

Lab 5

*School of Architecture, Civil and
Environmental Engineering*

EPFL, SS 2018-2019

http://disal.epfl.ch/teaching/signals_instruments_systems/

Lab 5 Outline

- Concepts:
 - Convolution
 - Sampling
 - Aliasing
 - Reconstruction
- Tools:
 - Matlab

Part 1: Continuous Convolution

$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau$$

- For each value of t :
 1. Flip (reflect) g 1) $g(\tau) \rightarrow g(-\tau)$
 2. Shift g by t 2) $g(-\tau) \rightarrow g(t - \tau)$
 3. Multiply f and g 3) $f(\tau) \cdot g(t - \tau)$
 4. Integrate over τ 4) $\int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau$
- Note that the result does **not** depend on τ !

Part 2: Discrete Convolution

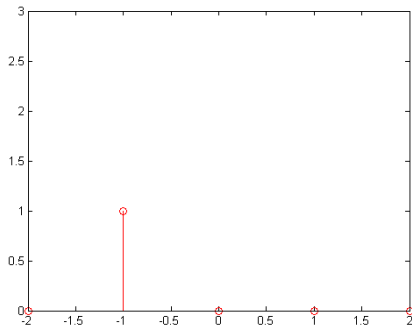
$$(f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau$$



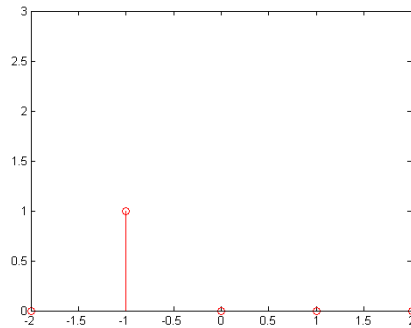
$$(f * g)(n) = \sum_{m=-\infty}^{\infty} f(m) \cdot g(n - m)$$

- Similar to the continuous version
- The integral becomes an infinite sum
- Matlab, operating on a computer, can only emulate continuity and therefore use the discrete version with an adjustable discretization level in time and amplitude

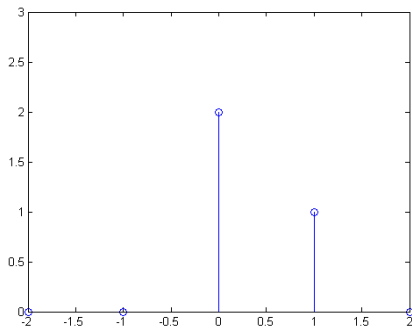
Example of Discrete Convolution



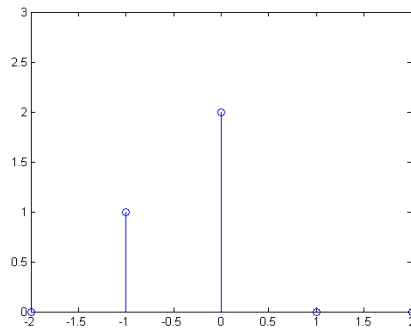
$f[m]$



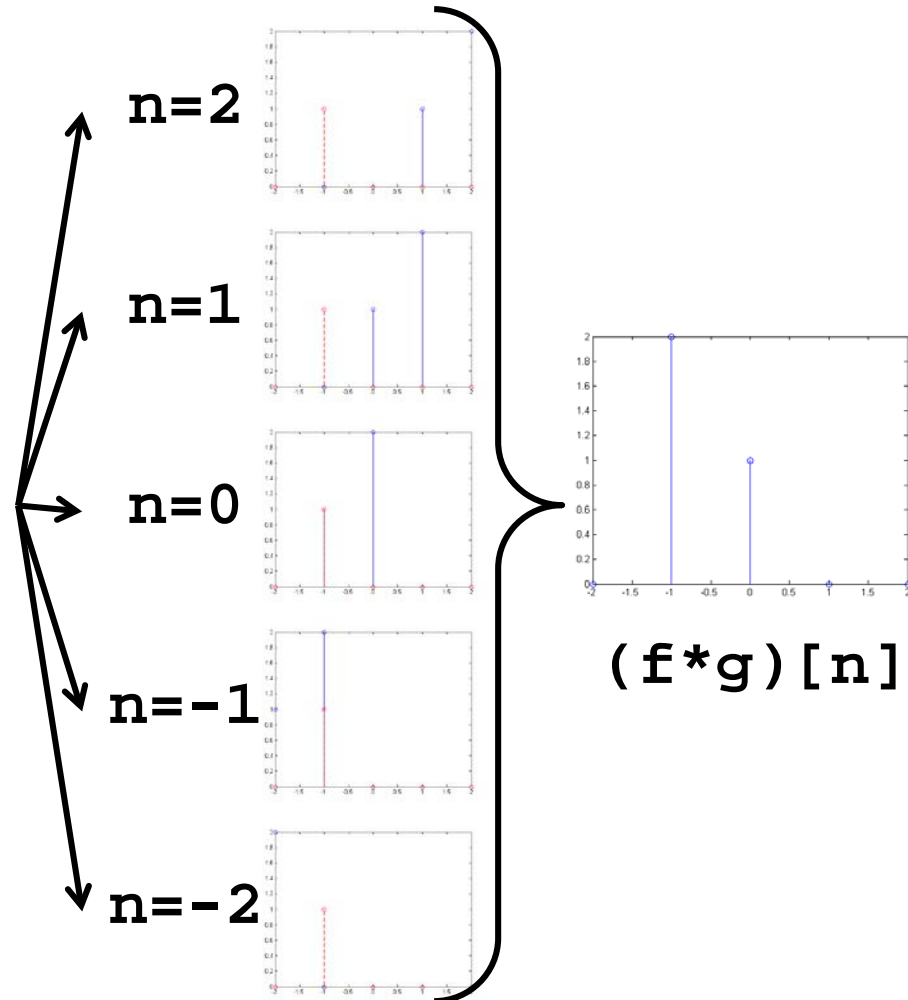
$f[-m]$



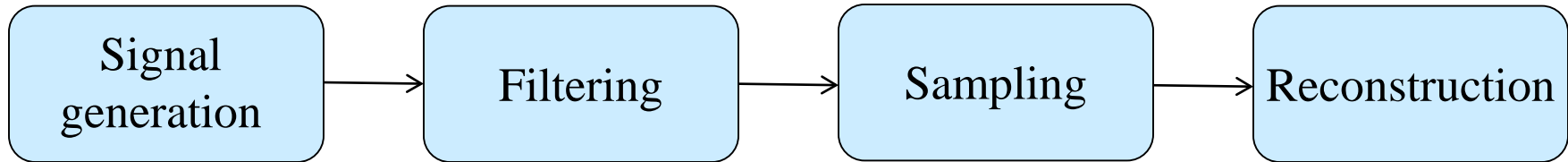
$g[m]$



$g[-m]$



Part 3: Sampling and reconstruction

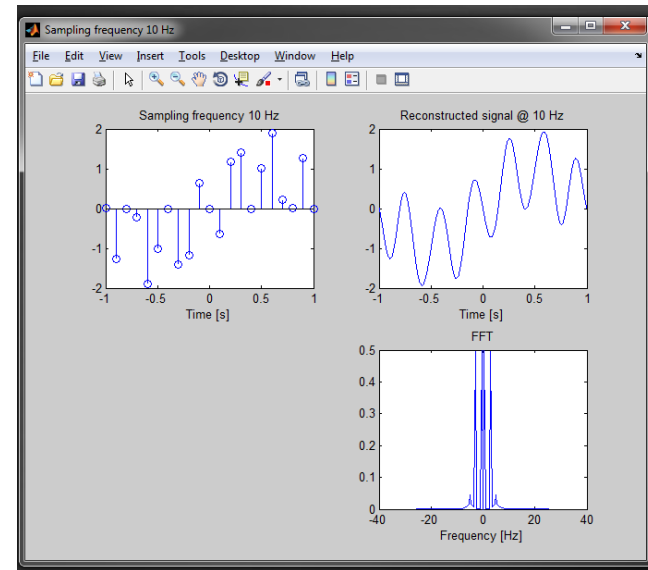
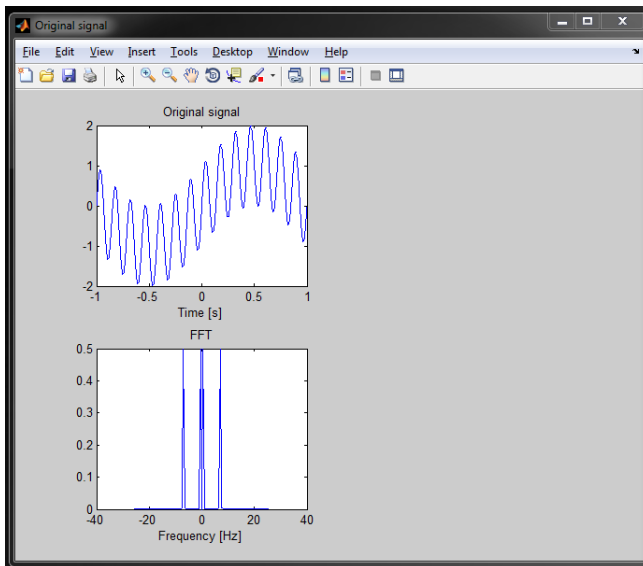


- $f(t) = \sum_i A_i \sin(2\pi f_i t)$
- frequencies
- amplitudes

- Not in this lab

- Sampling frequency

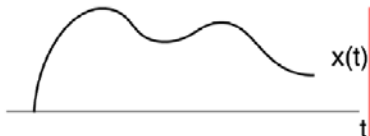
- Linear interpolation
- Whittaker-Shannon



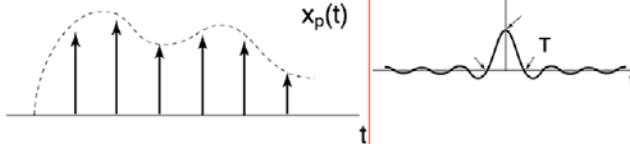
Reconstruction: Linear vs. Witteraker-Shannon

Graphic Illustration of Time-Domain Interpolation

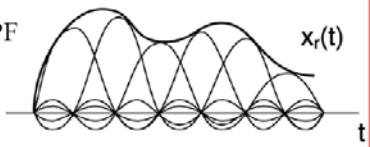
Original
CT signal



After sampling



After passing the LPF



Interpolation Methods

- Bandlimited Interpolation
- Zero-Order Hold
- First-Order Hold — Linear interpolation

