Signals, Instruments, and Systems – W9

An Introduction to Localization Systems and Methods

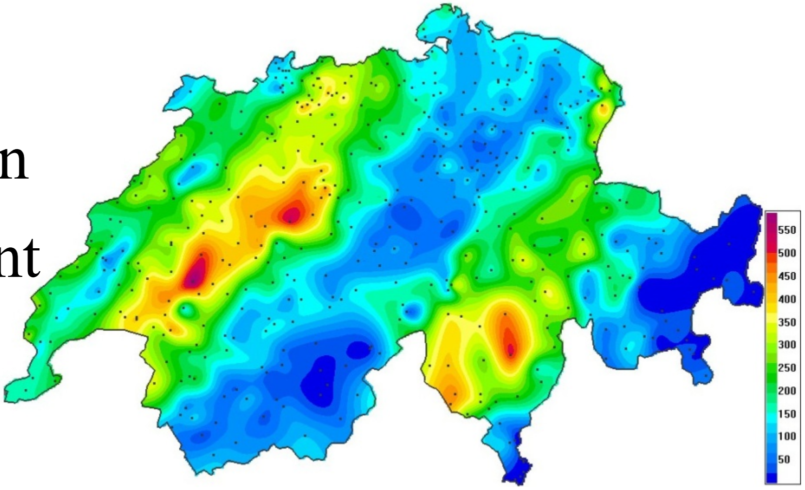
Outline

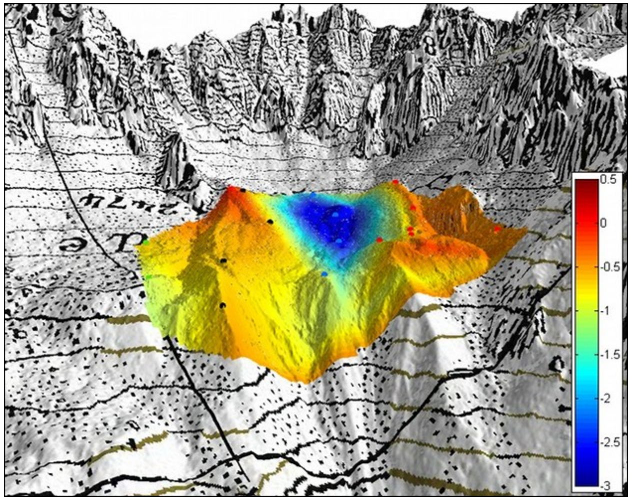
- Motivation
- Positioning systems
- Localization using proprioceptive sensors without uncertainties
- Localization using proprioceptive sensors with uncertainties

Motivation for Localization

Motivation

- Environmental data most often useless without location information
- Sophisticated localization equipment widely available
- → Ubiquity of localization information

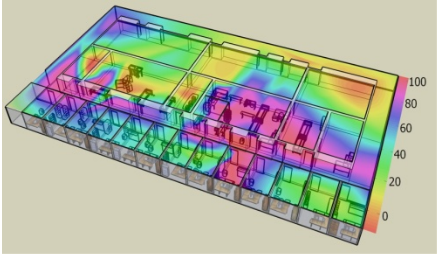


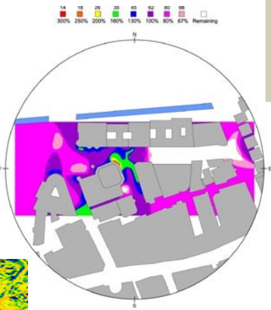


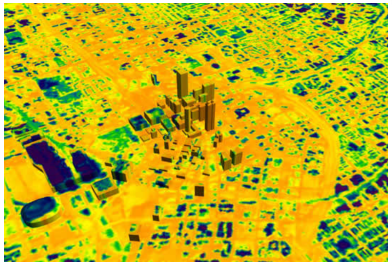
Picture: courtesy of Sensorscope

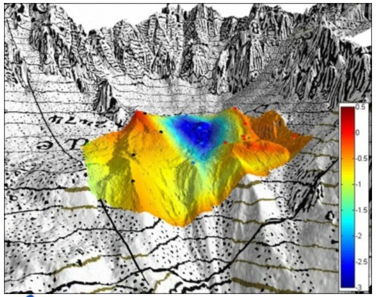
Motivation

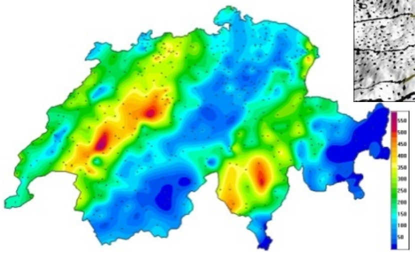
- Localization is required at different scales (1000's km → cm)
- Different environments
→ different methodologies











Pictures: courtesy of Sensorscope, NASA, NIST

Positioning Systems

Classification axes

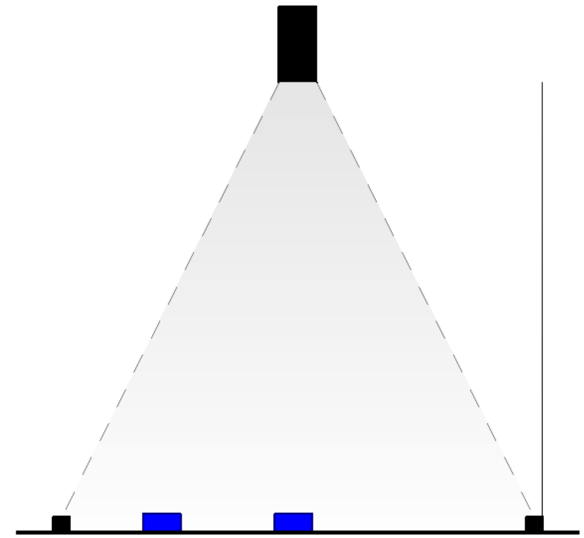
- Indoor vs. outdoor techniques
- Absolute vs. relative positioning systems
- Line-of-sight vs. non-line-of-sight
- Underlying physical principle and channel
- Positioning available on-board vs. off-board
- Scalability in terms of number of nodes

Selected Indoor Positioning Systems

- Overhead cameras and Motion Capture Systems (MCSs)
- Impulse Radio Ultra Wide Band (IR-UWB)
- Infrared (IR) + RF technology

2D Single- or Multi-Camera Systems

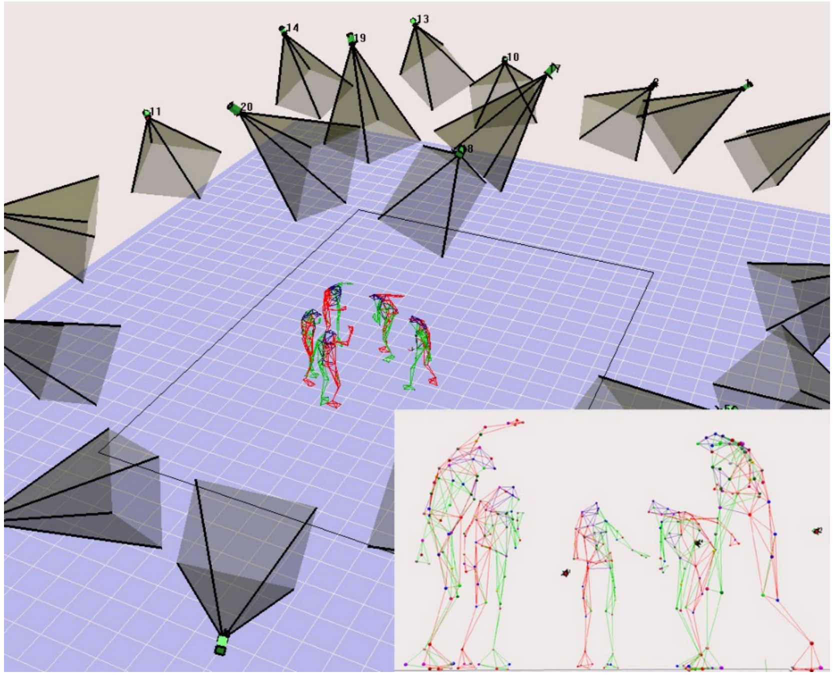
- Tracking objects with one (or more) overhead cameras
- Absolute positions/poses, available outside the robot/sensor
- Active, passive, or no markers
- Open-source software available (e.g., SwisTrack, developed at DISAL)
- Major issues: light, calibration



Performance 1 camera system	
Accuracy	~ 1 cm (2D)
Update rate	~ 20-100 Hz
# agents	~ 100
Area	~ 10 m^2

3D Multi-Camera Systems

- Called also Motion Capture System (MCS)
- 10-50 cameras
- mm accuracy
- Up to a few hundred Hz update, 2 ms latency
- 6D pose estimation of objects
- 4-5 passive markers per object to be tracked needed
- A few hundreds m^3 motion arena
- Open-source and markerless systems exist (but less reliable)



Coordinated ball (Prof. D'Andrea, ETHZ):
http://www.youtube.com/watch?v=hyGJBV1xnJI
Aggressive maneuver (Prof. Kumar, UPenn):
http://www.youtube.com/watch?v=geqip_0Vjec

IR-UWB System - Principles

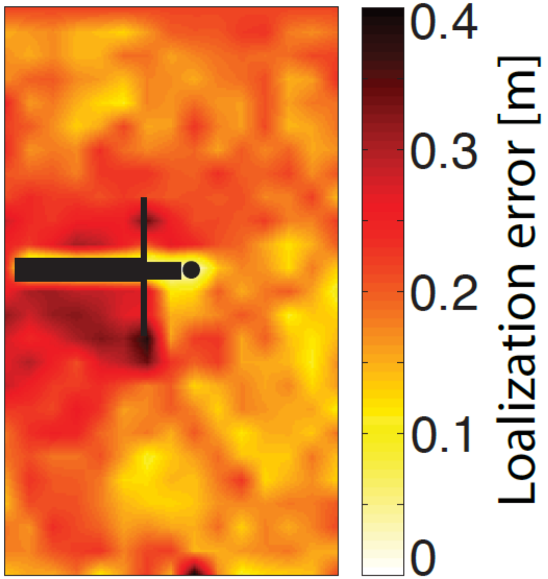
- Impulse Radio Ultra-Wide Band
- Based on time-of-flight (TDOA, Time Difference of Arrival)
- 6 - 8 GHz central frequency
- Very large bandwidth (>0.5GHz)
  → high material penetrability
- Fine time resolution
  → high theoretical ranging accuracy (order of cm)
- UWB tags (emitters, a few cm, low-power) and multiple synchronized receivers
- Emitters can be unsynchronized but then dealing with interferences not trivial (e.g., Ubisense system synchronized)
- Absolute positions available on the receiving system
- Positioning information can be fed back to tracked devices using a standard narrow-band channel
- Transceiver versions exist (e.g., Eliko system) thanks to progress in UWB chipsets

IR-UWB System – Performances

Ex. State-of-art system (e.g., Ubisense 7000 Series, Compact Tag)

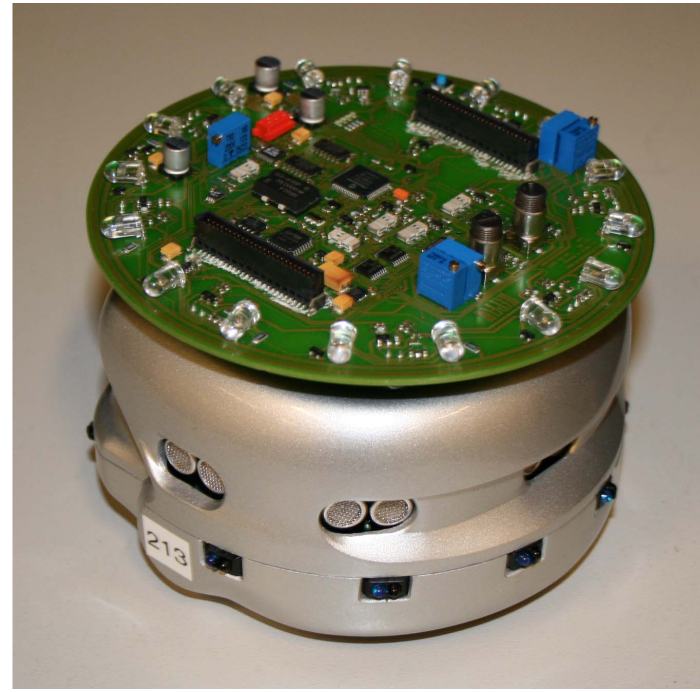
Accuracy	15 cm (3D)
Update rate	34 Hz / tag
# agents	~ 10000
Area	~ 1000 m^2

- Degraded accuracy performance if
  - Inter-emitter interferences
  - Non-Line-of-Sight (NLOS) bias
  - Multi-path



Infrared + Radio Technology

- Principle:
  - belt of IR emitters (LED) and receivers (photodiode)
  - IR LED used as antennas; modulated light (carrier 10.7 MHz), RF chip behind
  - Range: measurement of the Received Signal Strength Intensity (RSSI)
  - Bearing: signal correlation over multiple receivers
  - Measure range & bearing can be coupled with standard RF channel (e.g., 802.11) for heading assessment
  - Can also be used for 20 kbit/s IR com channel
  - Robot ID communicated with the IR channel (ad hoc protocol)



Infrared + Radio Technology

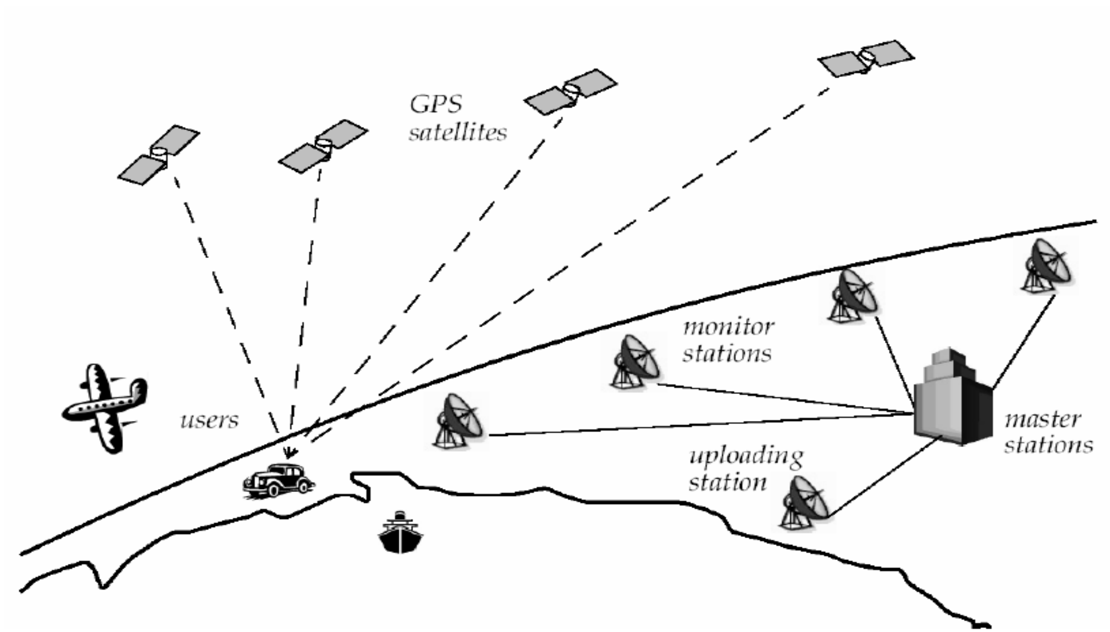
Performance summary:

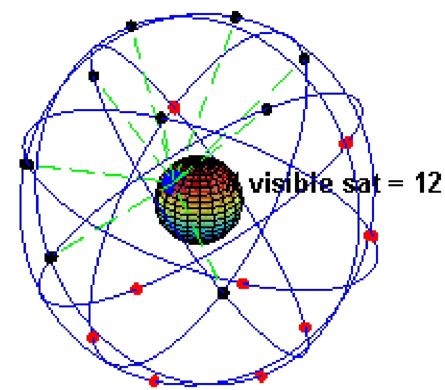
- Range: 3.5 m (extensible to a few m)
- Update frequency 25 Hz with 10 neighboring robots (or 250 Hz with 2); extensible to a few hundred Hz with TDMA schemes
- Accuracy range: <10%, generally decrease 1/d
- Accuracy bearing: < 10º
- LOS method
- Extension in 3D possible
- Larger range with more power consumption and dedicated optics; better bearing accuracy with more photodiodes

Selected Outdoor Positioning Techniques

- GPS
- Differential GPS (dGPS)

Global Positioning System





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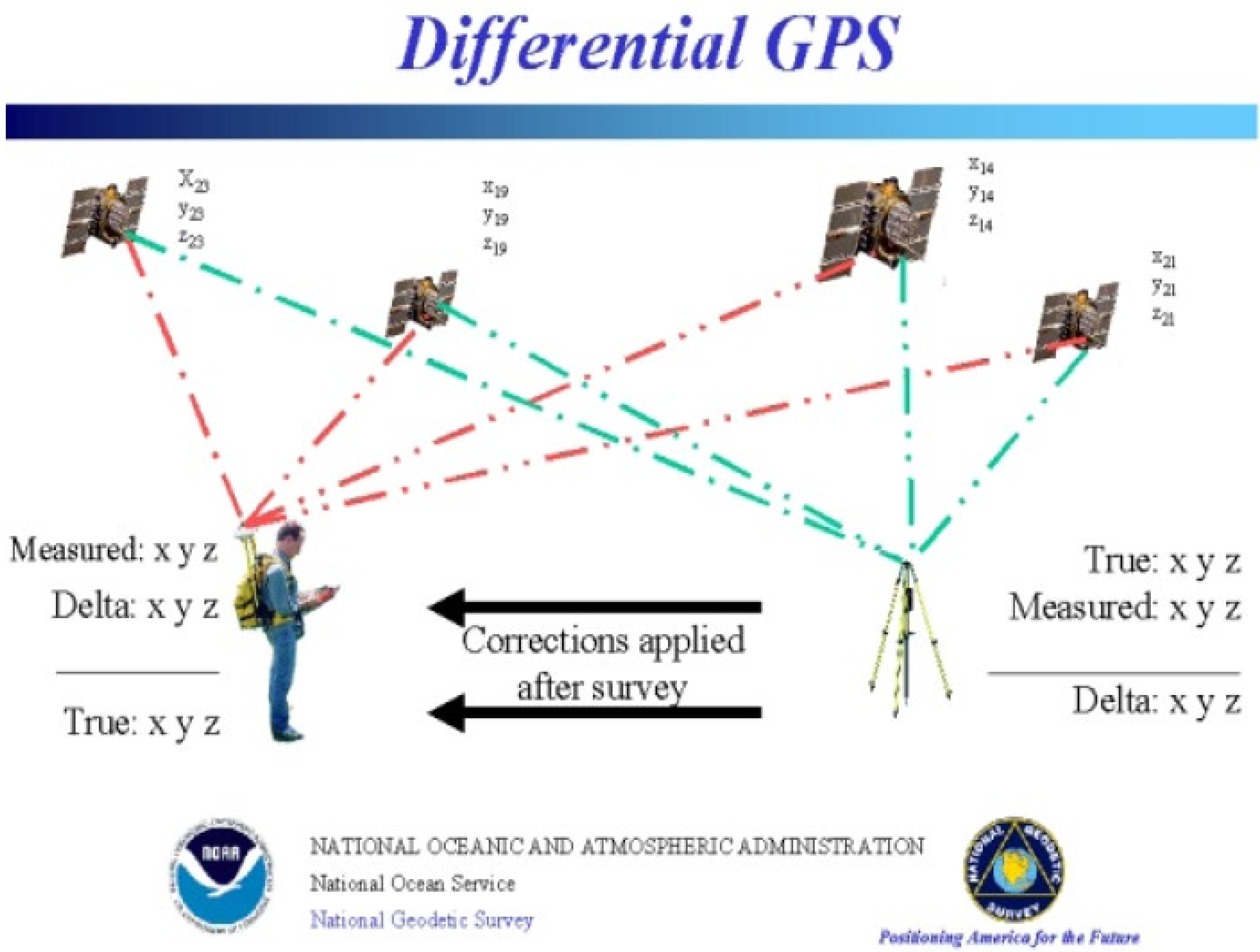
Note: the first and still most prominent example of GNSS systems (Global Navigation Satellite Systems)

Global Positioning System

- Initially 24 satellites (including three spares), 32 as of December 2012, orbiting the earth every 12 hours at a height of 20.190 km.
- Satellites synchronize their transmission (location + time stamp) so that signals are broadcasted at the same time (ground stations updating + atomic clocks on satellites)
- Real time update of the exact location of the satellites:
  - monitoring the satellites from a number of widely distributed ground stations
  - a master station analyses all the measurements and transmits the actual position to each of the satellites
- Location of any GPS receiver is determined through a time of flight measurement (*ns* accuracy!)
- Exact measurement of the time of flight
  - the receiver correlates a pseudocode with the same code coming from the satellite
  - the delay time for best correlation represents the time of flight.
  - quartz clock on the GPS receivers are not very precise
  - the range measurement with (at least) four satellites allows to identify the three values (x, y, z) for the position and the clock correction ΔT
- Recent commercial GPS receiver devices allows position accuracies down to a few meters with best satellite visibility conditions.
- 200-300 ms latency, so max 5 Hz GPS updates

dGPS



Differential GPS
X23
y23
z23
x19
y19
z19
x14
y14
z14
x21
y21
z21
Measured: x y z
Delta: x y z
True: x y z
Corrections applied
after survey
True: x y z
Measured: x y z
Delta: x y z
NATIONAL OCEANIC AND ATMOSPHERIC ADMINISTRATION
National Ocean Service
National Geodetic Survey
Positioning America for the Future

Odometry

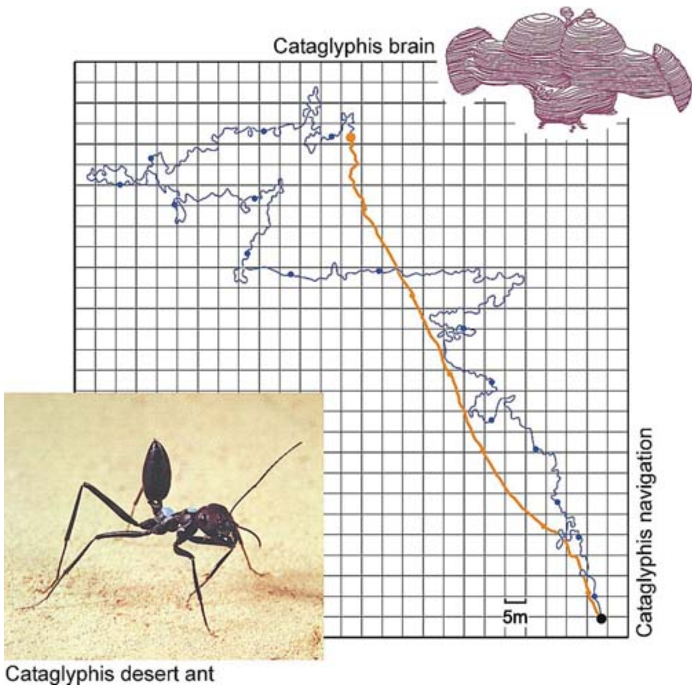
Definition

"Using proprioceptive sensory data influenced by the movement of actuators to estimate change in pose over time"

- Idea: navigating a room with the light turned off
- Start: initial position
- Actuators:
  - Legs
  - Wheels
  - Propeller
- Sensors (proprioceptive):
  - Wheel encoders (DC motors), step counters (stepper motors)
  - Inertial measurement units, accelerometers
  - Nervous systems, neural chains

Example of Navigation Heavily Leveraging Odometry

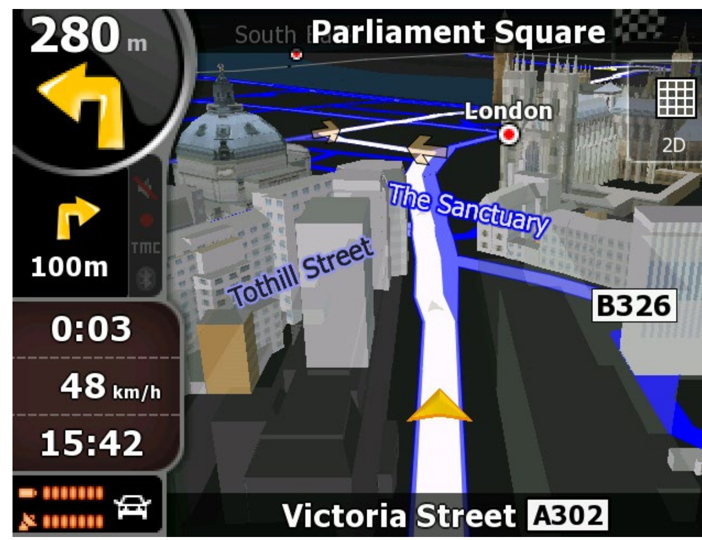
- Example: Cataglyphis desert ant
- Excellent study by Prof. R. Wehner (University of Zuerich, Emeritus)
- Individual foraging strategy
- Underlying mechanisms
  - Dead-reckoning (path integration on neural chains for leg control)
  - Internal compass (polarization of sun light)
  - Local search (around 1-2 m from the nest)
- Extremely accurate navigation: averaged error of a few tens of cm over 500 m path!



More examples

- Human in the dark
  - Very **bad** odometry sensors
  - $d_{Odometry} = O(1/m)$
- (Nuclear) Submarine
  - Very **good** odometry sensors
  - $d_{Odometry} = O(1/10^3\ km)$
- Navigation system in tunnel uses dead reckoning based on
  - Last velocity as measured by GPS
  - Car's odometer, compass

Picture: Courtesy of US Navy

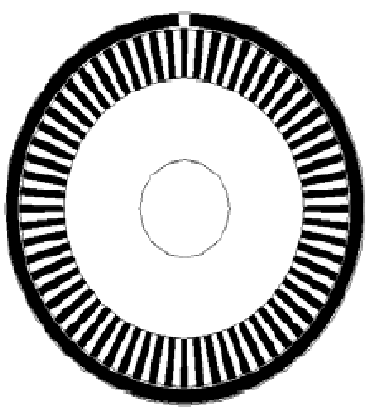


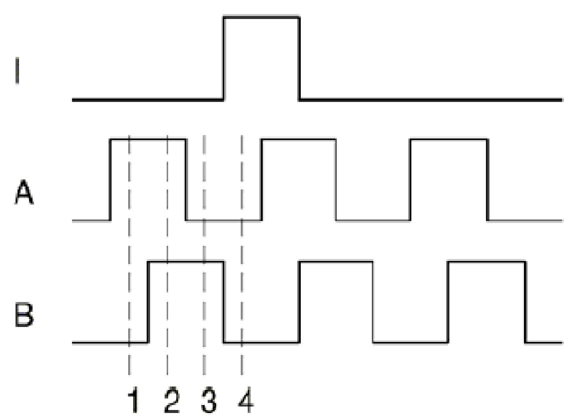
Picture: Courtesy of NavNGo

Odometry using Wheel Encoders or Step Counters

Optical Encoders

- Measure displacement (or speed) of the wheels
- Principle: mechanical light chopper consisting of photo-barriers (pair of light emitter and optical receiver) + pattern on a disc anchored to the motor shaft
- Quadrature encoder: 90º placement of 2 complete photo-barriers, 4x increase resolution + direction of movement
- Integrate wheel movements to get an estimate of the position -> odometry
- Typical resolutions: 64 - 4096 increments per revolution.
- Note: the e-puck is not endowed with wheel encoders but step counters for the stepper motors





State	Ch A	Ch B
S_1	High	Low
S_2	High	High
S_3	Low	High
S_4	Low	Low

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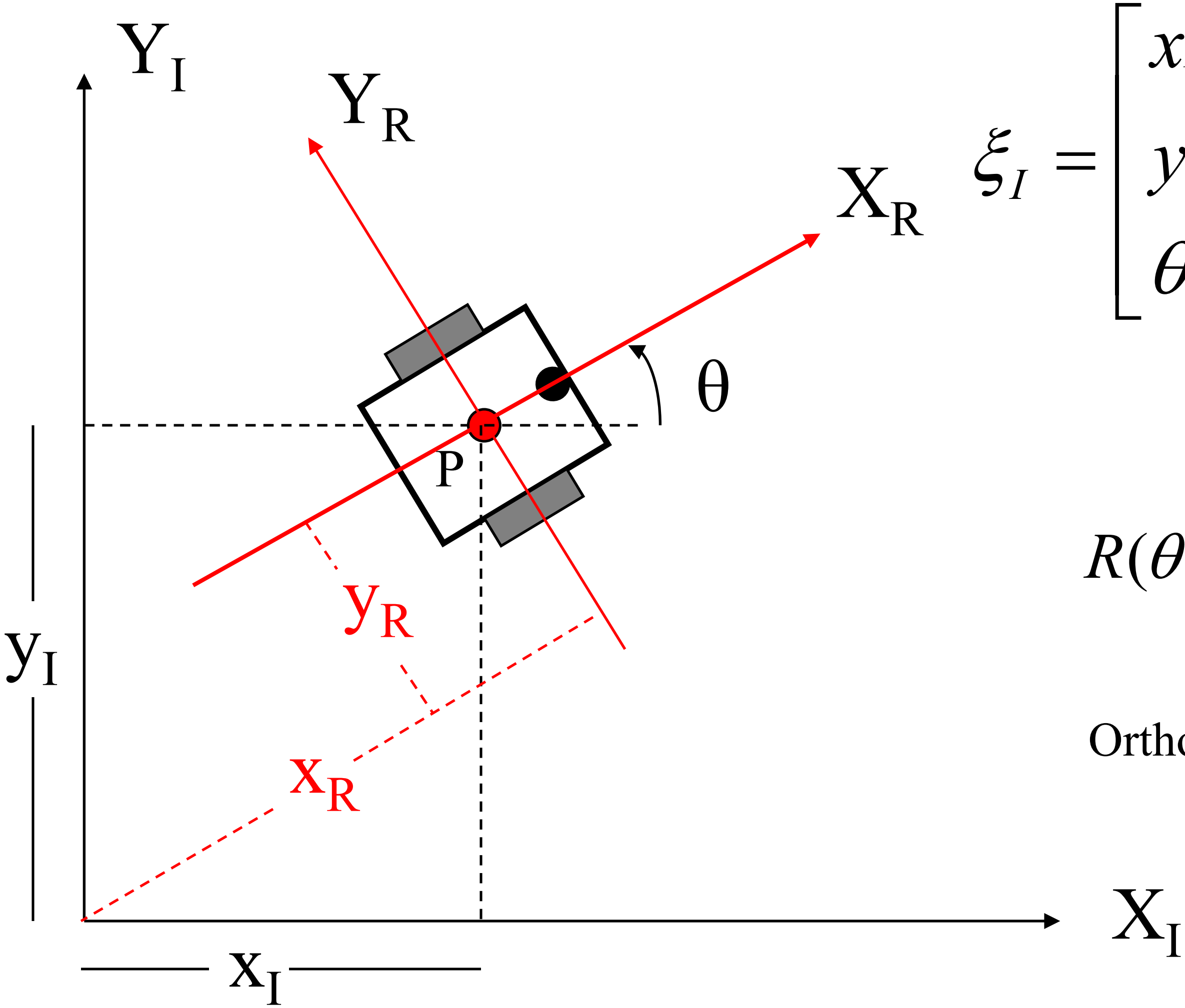
Pose (Position and Orientation) of a Differential-Drive Robot



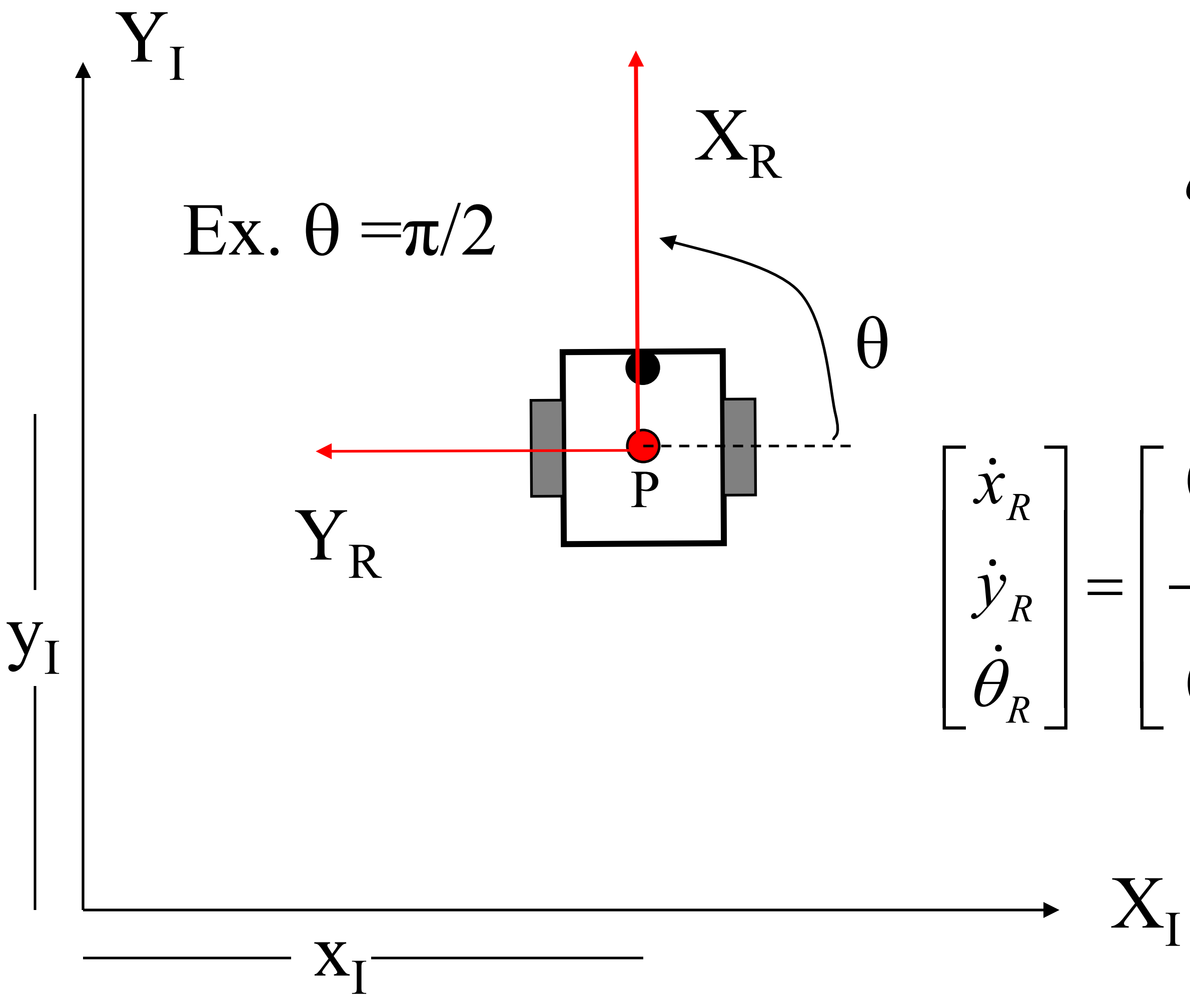
$$\xi_I = \begin{bmatrix} x_I \\ y_I \\ \theta \end{bmatrix} \quad \xi_R = \begin{bmatrix} x_R \\ y_R \\ \theta \end{bmatrix} = R(\theta)\xi_I$$

$$R(\theta) = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Orthogonal Rotation Matrix

From *Introduction to Autonomous Mobile Robots*, Siegwart R. and Nourbakhsh I. R.

Absolute and Relative Motion of a Differential-Drive Robot



$$\dot{\xi}_R = R(\theta)\dot{\xi}_I$$

$$\begin{bmatrix} \dot{x}_R \\ \dot{y}_R \\ \dot{\theta}_R \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_I \\ \dot{y}_I \\ \dot{\theta}_I \end{bmatrix} = \begin{bmatrix} \dot{y}_I \\ -\dot{x}_I \\ \dot{\theta}_I \end{bmatrix}$$

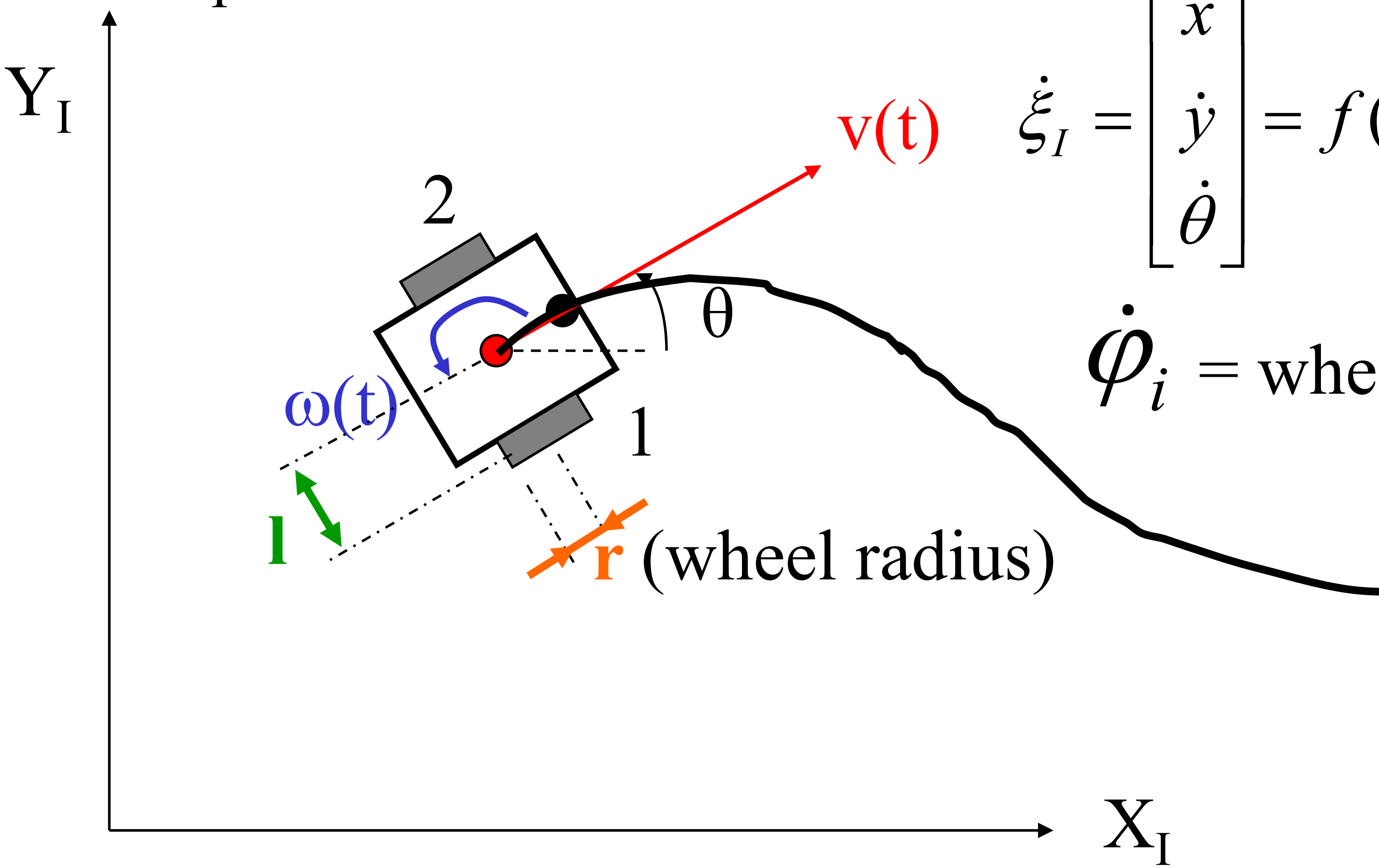
Forward Kinematic Model

How does the robot move given the wheel speeds and geometry?

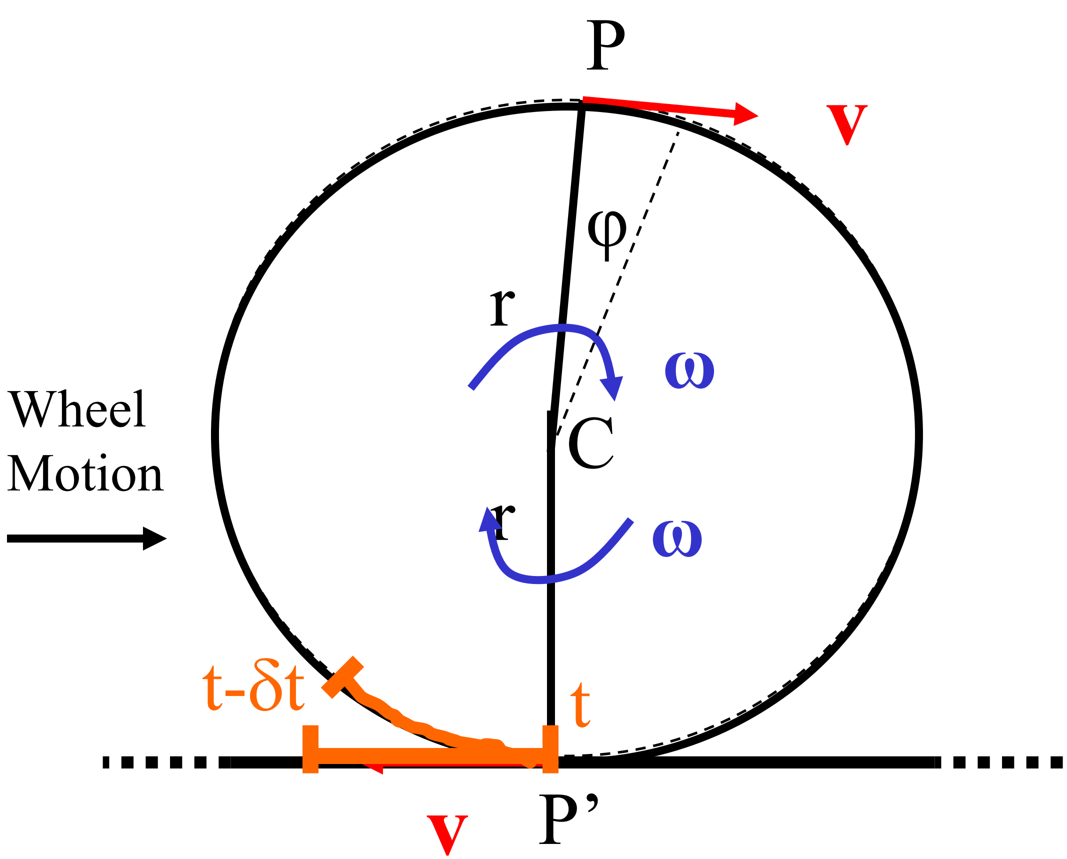
- Assumption: no wheel slip (rolling mode only)!
- In miniature robots no major dynamic effects due to low mass-to-power ratio

$$\dot{\xi}_I = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = f(l, r, \theta, \dot{\varphi}_1, \dot{\varphi}_2)$$

$$\dot{\varphi}_i = \text{wheel i speed}$$



Recap ME/PHY Fundamentals



$$v = \omega r = \dot{\varphi} r$$

v = tangential speed
ω = rotational speed
r = rotation radius
φ = rotation angle
C = rotation center
P = peripheral point

P'= contact point at time t

Rolling!

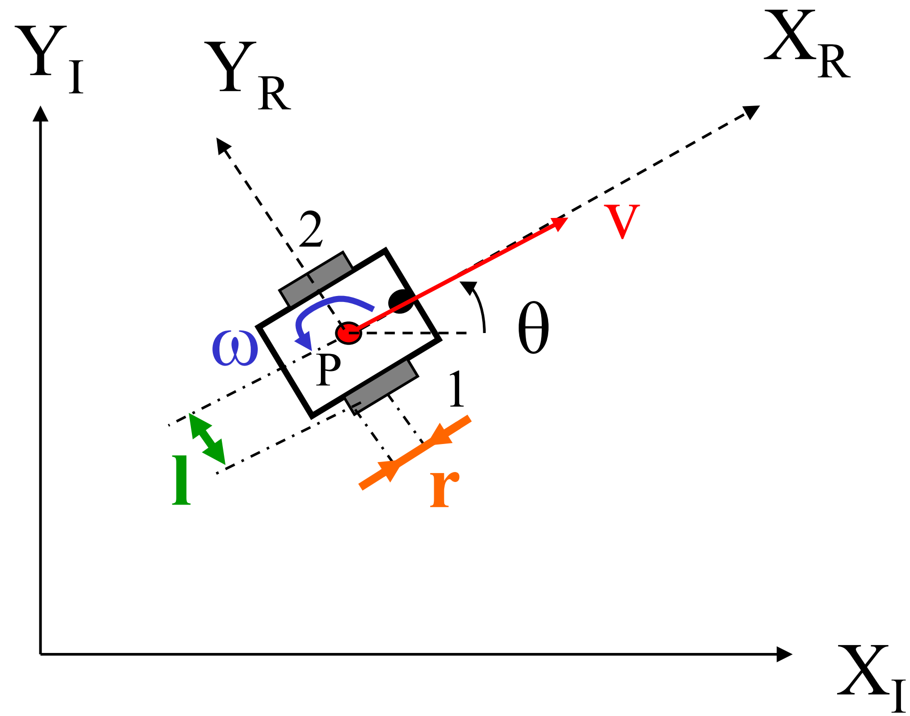
Forward Kinematic Model

Linear speed = average wheel speed 1 and 2:

$$v = \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2}$$

Rotational speed = sum of rotation speeds (wheel 1 forward speed -> ω anti-clockwise, wheel 2 forward speed ω clockwise):

$$\omega = \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l}$$



Idea: linear superposition of individual wheel contributions

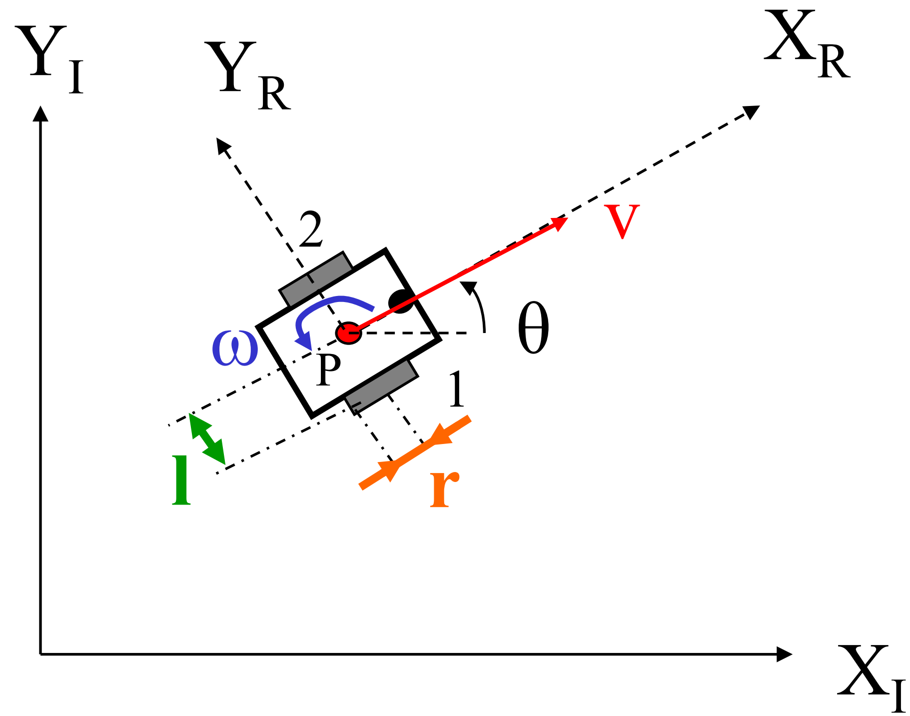
Forward Kinematic Model

1. $\dot{x}_R = v = \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2}$

2. $\dot{y}_R = 0$

3. $\dot{\theta}_R = \omega = \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l}$

4. $\dot{\xi}_I = R^{-1}(\theta)\dot{\xi}_R$



$$\dot{\xi}_I = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

Odometry

- Given our absolute pose over time, we can calculate the robot pose after some time *t* through integration
- Given the kinematic forward model, and assuming no slip on both wheels

$$\xi_I(T) = \xi_{I0} + \int_0^T \dot{\xi}_I dt = \xi_{I0} + \int_0^T R^{-1}(\theta)\dot{\xi}_R dt$$

- Given an initial pose ξ_{I0}, after time T, the pose of the vehicle will be $\xi_I(T)$
- $\xi_I(T)$ computable with wheel speed 1, wheel speed 2, and parameters r and l

Wheel-Based Odometry in Practice

From Model to Practice

From s. 30:

$$\dot{\xi}_I = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r\dot{\varphi}_1}{2} + \frac{r\dot{\varphi}_2}{2} \\ 0 \\ \frac{r\dot{\varphi}_1}{2l} + \frac{-r\dot{\varphi}_2}{2l} \end{bmatrix}$$

$v_r = r\dot{\varphi}_1$ (right wheel speed)

$v_l = r\dot{\varphi}_2$ (left wheel speed)

$b = 2l$ (inter-wheel distance)

With x, y, and θ in the inertial frame:

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{v_r + v_l}{2} \\ 0 \\ \frac{v_r - v_l}{b} \end{bmatrix}$$

From Model to Practice

$$\begin{cases} \dot{x} = \frac{v_r+v_l}{2}\cos\theta \\ \dot{y} = \frac{v_r+v_l}{2}\sin\theta \\ \dot{\theta} = \frac{v_r - v_l}{b} \end{cases}$$

- Assume small time interval Δt
- Assume v_r and v_l constant in time interval Δt
- Transform differential in difference equations and approximate over Δt
- $\theta = \theta(\mathrm{t})$ -> rotational matrix in the middle of interval Δt -> $\tilde{\theta} = \theta + \Delta\theta/2$

$$\begin{cases} \frac{\Delta x}{\Delta t} = \frac{v_r+v_l}{2}\cos\tilde{\theta} \\ \frac{\Delta y}{\Delta t} = \frac{v_r+v_l}{2}\sin\tilde{\theta} \\ \frac{\Delta\theta}{\Delta t} = \frac{v_r - v_l}{b} \end{cases}$$

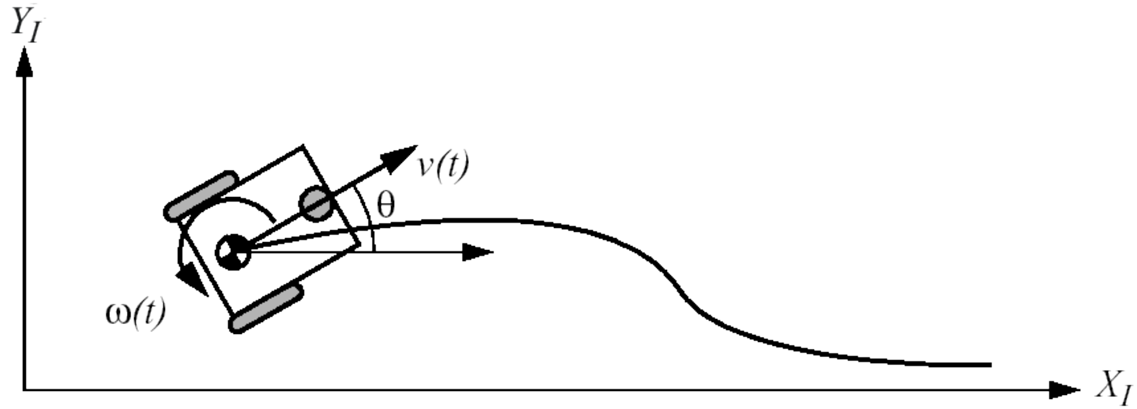
$$\begin{cases} \Delta x = \frac{v_r\Delta t+v_l\Delta t}{2}\cos\tilde{\theta} = \frac{\Delta s_r+\Delta s_l}{2}\cos\tilde{\theta} \\ \Delta y = \frac{v_r\Delta t+v_l\Delta t}{2}\sin\tilde{\theta} = \frac{\Delta s_r+\Delta s_l}{2}\sin\tilde{\theta} \\ \Delta\theta = \frac{v_r\Delta t-v_l\Delta t}{b} = \frac{\Delta s_r-\Delta s_l}{b} \end{cases}$$

Pose Variation During Δt

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

$$\begin{cases} \Delta x = \Delta s\cos(\theta + \frac{\Delta\theta}{2}) \\ \Delta y = \Delta s\sin(\theta + \frac{\Delta\theta}{2}) \\ \Delta\theta = \frac{\Delta s_r - \Delta s_l}{b} \end{cases}$$

$$p = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \xrightarrow{t'=t+\Delta t} p' = \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix}$$



b = inter-wheel distance
Δs_r = traveled distance right wheel
Δs_l = traveled distance left wheel
Δθ = orientation change of the vehicle

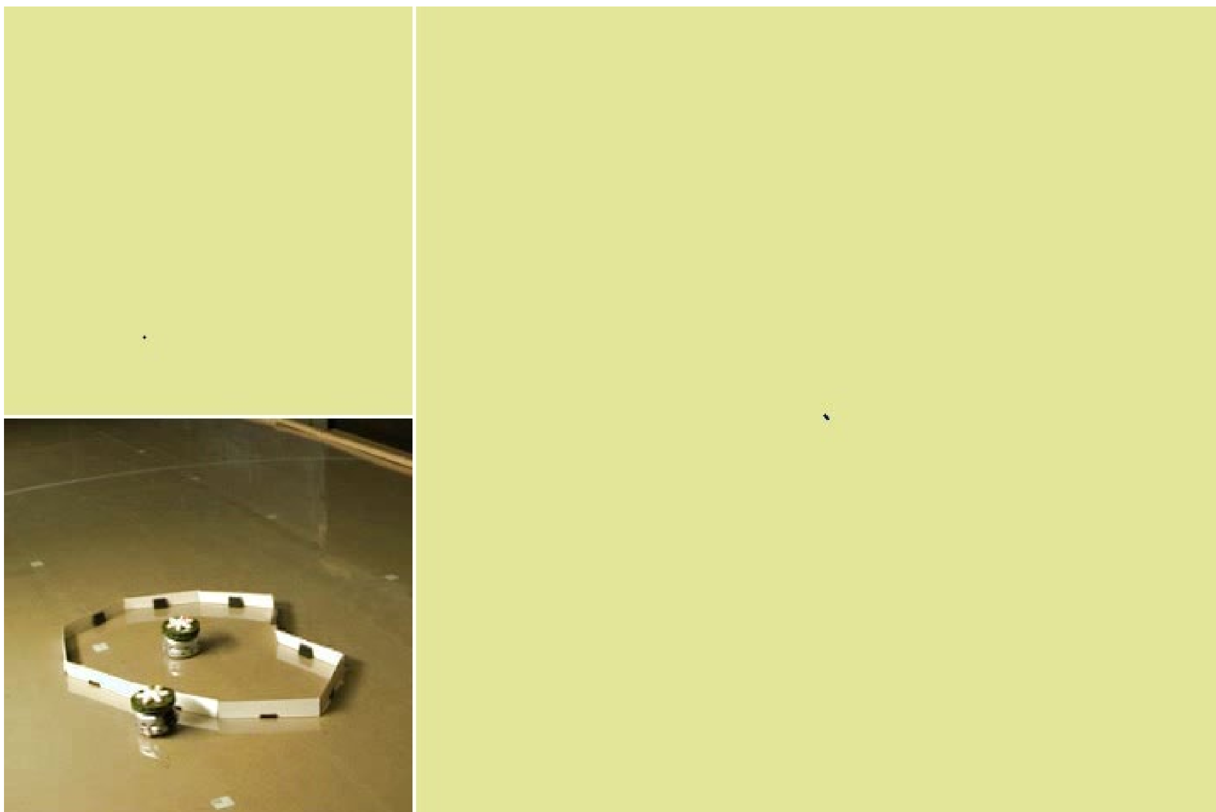
$$p' = f(x, y, \theta, \Delta s_r, \Delta s_l, b) = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta s\cos(\theta + \Delta\theta/2) \\ \Delta s\sin(\theta + \Delta\theta/2) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2}\cos(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r + \Delta s_l}{2}\sin(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

Localization Uncertainities in Odometry

Deterministic Error Sources

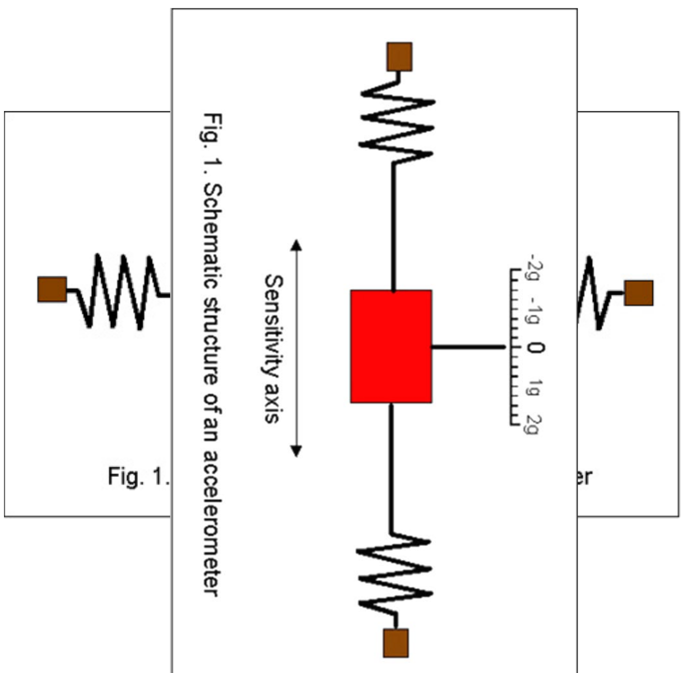
- Limited encoder resolution
- Wheel misalignment and small differences in wheel diameter

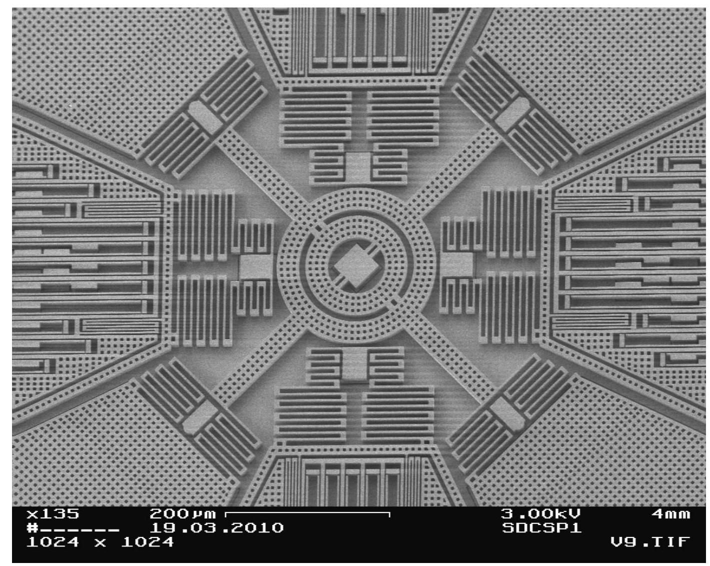
➢ Can be fixed by calibration



Non-Deterministic Error Sources

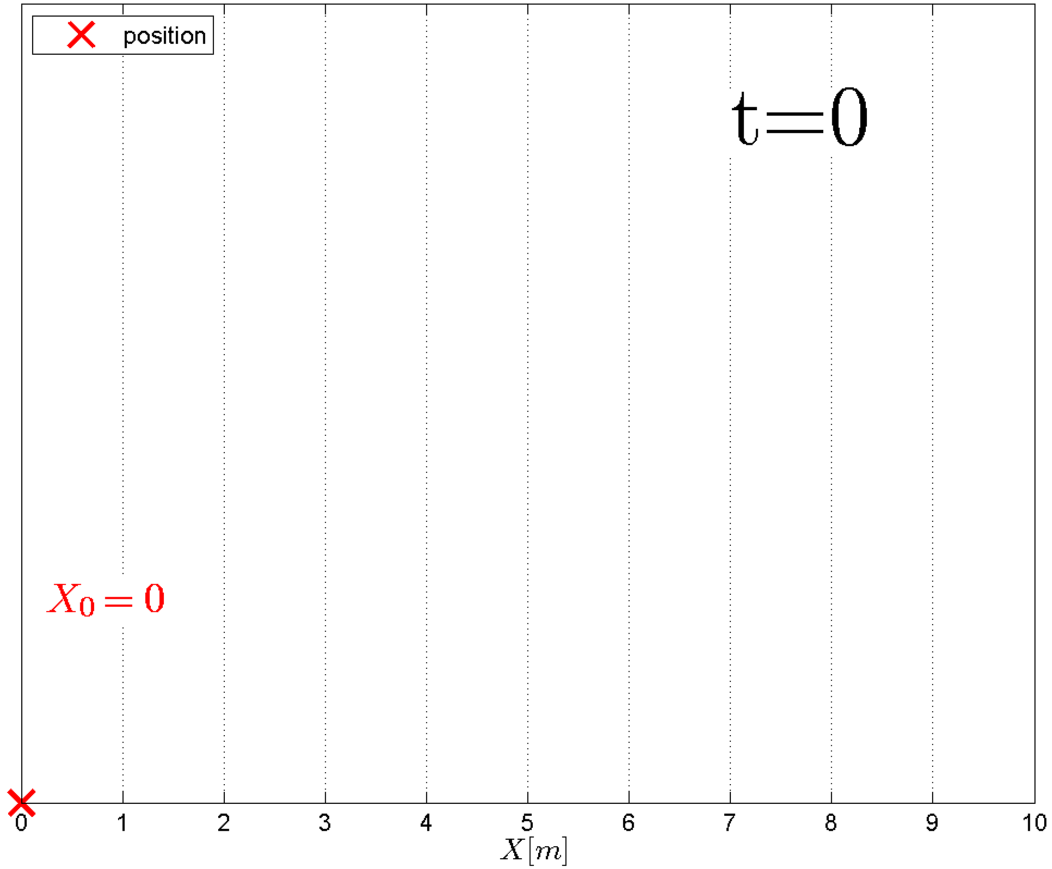
- From Week 5: no deterministic prediction possible → we have to describe them probabilistically
- Example: accelerometer-based odometry

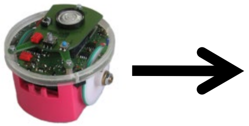




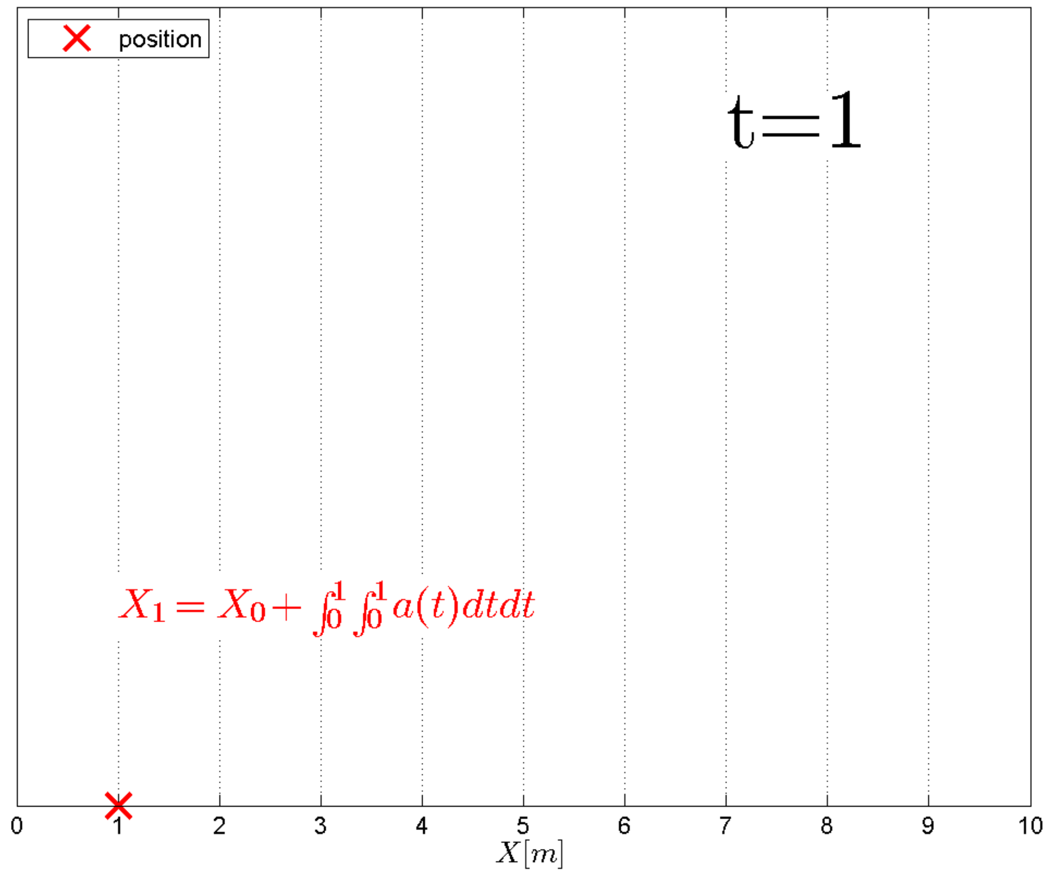
MEMS-Based accelerometer (e.g., on e-puck)

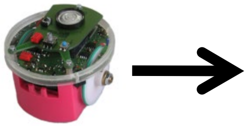
Odometry in 1D



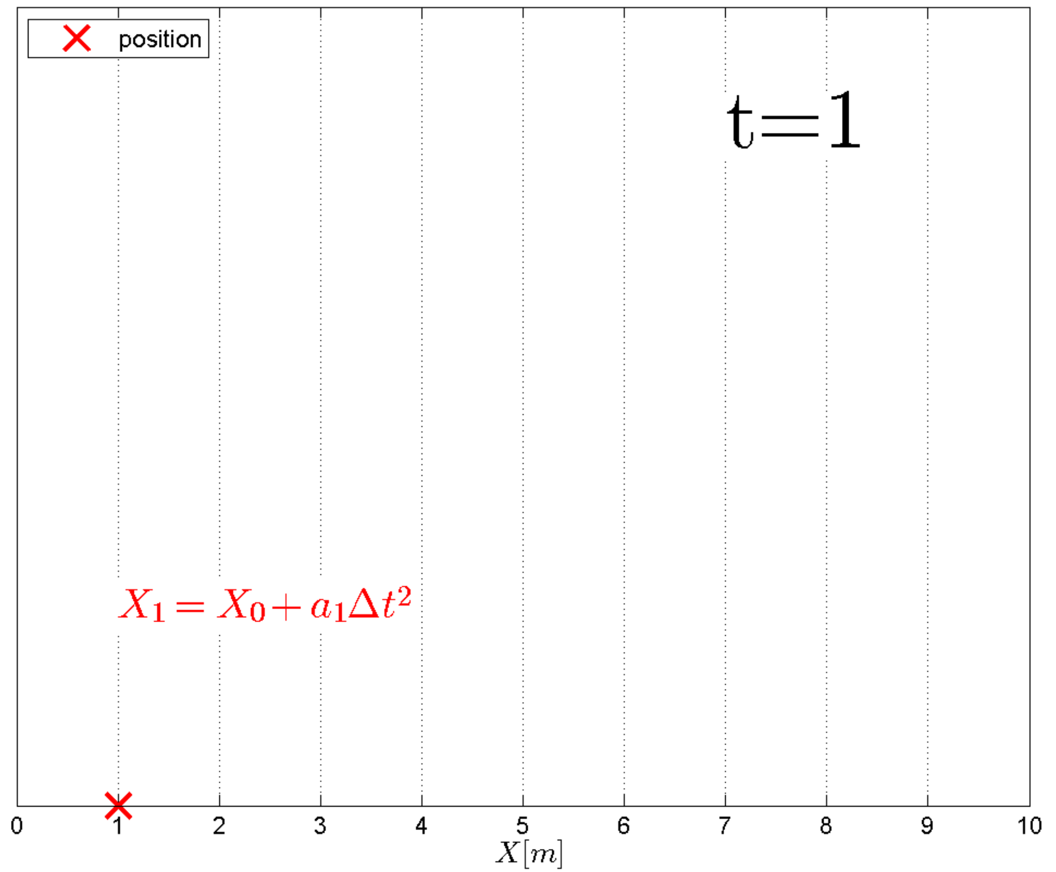


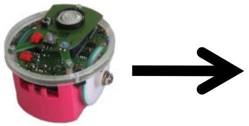
Odometry in 1D



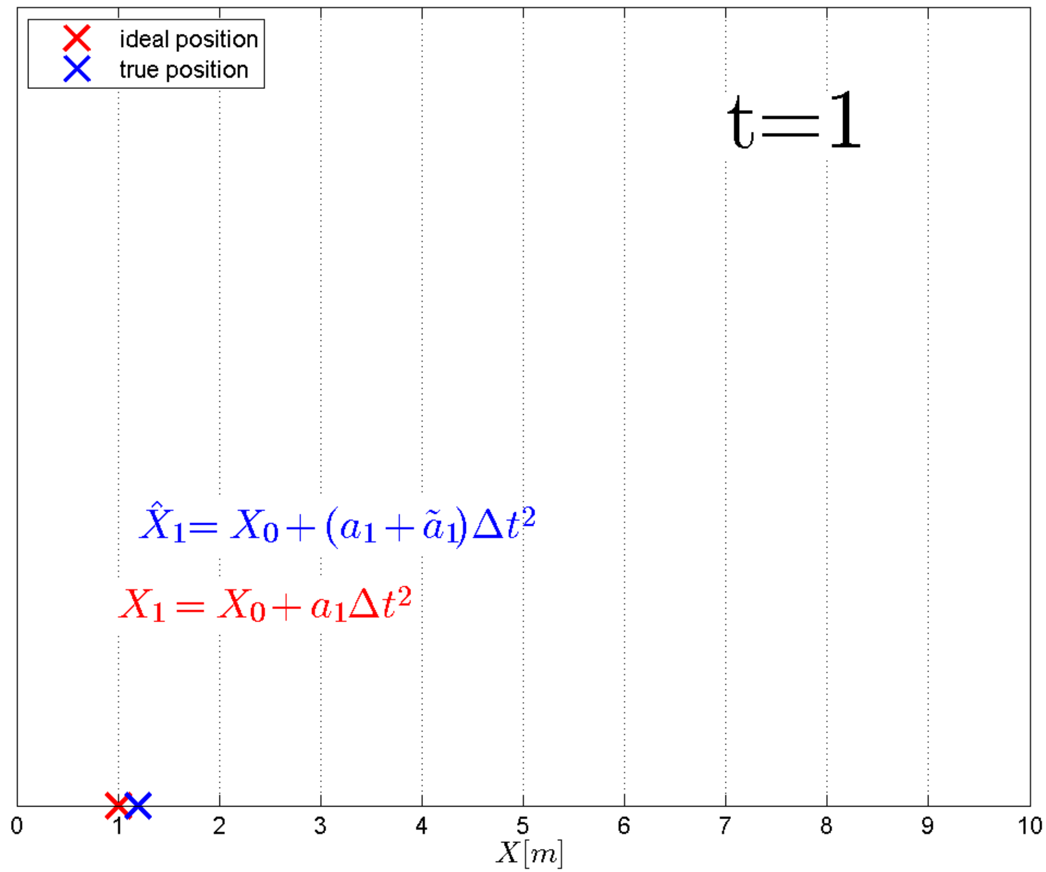


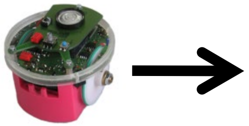
Odometry in 1D



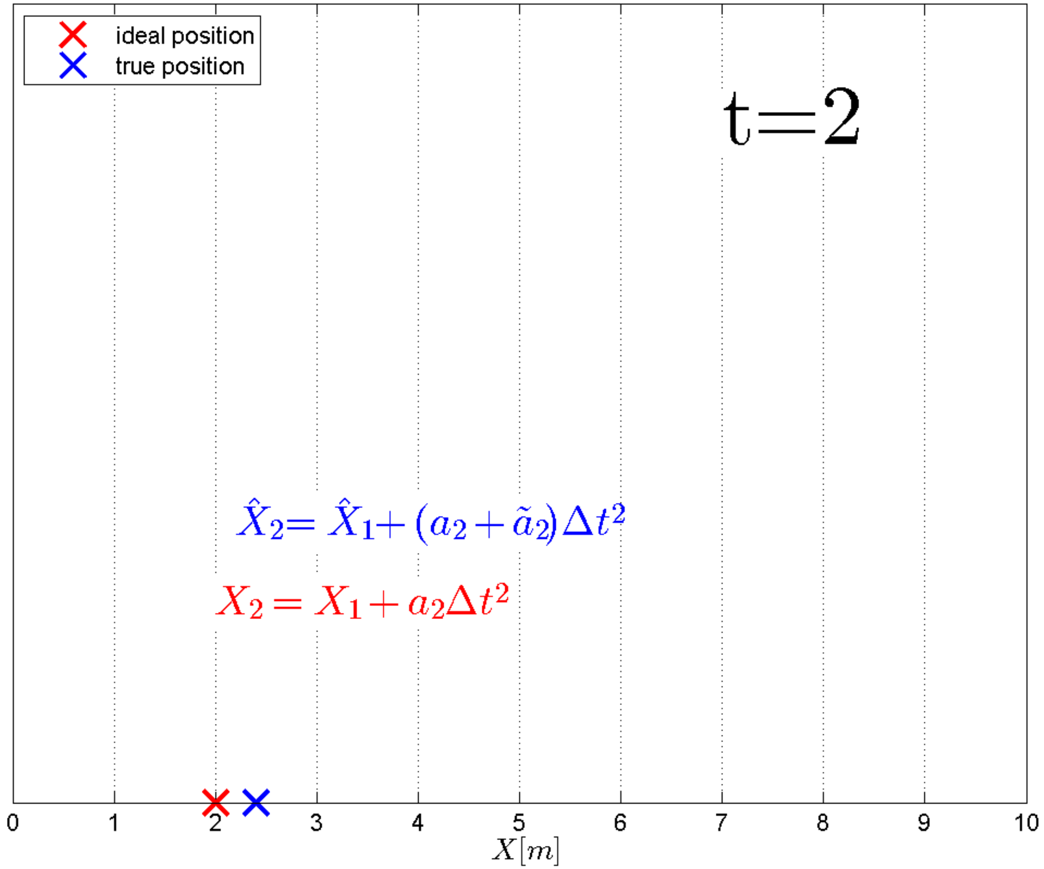


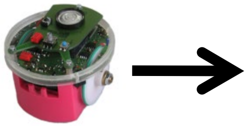
Odometry in 1D



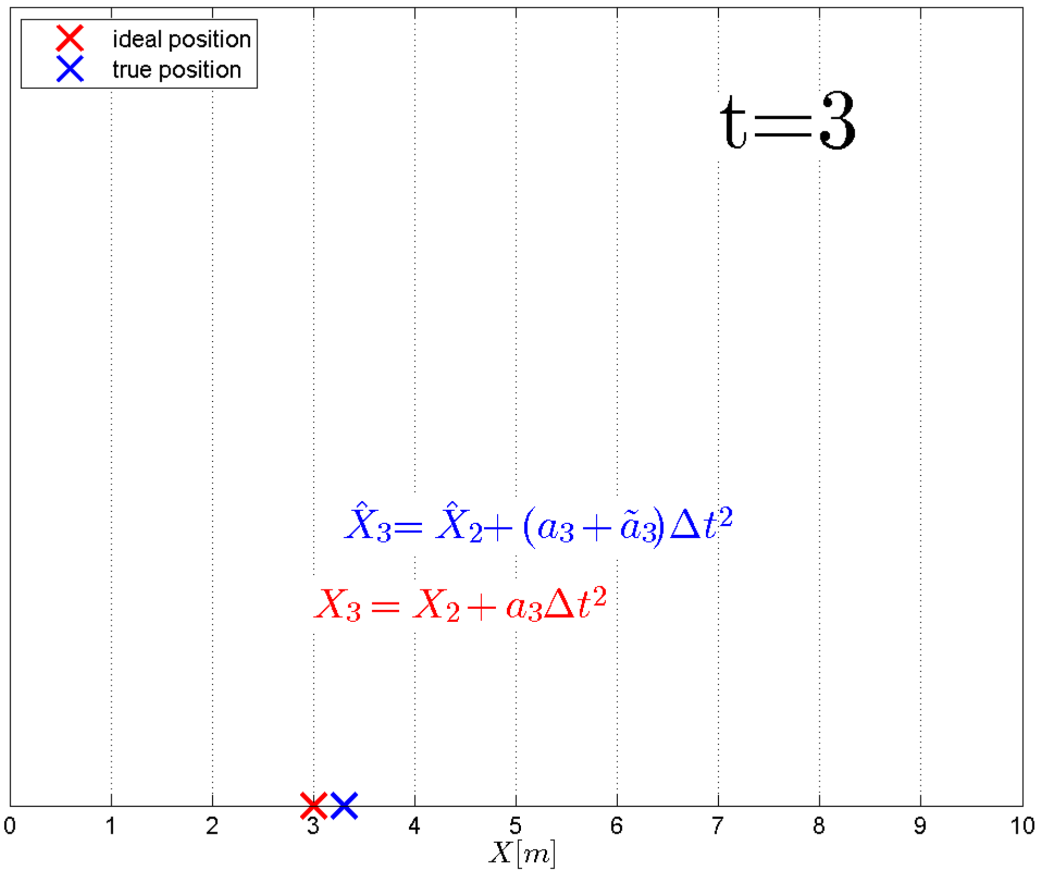


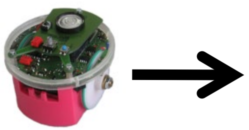
Odometry in 1D



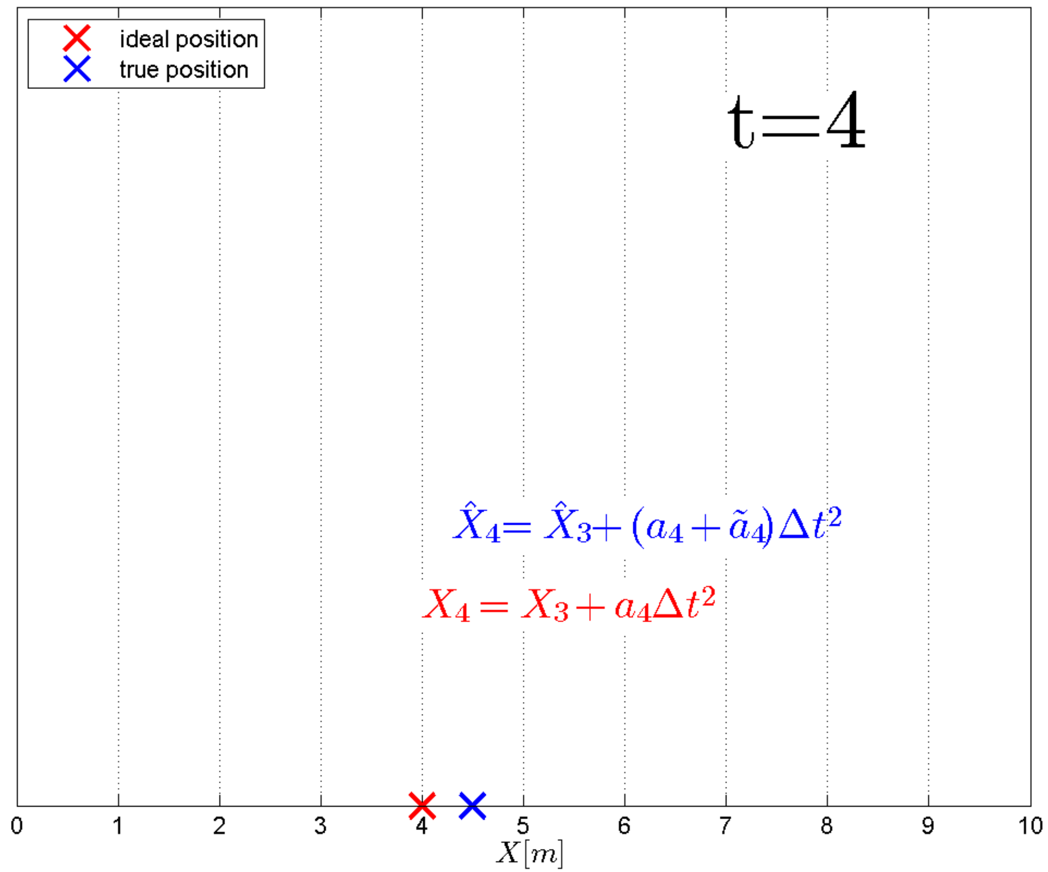


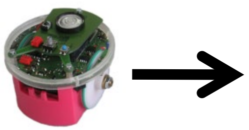
Odometry in 1D



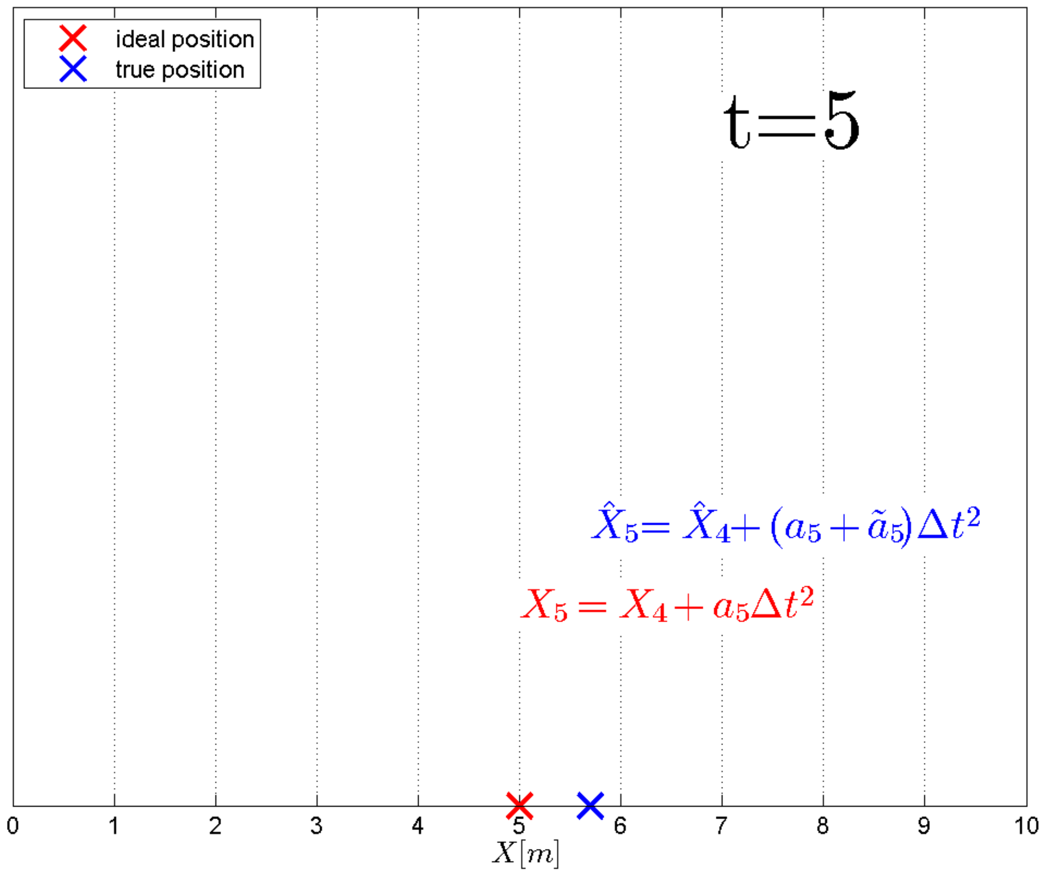


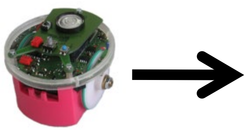
Odometry in 1D





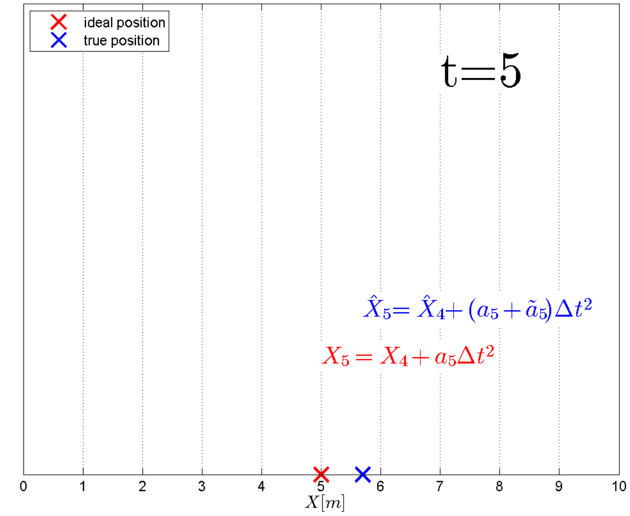
Odometry in 1D

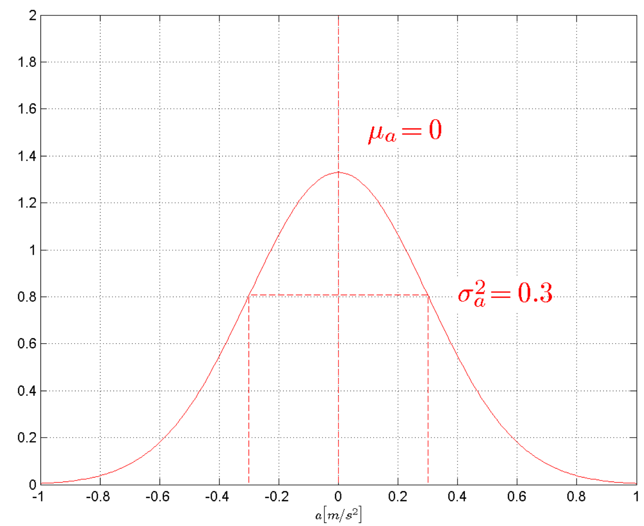




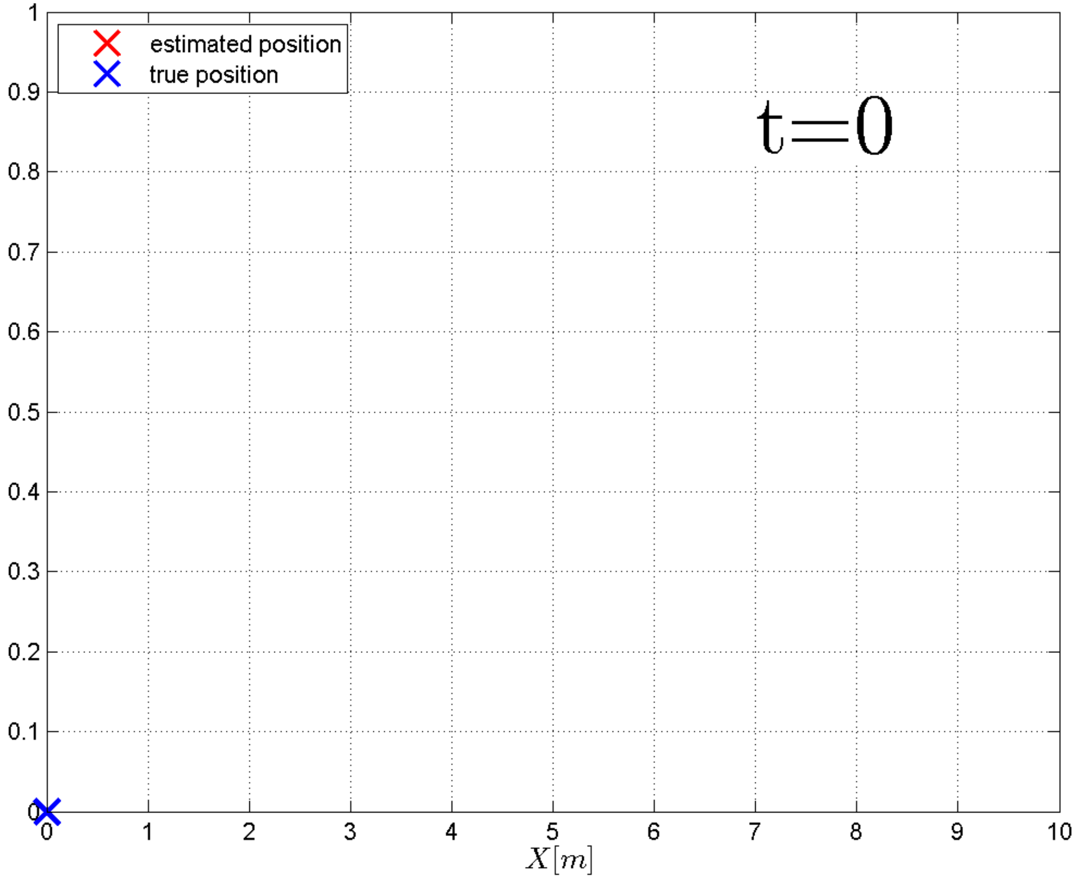
1D Odometry: Error Modeling

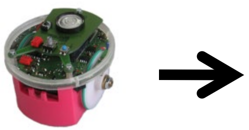
- Error happens!
- Odometry error is cumulative.
  → grows without bound
- We need to be aware of it.
  → We need to model odometry error.
  → We need to model sensor error.
- Multiple independent source of errors with arbitrary distribution combined → Central Limit Theorem → Gaussian assumption reasonable
- Acceleration is random variable A drawn from "mean-free" Gaussian ("Normal") distribution.
  → Position X is random variable with Gaussian distribution.



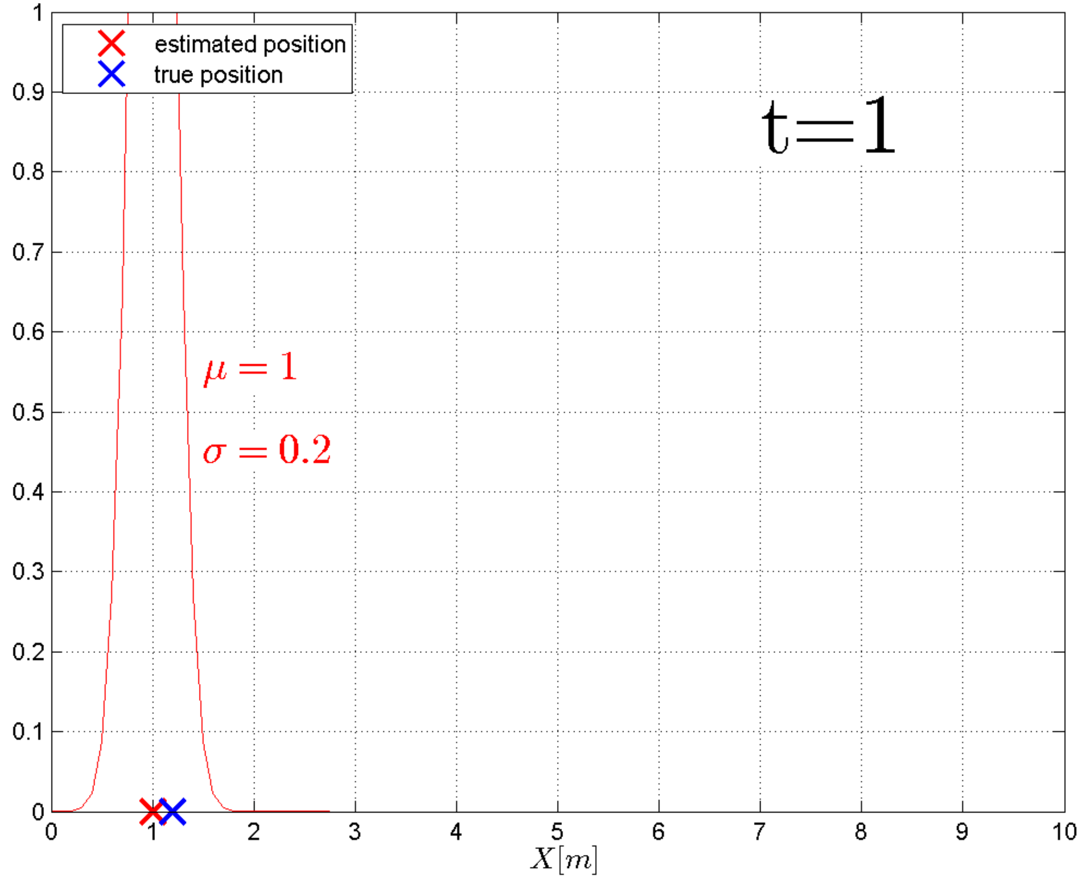


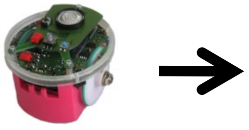
1D Odometry with Gaussian Uncertainty



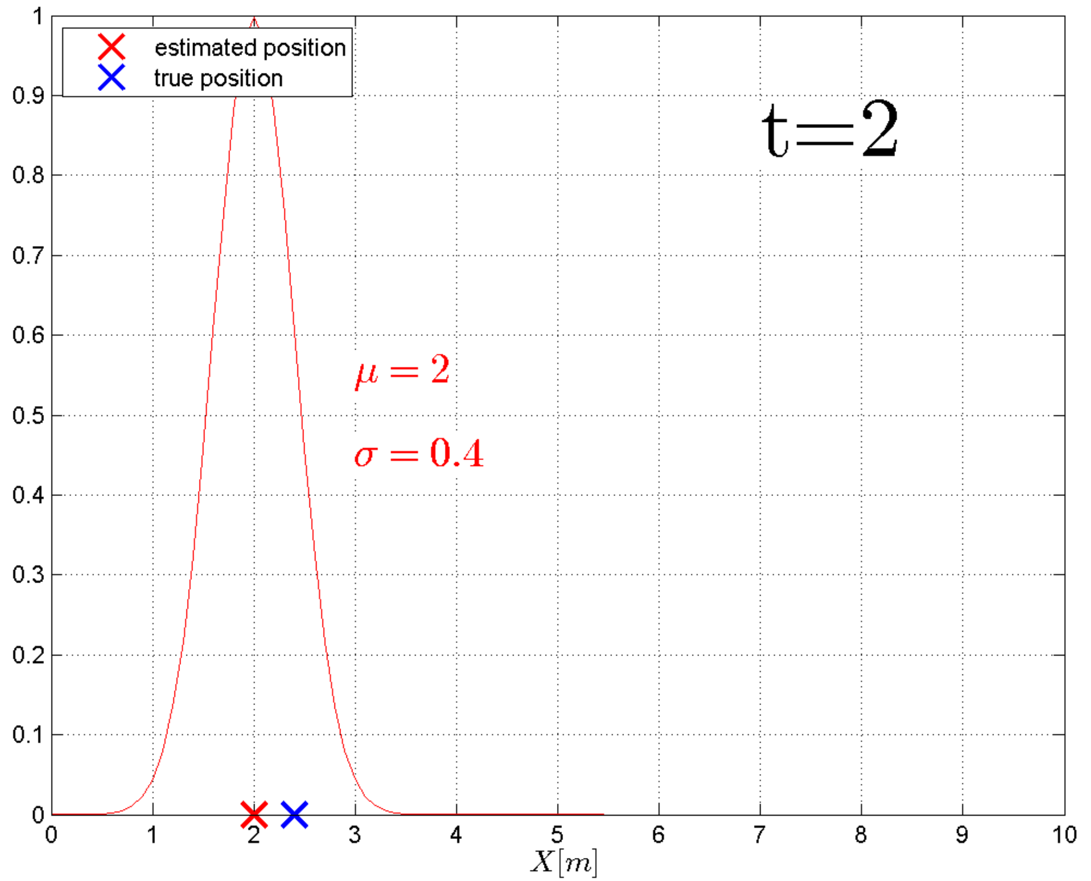


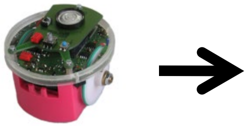
1D Odometry with Gaussian Uncertainty



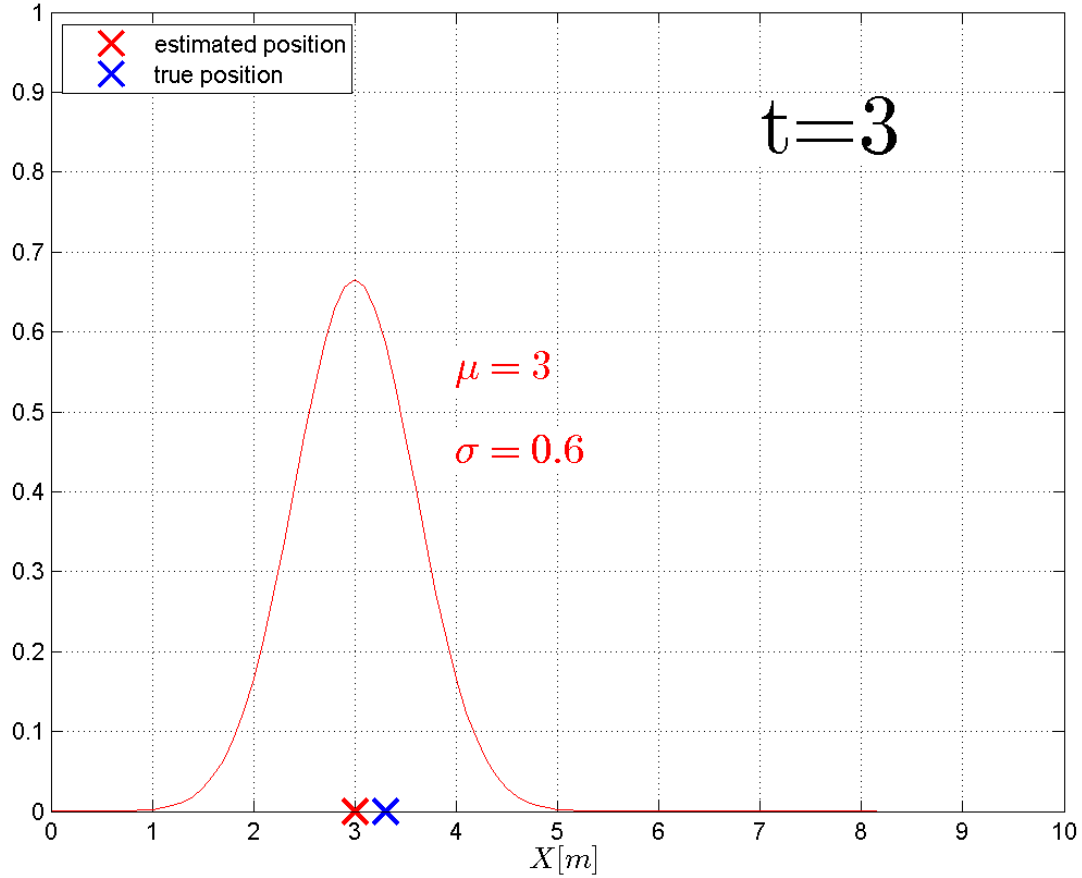


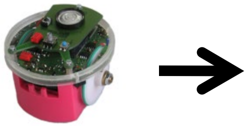
1D Odometry with Gaussian Uncertainty



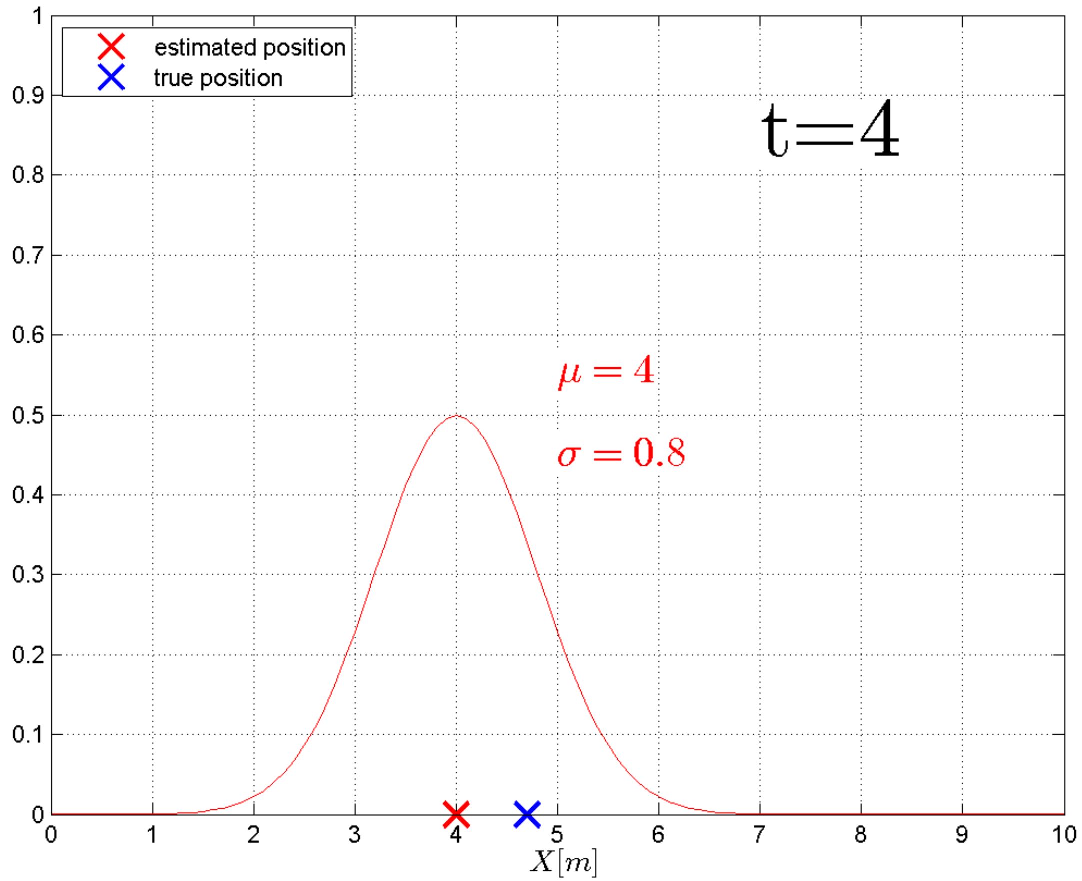


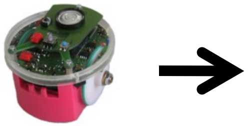
1D Odometry with Gaussian Uncertainty



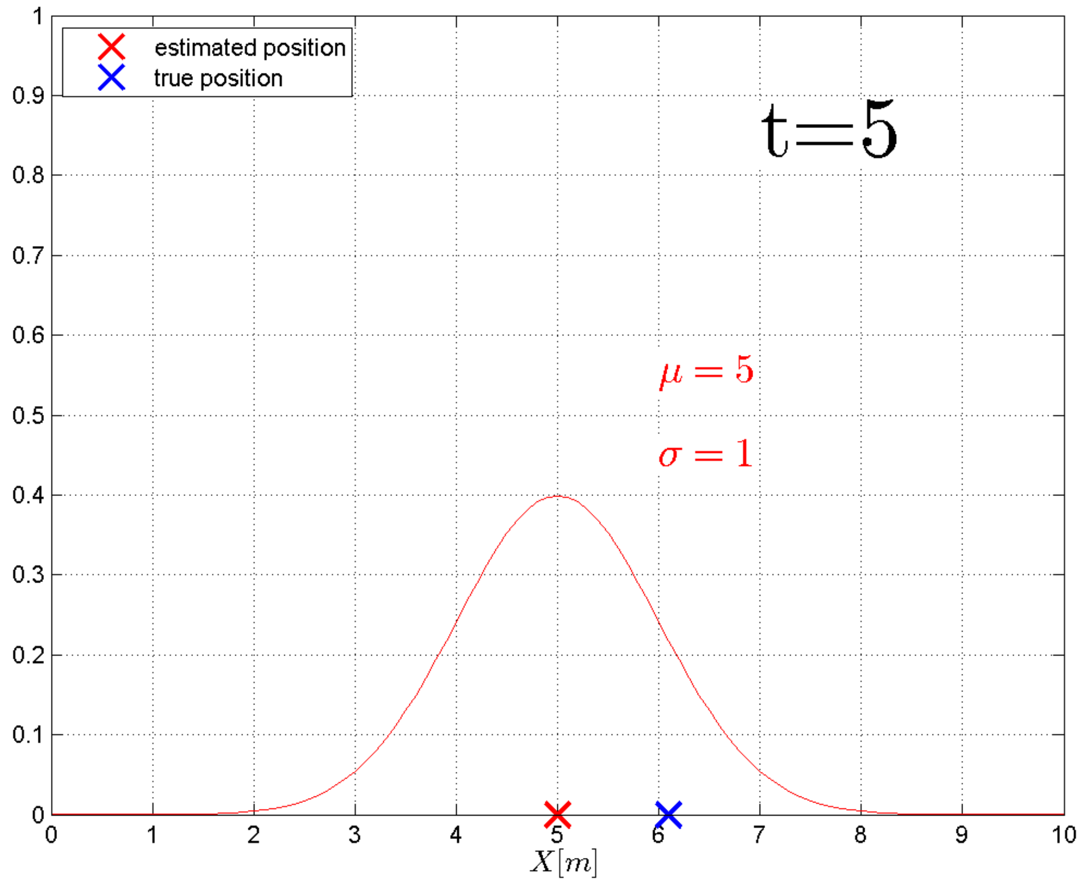


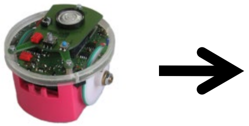
1D Odometry with Gaussian Uncertainty





1D Odometry with Gaussian Uncertainty





Non-Deterministic Uncertainities in Odometry based on Wheel Encoders

Nondeterministic Error Sources

- Variation of the contact point of the wheel
- Unequal floor contact (e.g., wheel slip, nonplanar surface)

- Wheels cannot be assumed to roll perfectly
- Measured encoder values do not perfectly reflect the actual motion
- Pose error is cumulative and incrementally increases
- Probabilistic modeling for assessing quantitatively the error

Odometric Error Types

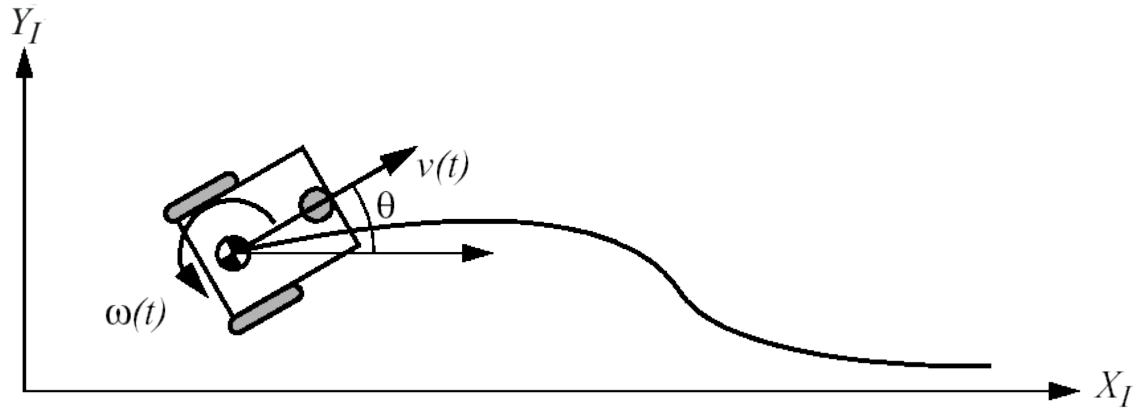
- Range error: sum of the wheel movements
- Turn error: difference of wheel motion
- Drift error: difference between wheel errors lead to heading error

Pose Variation During Δt

$$\Delta s = \frac{\Delta s_r + \Delta s_l}{2}$$

$$\begin{cases} \Delta x = \Delta s \cos(\theta + \frac{\Delta\theta}{2}) \\ \Delta y = \Delta s \sin(\theta + \frac{\Delta\theta}{2}) \\ \Delta\theta = \frac{\Delta s_r - \Delta s_l}{b} \end{cases}$$

$$p = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} \xrightarrow{t' = t + \Delta t} p' = \begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix}$$



b = 2l = inter-wheel distance

Δs_r = traveled distance right wheel

Δs_l = traveled distance left wheel

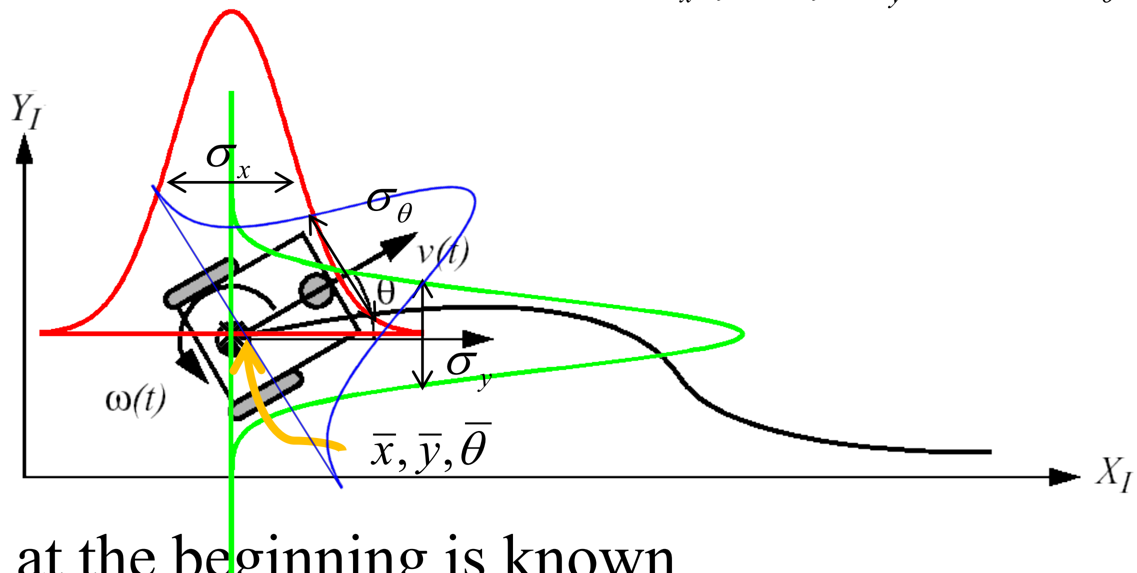
Δθ = orientation change of the vehicle

$$p' = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta s \cos(\theta + \Delta\theta/2) \\ \Delta s \sin(\theta + \Delta\theta/2) \\ \Delta\theta \end{bmatrix} = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{\Delta s_r + \Delta s_l}{2} \cos(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r + \Delta s_l}{2} \sin(\theta + \frac{\Delta s_r - \Delta s_l}{2b}) \\ \frac{\Delta s_r - \Delta s_l}{b} \end{bmatrix}$$

Noise modeling

Model error in each dimension with a Gaussian $x \rightarrow \bar{x}, \sigma_x; y \rightarrow \bar{y}, \sigma_y; \theta \rightarrow \bar{\theta}, \sigma_\theta$

$$\Sigma_p = \begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{x\theta}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{y\theta}^2 \\ \sigma_{\theta x}^2 & \sigma_{\theta y}^2 & \sigma_{\theta\theta}^2 \end{bmatrix}$$



Assumptions:

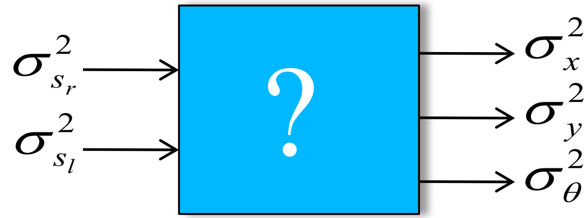
- Covariance matrix Σ_p at the beginning is known
- Errors of the two individual wheels are independent
- Errors are independent of direction of motion
- Errors are proportional to the distance traveled (k_r, k_l model parameters)

$$\sum_\Delta = \mathrm{cov}(\Delta s_r, \Delta s_l) = \begin{bmatrix} k_r|\Delta s_r| & 0 \\ 0 & k_l|\Delta s_l| \end{bmatrix} = \begin{bmatrix} \sigma_{s_r}^2 & 0 \\ 0 & \sigma_{s_l}^2 \end{bmatrix}$$

Actuator Noise → Pose Noise

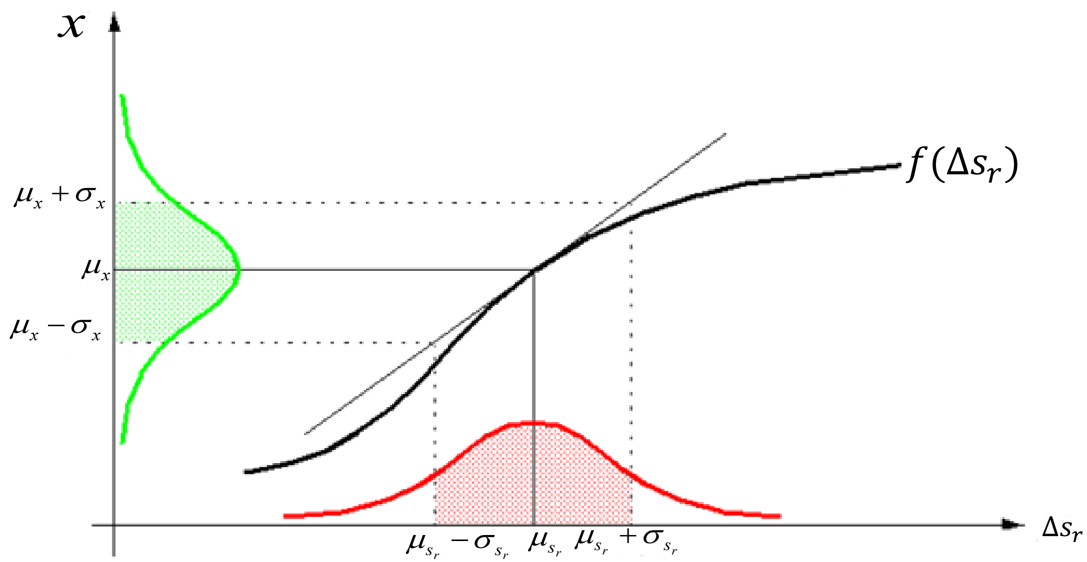
- How is the actuator noise (2D) propagated to the pose (3D)?

$$\Sigma_\Delta = \begin{bmatrix} \sigma_{s_r}^2 & 0 \\ 0 & \sigma_{s_l}^2 \end{bmatrix}$$



$$\begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{x\theta}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{y\theta}^2 \\ \sigma_{\theta x}^2 & \sigma_{\theta y}^2 & \sigma_{\theta\theta}^2 \end{bmatrix} = \Sigma_p$$

- 1D to 1D example $N(\mu_{s_r}, \sigma_{s_r}) \rightarrow N(\mu_x, \sigma_x)$



- We need to linearize → Taylor Series

$$x \approx f(\Delta s_r)\Big|_{\Delta s_r = \mu_{s_r}} \approx f(\Delta s_r) + \frac{1}{1!}\frac{\partial f}{\partial \Delta s_r}(\Delta s_r - \mu_{s_r}) + \frac{1}{2!}\frac{\partial^2 f}{\partial \Delta {s_r}^2}(\Delta s_r - \mu_{s_r})^2 + \cdots$$

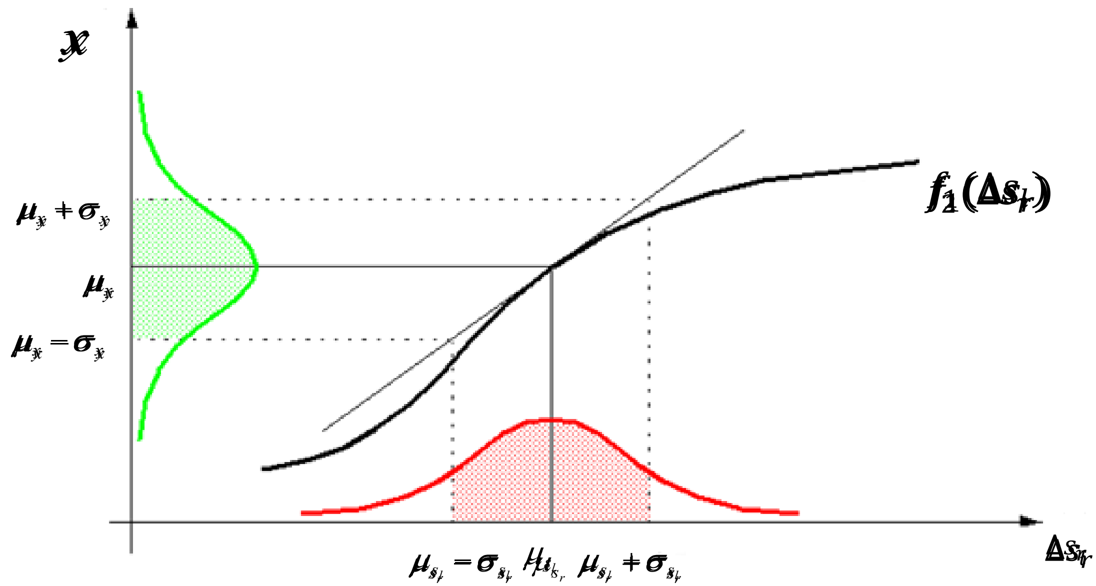
Actuator Noise → Pose Noise

$$x \approx f_1(\Delta s_r)\Big|_{\Delta s_r=\mu_{s_r}} \approx f_1(\Delta s_r) + \frac{\partial f_1}{\partial \Delta s_r}(\Delta s_r - \mu_{s_r})$$

$$x \approx f_1(\Delta s_l)\Big|_{\Delta s_l=\mu_{s_l}} \approx f_1(\Delta s_l) + \frac{\partial f_1}{\partial \Delta s_l}(\Delta s_l - \mu_{s_l})$$

$$y \approx f_2(\Delta s_r)\Big|_{\Delta s_r=\mu_{s_r}} \approx f_2(\Delta s_r) + \frac{\partial f_2}{\partial \Delta s_r}(\Delta s_r - \mu_{s_r})$$

...



$$F_{\Delta rl} = \begin{bmatrix} \frac{\partial f_1}{\partial \Delta s_r} & \frac{\partial f_1}{\partial \Delta s_l} \\ \frac{\partial f_2}{\partial \Delta s_r} & \frac{\partial f_2}{\partial \Delta s_l} \\ \frac{\partial f_3}{\partial \Delta s_r} & \frac{\partial f_3}{\partial \Delta s_l} \end{bmatrix}$$ Jacobian

$$\Sigma_\Delta = \begin{bmatrix} k_r|\Delta s_r| & 0 \\ 0 & k_l|\Delta s_l| \end{bmatrix}$$

- General error propagation law

$$\Sigma_{\Delta rl} = F_{\Delta rl}\Sigma_\Delta F_{\Delta rl}^T$$

Actuator Noise → Pose Noise

How does the pose covariance Σ_p evolve over time?

- Initial covariance of vehicle at t=0:

$$\Sigma_p^{(t=0)} = \begin{bmatrix} \sigma_{xx}^2 & \sigma_{xy}^2 & \sigma_{x\theta}^2 \\ \sigma_{yx}^2 & \sigma_{yy}^2 & \sigma_{y\theta}^2 \\ \sigma_{\theta x}^2 & \sigma_{\theta y}^2 & \sigma_{\theta\theta}^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

- Additional noise at each time step Δt: $\Sigma_{\Delta rl} = F_{\Delta rl} \Sigma_{\Delta} F_{\Delta rl}^T$
- Covariance at t=1Δt: $\Sigma_p^{(t=1\Delta t)} = \Sigma_p^{(t=0)} + \Sigma_{\Delta rl} = \Sigma_{\Delta rl}$
- Covariance at t=2Δt:

$$\Sigma_p^{(t=2\Delta t)} = F_p \Sigma_p^{(t=1\Delta t)} F_p^T + F_{\Delta rl} \Sigma_{\Delta} F_{\Delta rl}^T$$

$$F_p = \begin{bmatrix} \frac{\partial f_1}{\partial_x} & \frac{\partial f_1}{\partial_y} & \frac{\partial f_1}{\partial_\theta} \\ \frac{\partial f_2}{\partial_x} & \frac{\partial f_2}{\partial_y} & \frac{\partial f_2}{\partial_\theta} \\ \frac{\partial f_3}{\partial_x} & \frac{\partial f_3}{\partial_y} & \frac{\partial f_3}{\partial_\theta} \end{bmatrix}$$

Actuator Noise → Pose Noise

Algorithm

Precompute:

- Determine actuator noise Σ_Δ
- Compute mapping actuator-to-pose noise incremental $F_{\Delta rl}$
- Compute mapping pose propagation noise over step F_p

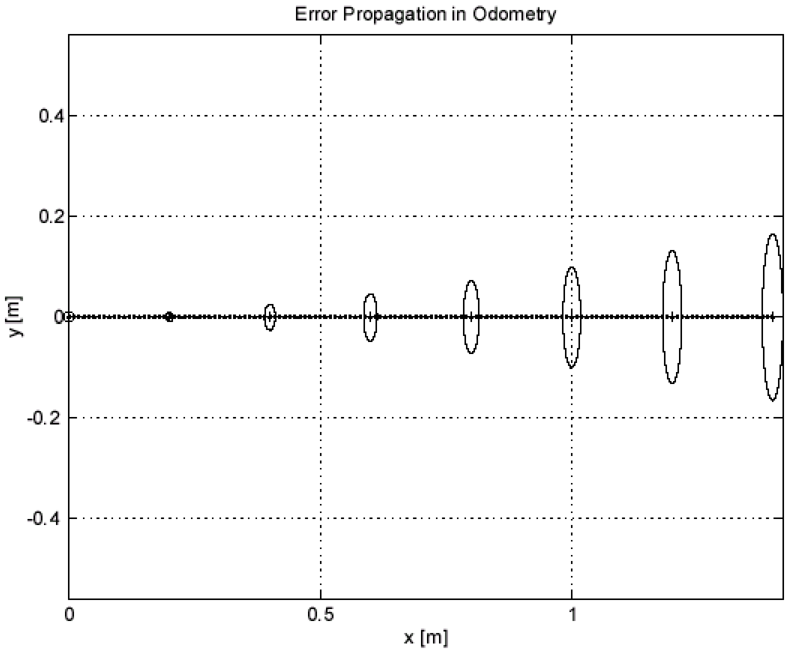
Initialize:

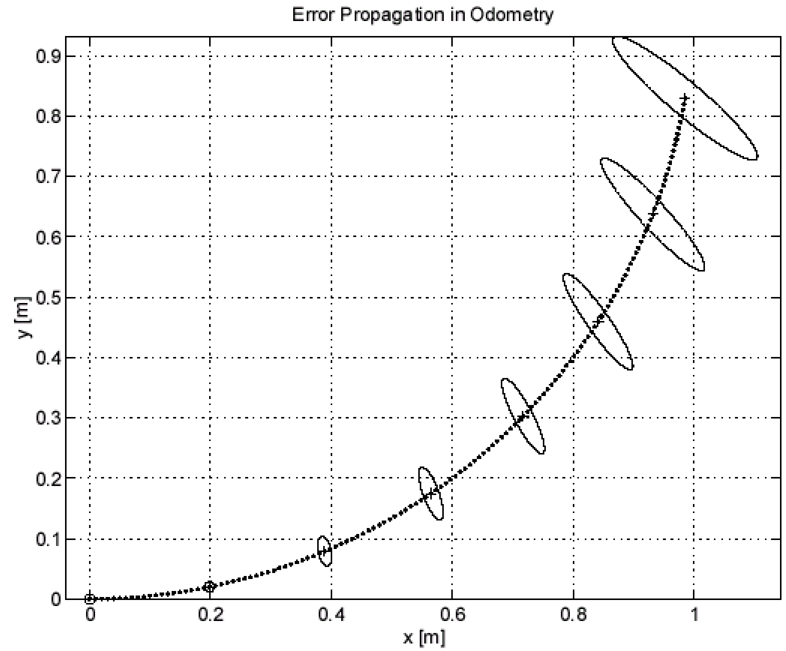
- Initialize $\Sigma_p^{(t=0)} = [0]$

Iterate:

$$\Sigma_p^{(t=(k+1)\Delta t)} = F_p \Sigma_p^{(t=k\Delta t)} F_p^T + F_{\Delta rl} \Sigma_\Delta F_{\Delta rl}^T$$

Classical 2D Representation





Courtesy of R. Siegwart and R. Nourbakhsh

Ellipses: typical 3σ bounds

Conclusion

Take Home Messages

- There are several localization techniques for indoor and outdoor systems
- Each of the localization methods/positioning system has advantage and drawbacks
- Odometry is an absolute positioning method using only proprioceptive sensors but affected by a cumulative error
- Localization error in odometry can be both deterministic and non-deterministic
- Deterministic errors can be mitigated by calibration, non-deterministic error can be modeled and taken into account
- The impact of non-deterministic error on the pose of the robot can be estimated by leveraging an approximation of the kinematic forward model of the vehicle and the error propagation law

Additional Literature – Week 9

Pointers on odometry

http://rossum.sourceforge.net/papers/DiffSteer/

http://rossum.sourceforge.net/tools/MotionApplet/MotionApplet.html

Books

- Weston J. and Titterton D, "Strapdown Inertial Navigation", IET, 2005
- Siegwart R., Nourbakhsh I., and Scaramuzza D., "Introduction to Autonomous Mobile Robots, second Edition", MIT Press, 2011.
- Borenstein J., Everett H. R., and Feng L. "Navigating Mobile Robots: Systems and Techniques", A. K. Peters, Ltd., 1996.