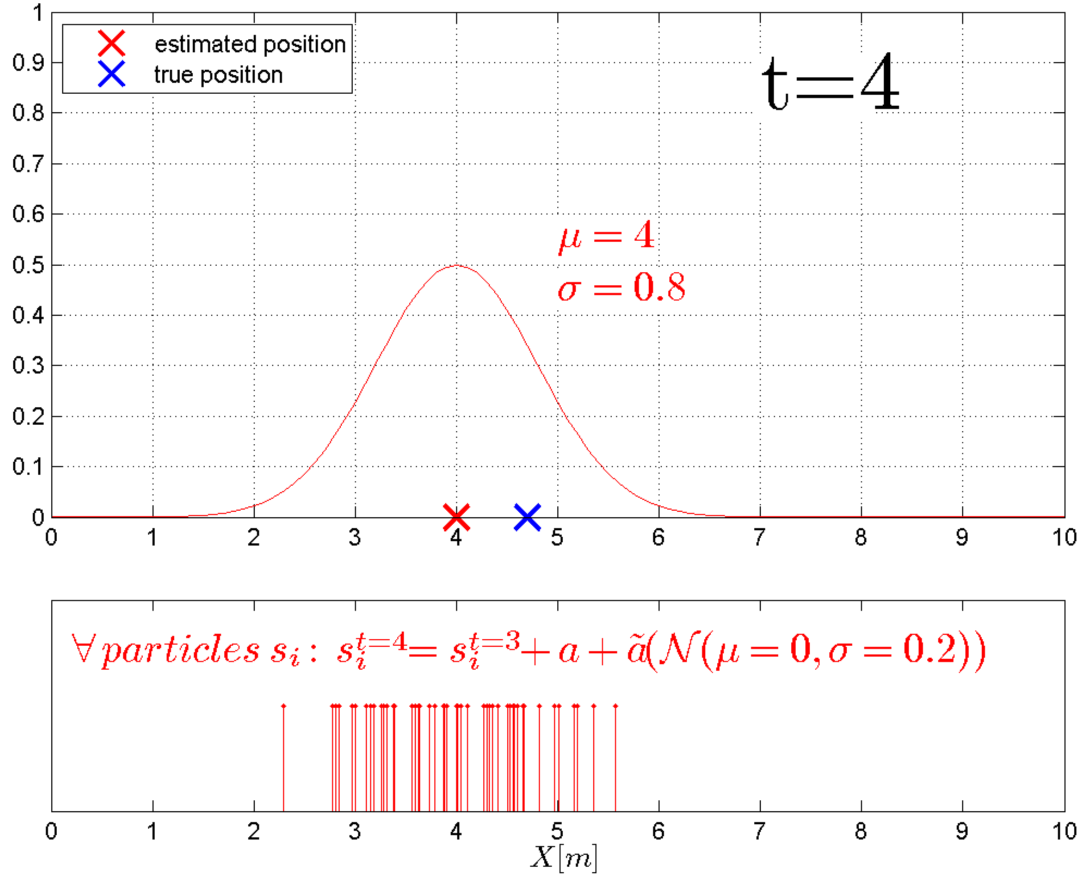

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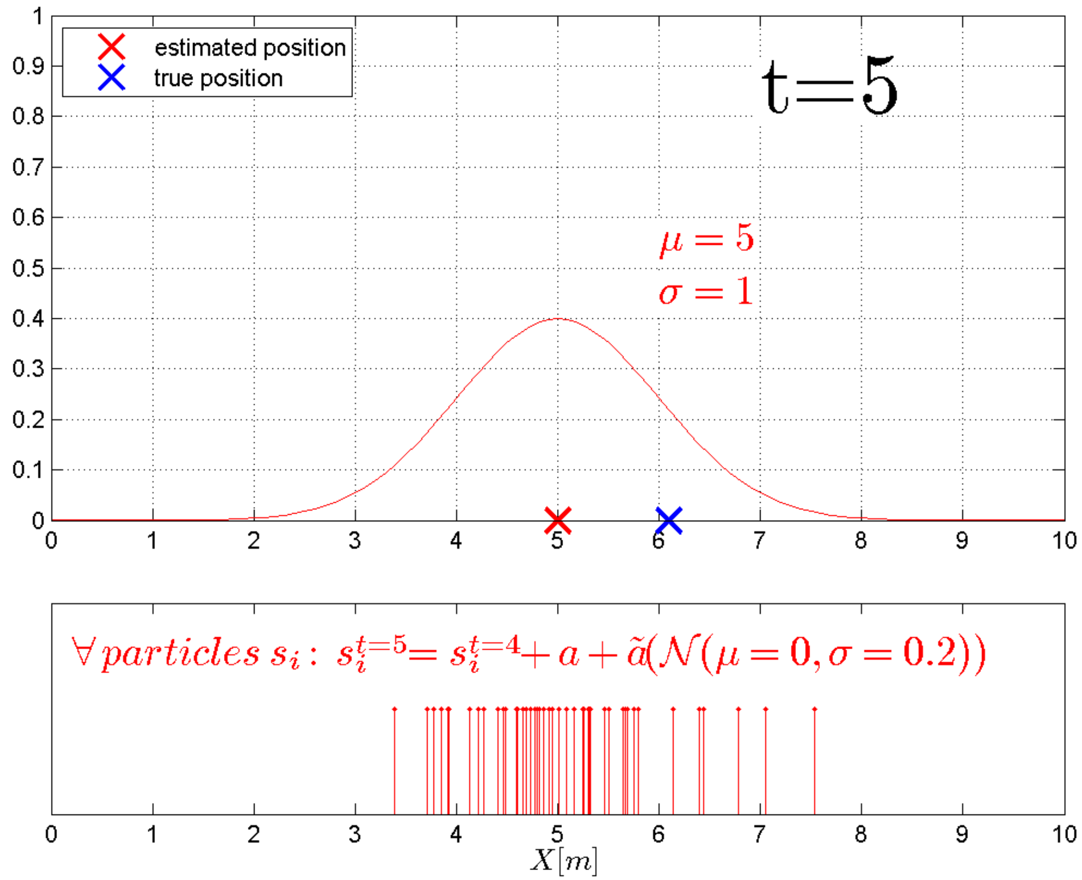

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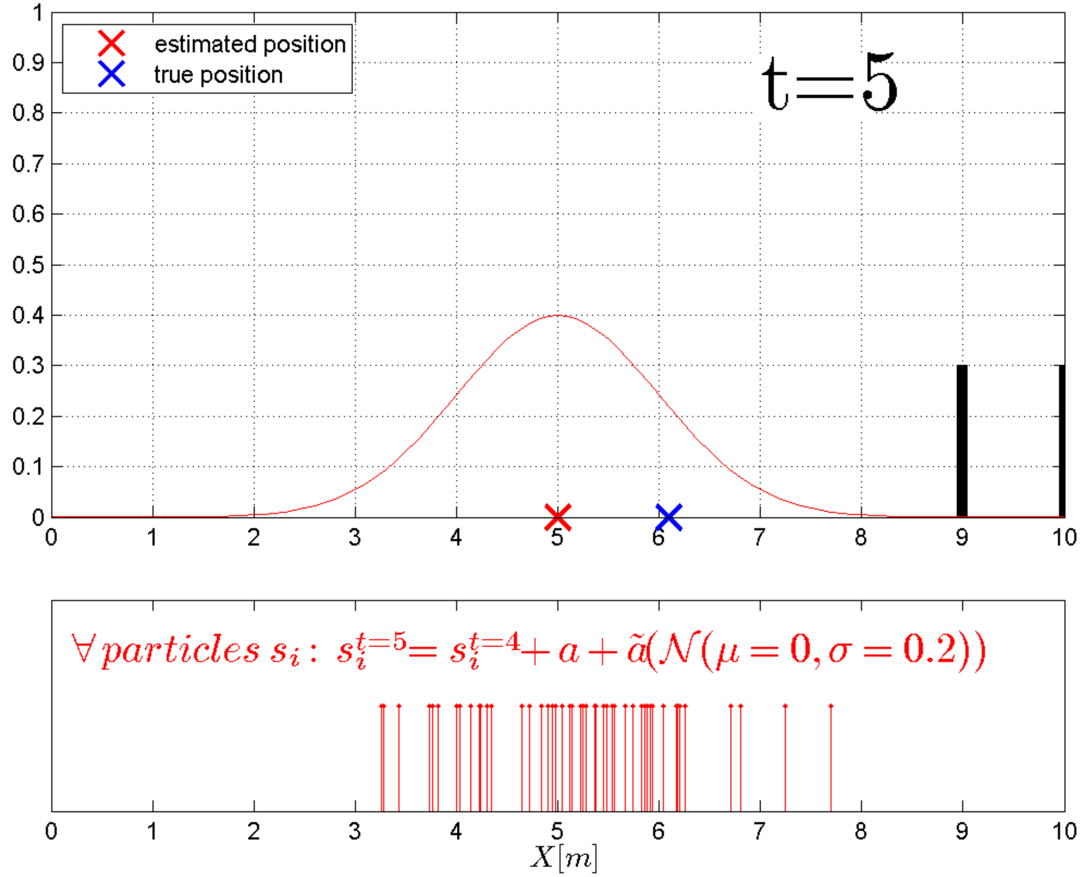

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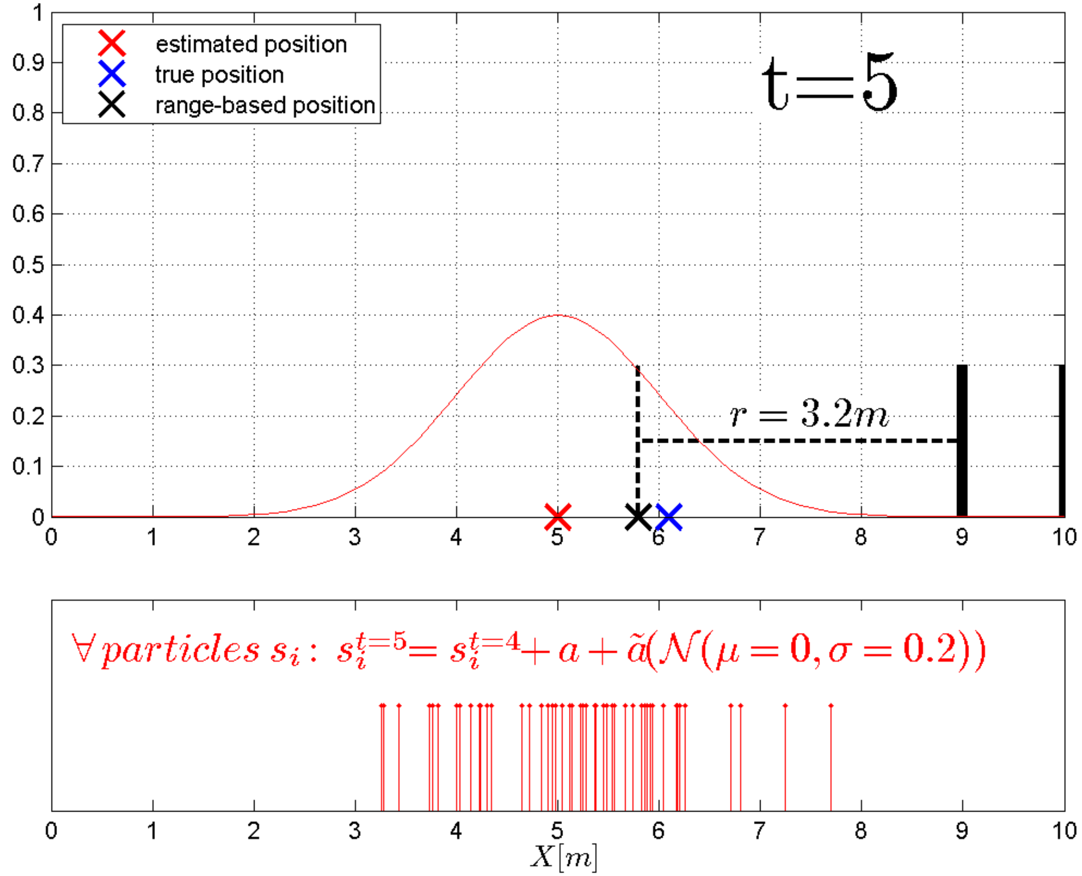

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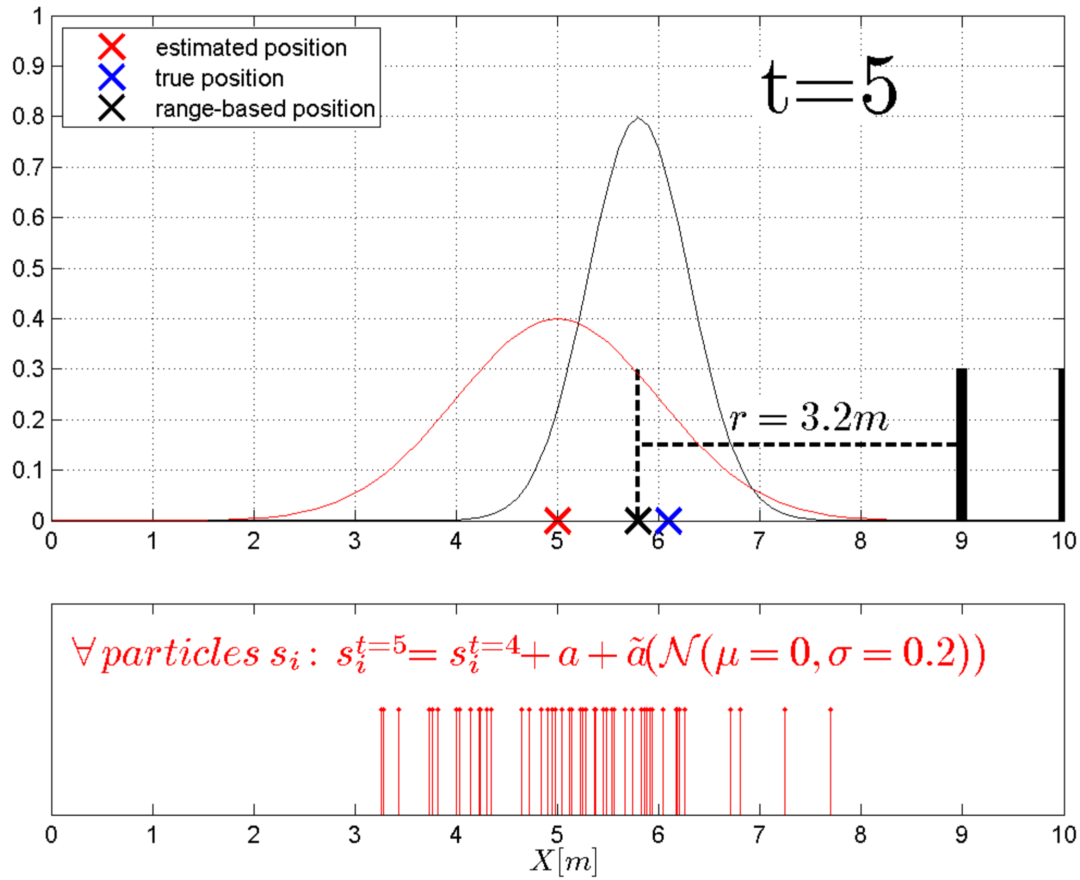

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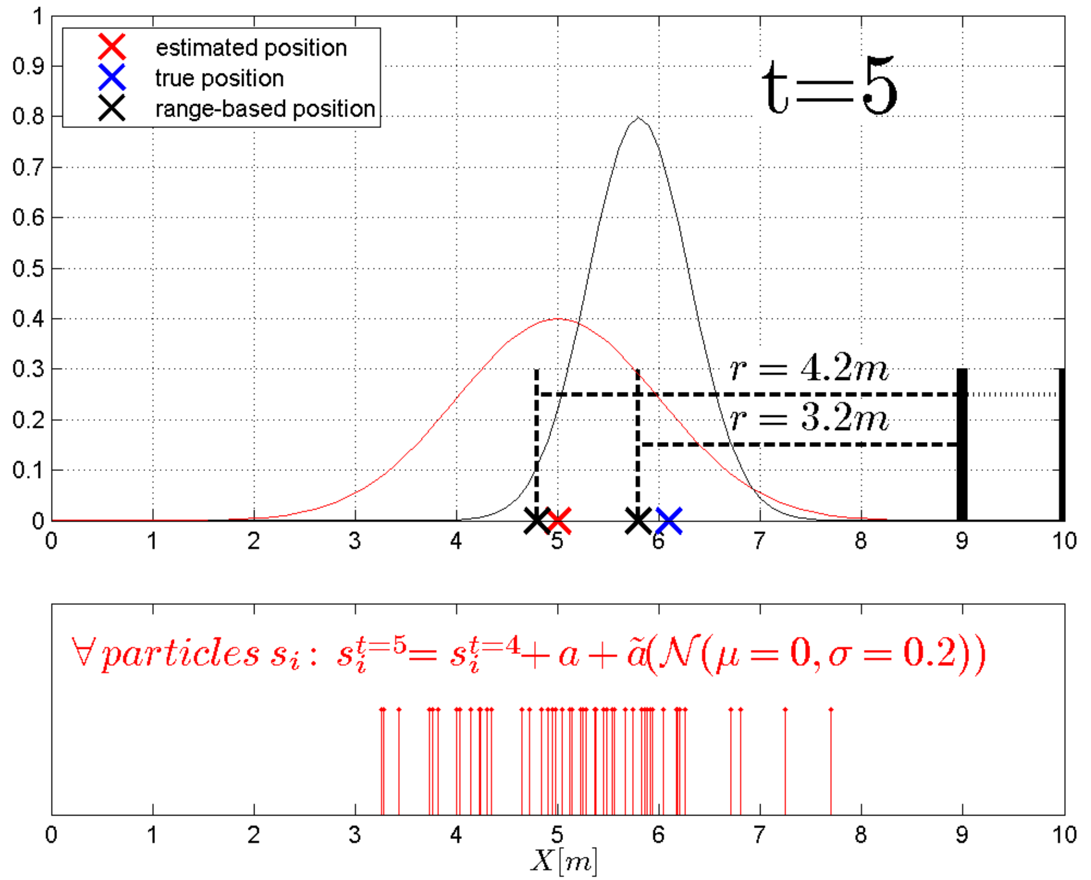

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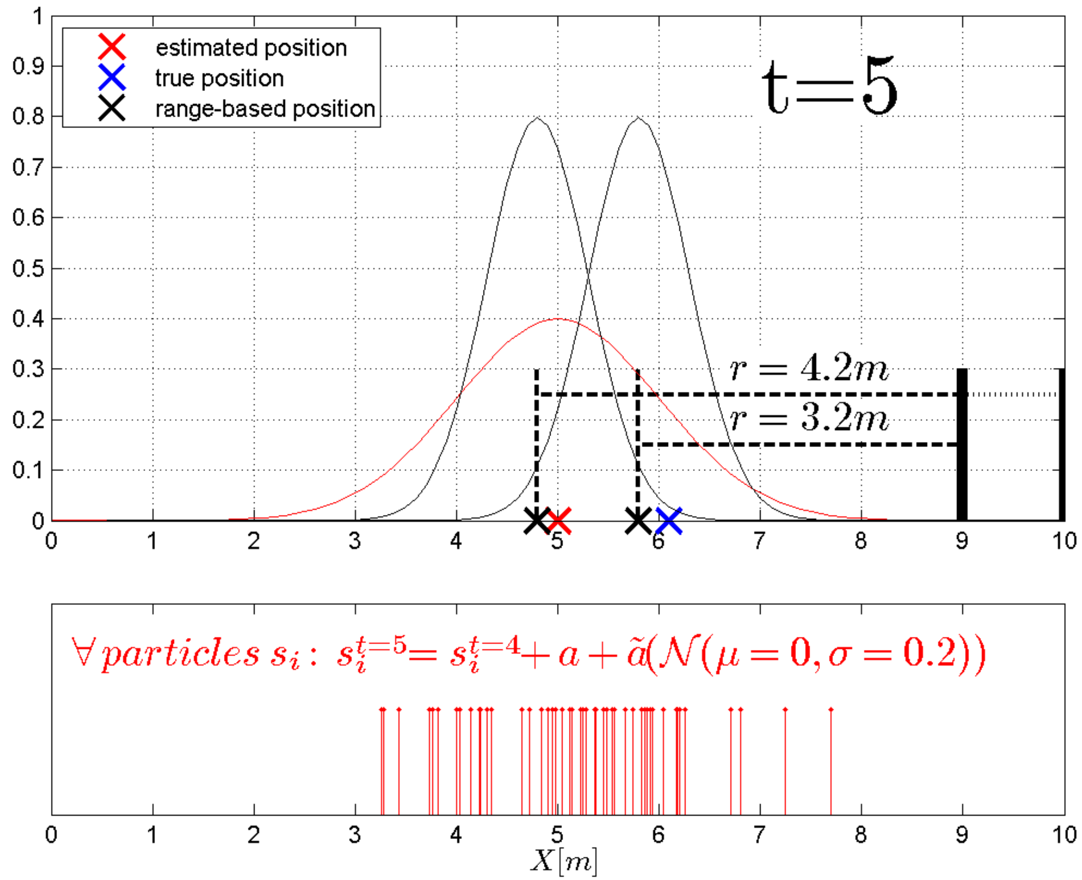

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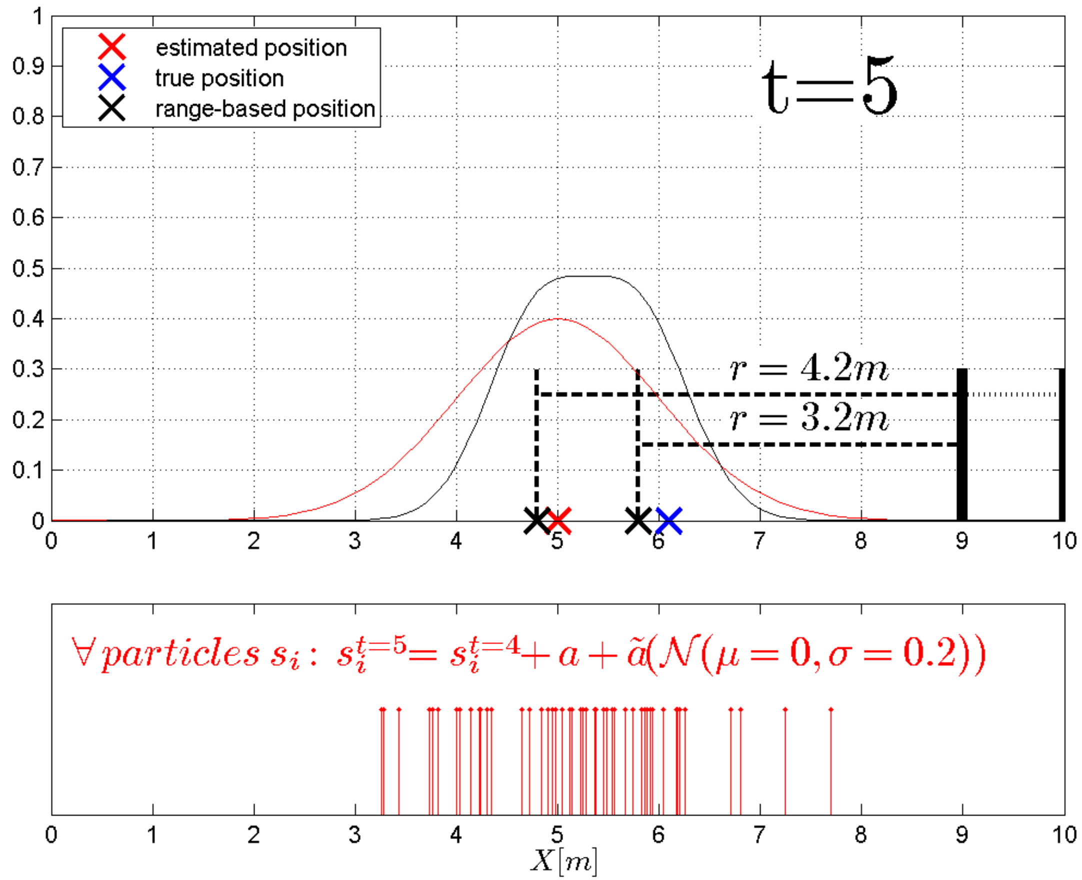

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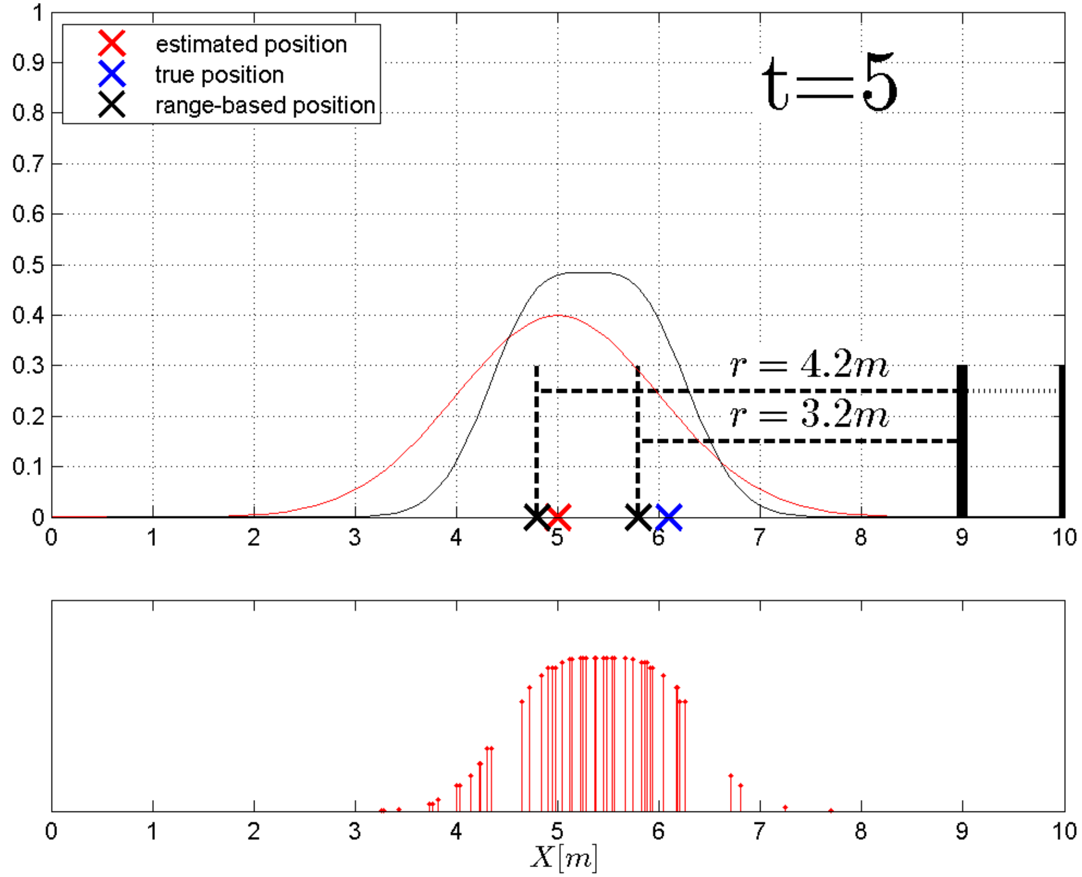
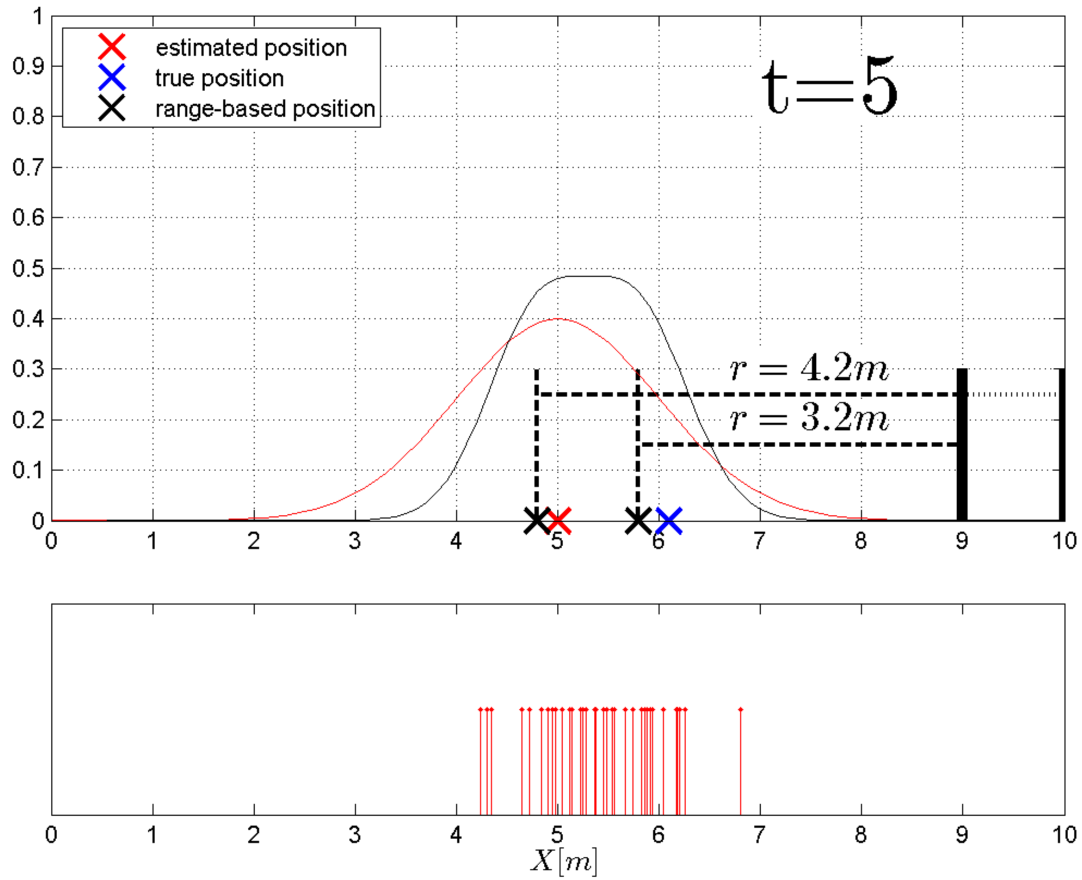


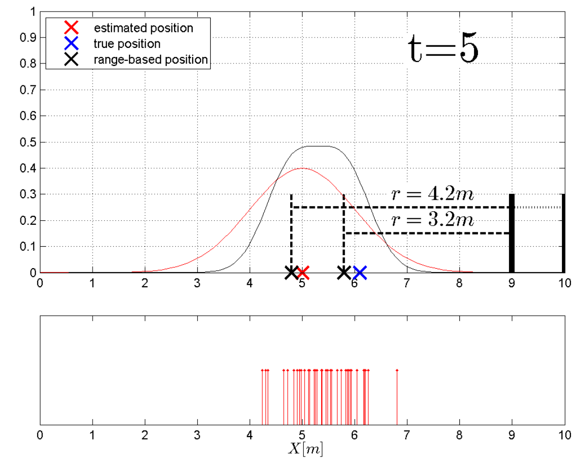
Particle Filter Localization



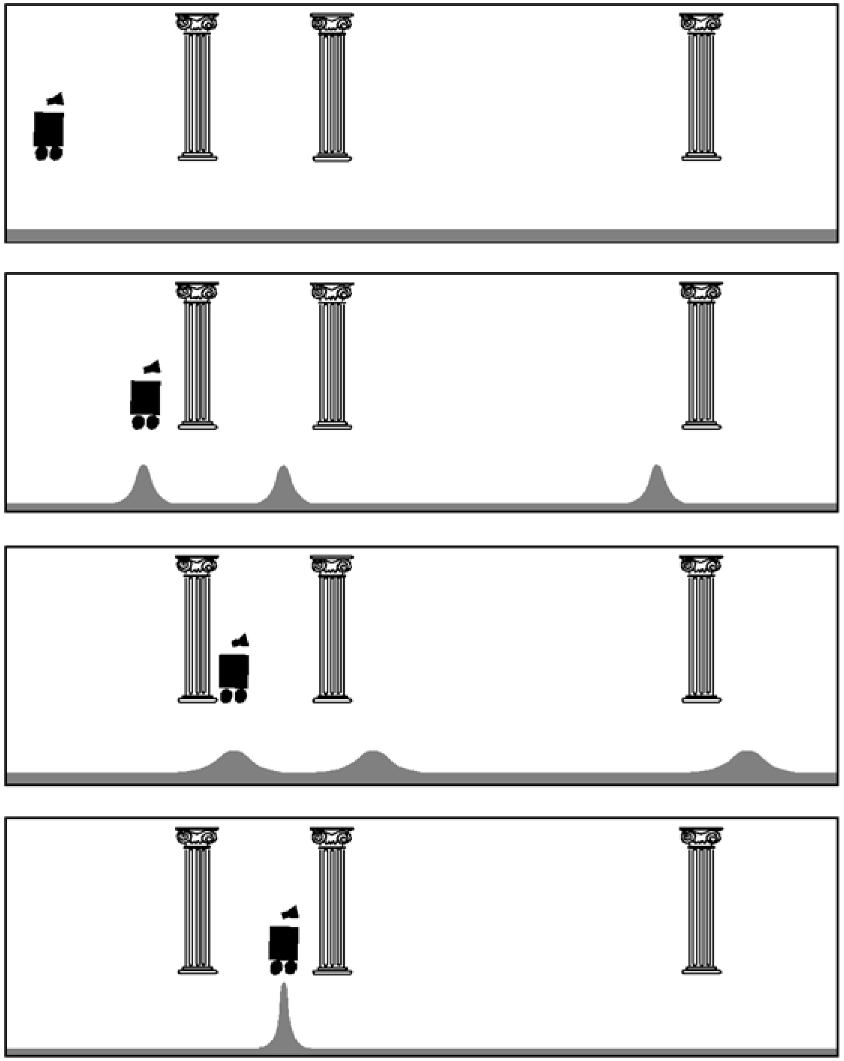
Particle Filter Localization

Belief representation through particle distribution

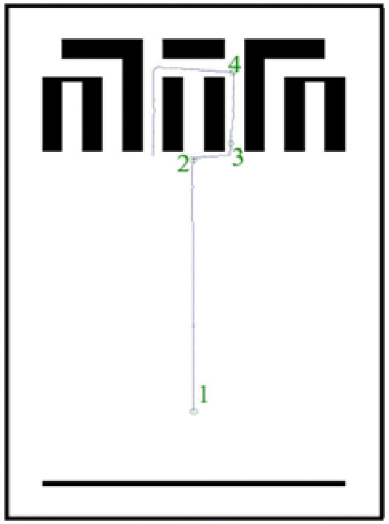
- Advantages:
  - Can model arbitrary beliefs
  - No assumptions on noise characteristic
- Disadvantages:
  - No unique solution
  - Not continuous
  - Computationally expensive
  - Tuning required

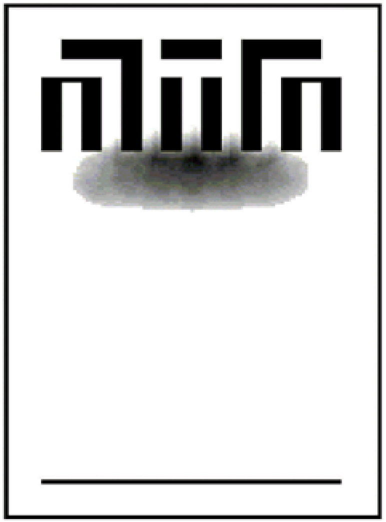


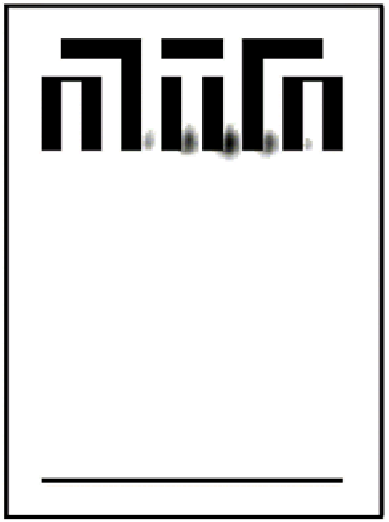
Particle Filter Localization

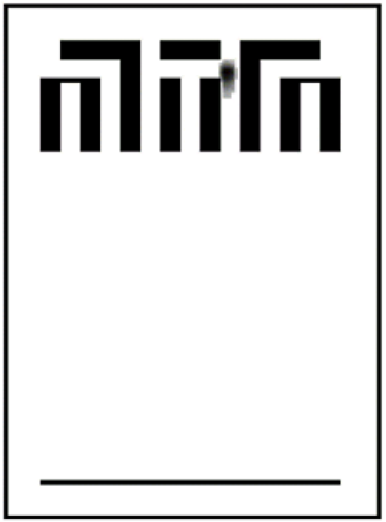


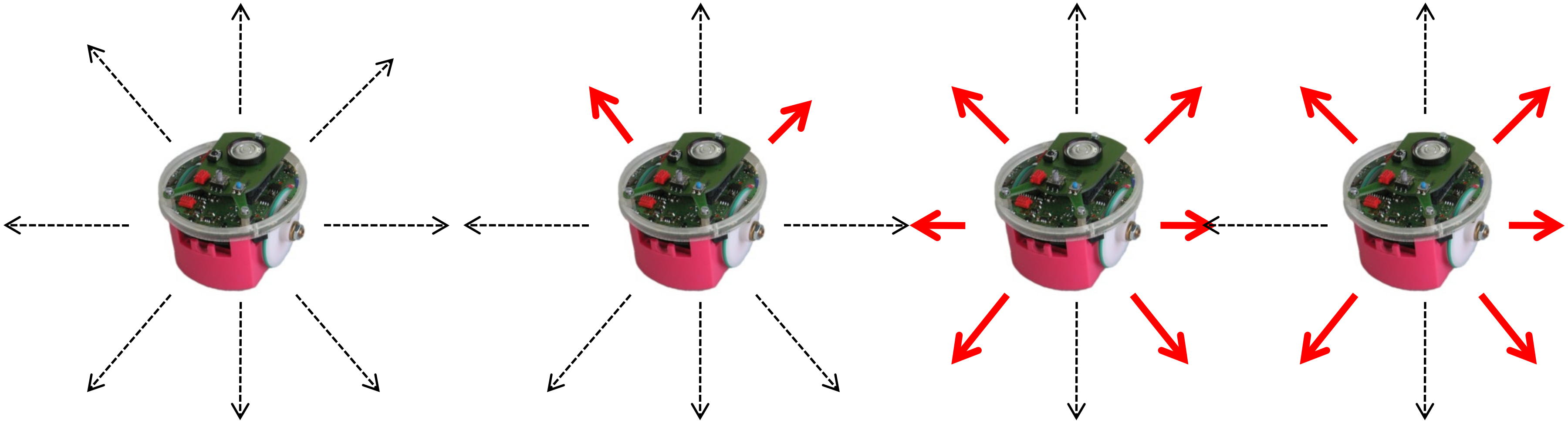
Particle Filter Localization











Conclusion

Take Home Messages

- Estimation methods, including Kalman filter techniques are applicable to a large variety of problems involving noisy processes and noisy sensing, not just localization
- Feature-based localization is a way to compensate odometry limitations by leveraging exteroceptive sensors in addition to proprioceptive ones
- A Kalman filter is a computationally efficient, optimal recursive data processing algorithm that allows fusion of multiple estimates coming from either process or sensing models
- A Kalman filter assumes linear motion and sensor models characterized by white Gaussian noise: many problems in robot localization do not fulfill these assumptions
- Particle filters allow for working around limitations of Kalman filters (in localization problems and beyond) at the price of additional complexity and computational cost

Additional Literature – Week 10

Pointers

http://www.probabilistic-robotics.org/

Books

- Siegwart R., Nourbakhsh I., and Scaramuzza D., “Introduction to Autonomous Mobile Robots, second Edition”, MIT Press, 2011.
- Thrun S., Burgard W., and Fox D., Probabilistic Robotics, MIT Press, 2005.
- Choset H., Lynch K. M., Hutchinson S., Kantor G., Burgard W., Kavraki L., and Thrun S., “Principles of Robot Motion”. MIT Press, 2005.
- Borenstein J., Everett H. R., and Feng L. “Navigating Mobile Robots: Systems and Techniques”, A. K. Peters, Ltd., 1996.