

Lab 1

*School of Architecture, Civil and
Environmental Engineering*

EPFL, SS 2020-2021

http://disal.epfl.ch/teaching/signals_instruments_systems/

Lab 1 Outline

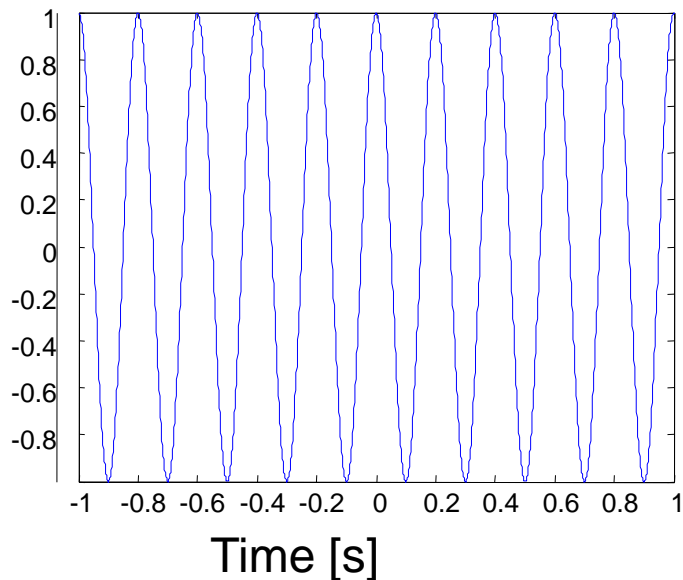
- Concept:
 - Signals (Part 0)
 - Fourier Transforms (Part 1)
 - Fourier Series (Part 2)
 - Convolution (Part 3, 4)
- Tools:
 - Matlab

Reminder: Fourier Transform

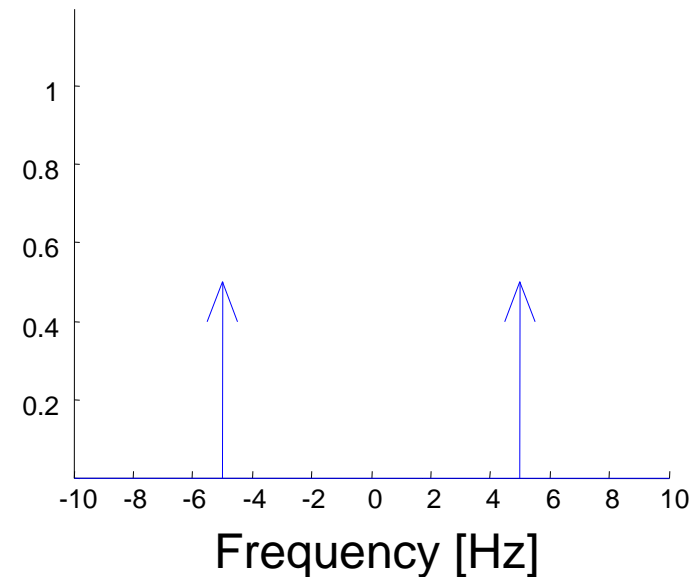
$$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i2\pi\xi t} dt$$

$$f(t) = \int_{-\infty}^{\infty} \hat{f}(\xi) \cdot e^{i2\pi t\xi} d\xi$$

Reminder: Fourier Transform

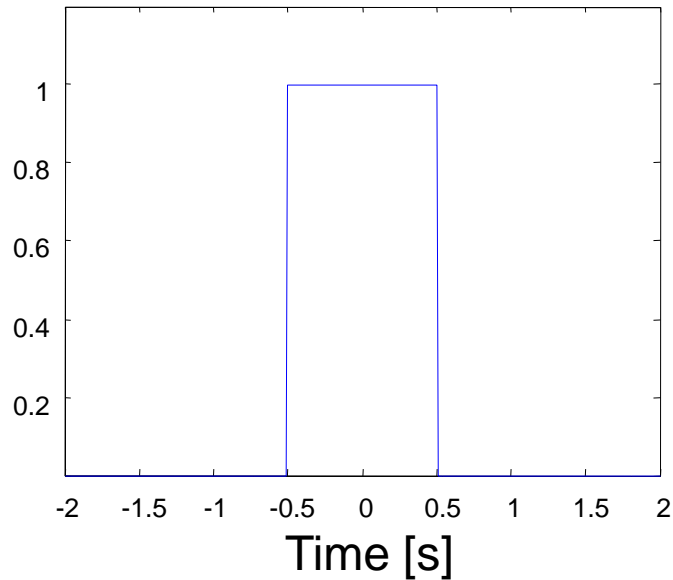


$$f(t) = \cos(2\pi at)$$

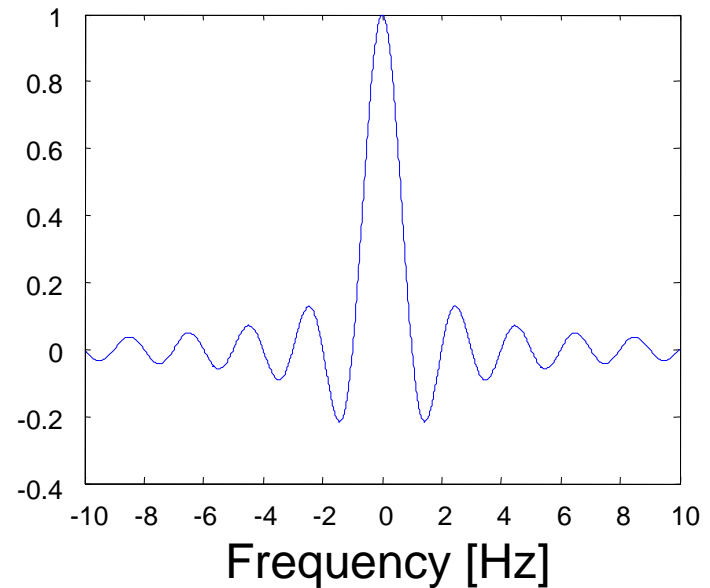


$$\hat{f}(\xi) = \frac{\delta(\xi - a) + \delta(\xi + a)}{2}$$

Reminder: Fourier Transform



$$f(t) = \text{rect}(t)$$



$$\hat{f}(\xi) = \text{sinc}(\xi)$$

Fourier Properties Table

	Function	Fourier transform unitary, ordinary frequency	Fourier transform unitary, angular frequency	Fourier transform non-unitary, angular frequency
	$f(x)$	$\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x)e^{-2\pi i x \xi} dx$	$\hat{f}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x)e^{-i\omega x} dx$	$\hat{f}(\nu) = \int_{-\infty}^{\infty} f(x)e^{-i\nu x} dx$
101	$a \cdot f(x) + b \cdot g(x)$	$a \cdot \hat{f}(\xi) + b \cdot \hat{g}(\xi)$	$a \cdot \hat{f}(\omega) + b \cdot \hat{g}(\omega)$	$a \cdot \hat{f}(\nu) + b \cdot \hat{g}(\nu)$
102	$f(x - a)$	$e^{-2\pi i a \xi} \hat{f}(\xi)$	$e^{-i a \omega} \hat{f}(\omega)$	$e^{-i a \nu} \hat{f}(\nu)$
103	$e^{2\pi i a x} f(x)$	$\hat{f}(\xi - a)$	$\hat{f}(\omega - 2\pi a)$	$\hat{f}(\nu - 2\pi a)$
104	$f(ax)$	$\frac{1}{ a } \hat{f}\left(\frac{\xi}{a}\right)$	$\frac{1}{ a } \hat{f}\left(\frac{\omega}{a}\right)$	$\frac{1}{ a } \hat{f}\left(\frac{\nu}{a}\right)$
105	$\hat{f}(x)$	$f(-\xi)$	$f(-\omega)$	$2\pi f(-\nu)$
106	$\frac{d^n f(x)}{dx^n}$	$(2\pi i \xi)^n \hat{f}(\xi)$	$(i\omega)^n \hat{f}(\omega)$	$(i\nu)^n \hat{f}(\nu)$
107	$x^n f(x)$	$\left(\frac{i}{2\pi}\right)^n \frac{d^n \hat{f}(\xi)}{d\xi^n}$	$i^n \frac{d^n \hat{f}(\omega)}{d\omega^n}$	$i^n \frac{d^n \hat{f}(\nu)}{d\nu^n}$
108	$(f * g)(x)$	$\hat{f}(\xi)\hat{g}(\xi)$	$\sqrt{2\pi} \hat{f}(\omega)\hat{g}(\omega)$	$\hat{f}(\nu)\hat{g}(\nu)$
109	$f(x)g(x)$	$(\hat{f} * \hat{g})(\xi)$	$\frac{(\hat{f} * \hat{g})(\omega)}{\sqrt{2\pi}}$	$\frac{1}{2\pi} (\hat{f} * \hat{g})(\nu)$

(Continuous) Fourier Transform Implementation

- Use of Matlab symbolics tool to compute the Fourier transform analytically.
- The result is true almost everywhere
- To create a symbolic variable, use the command *syms*

Reminder: Discrete Fourier Transform

$$X[k] = \sum_{n=0}^{N-1} x[n] \cdot e^{-\frac{2\pi i}{N}kn}, k = 0, \dots, N-1$$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] \cdot e^{\frac{2\pi i}{N}kn}, k = 0, \dots, N-1$$

- Input and output sequences are both finite (the integral becomes a finite sum)
- Different from the Discrete-Time Fourier Transform (DTFT)
- Efficient implementation is done using Fast Fourier Algorithms (FFT)

Continuous Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau$$

- For each value of t :
 1. Flip (reflect) g 1) $g(\tau) \rightarrow g(-\tau)$
 2. Shift g by t 2) $g(-\tau) \rightarrow g(t - \tau)$
 3. Multiply f and g 3) $f(\tau) \cdot g(t - \tau)$
 4. Integrate over τ 4) $\int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau$
- Note that the result does **not** depend on τ !

Discrete Convolution

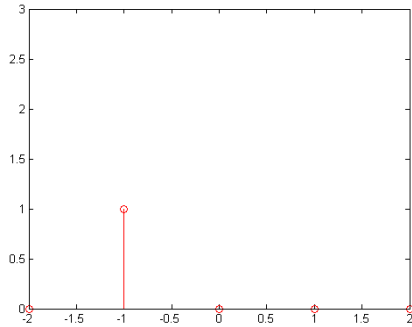
$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau$$



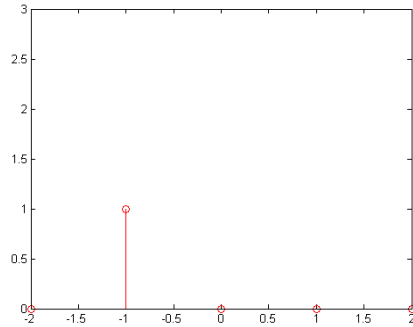
$$(f * g)(n) = \sum_{m=-\infty}^{\infty} f(m) \cdot g(n - m)$$

- Similar to the continuous version
- The integral becomes an infinite sum
- Matlab, operating on a computer, can only emulate continuity and therefore use the discrete version with an adjustable discretization level in time and amplitude

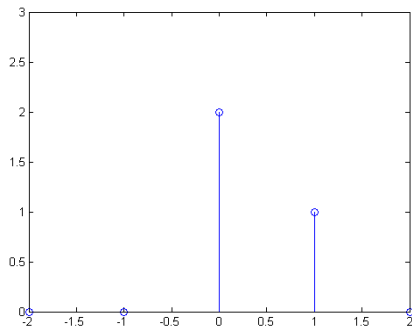
Example of Discrete Convolution



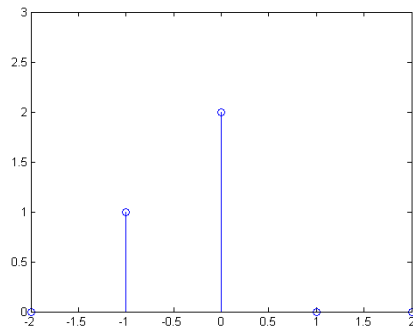
$f[m]$



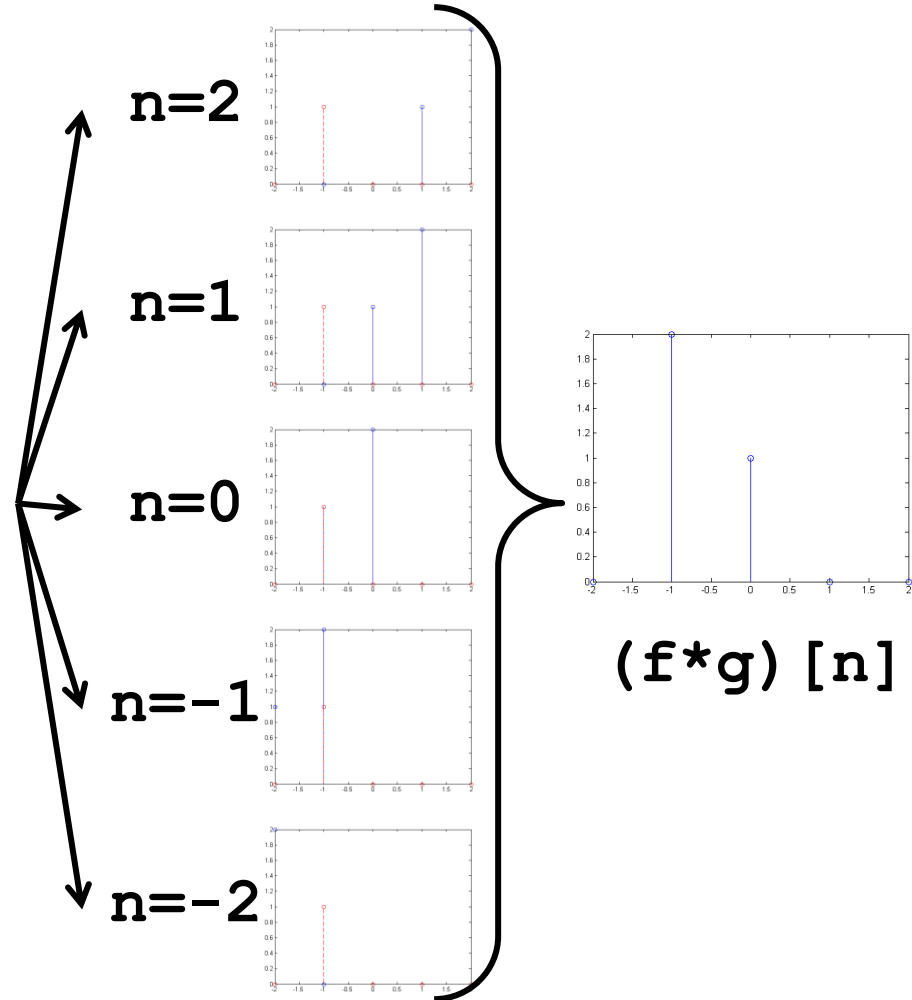
$f[-m]$



$g[m]$



$g[-m]$



Feedback form

Please fill the feedback form for Lab 1!