Signals, Instruments, and Systems – W11

An Introduction to Localization Systems and Methods
Outline

• Motivation

• Positioning systems

• Localization using proprioceptive sensors without uncertainties

• Localization using proprioceptive sensors with uncertainties

• Localization using proprioceptive and exteroceptive sensors
Motivation for Localization
Motivation

- Environmental data most often useless without location information
- Sophisticated localization equipment widely available
- → Ubiquity of localization information

*Picture: courtesy of Sensorscope*
Motivation

- Localization is required at different scales (1000’s km → cm)
- Different environments → different methodologies

Pictures: courtesy of Sensorscope, NASA, NIST
Classification axes

- Indoor vs. outdoor techniques
- Absolute vs. relative positioning systems
- Line-of-sight vs. non-line-of-sight
- Underlying physical principle and channel
- Positioning available on-board vs. off-board
- Scalability in terms of number of nodes
Selected Indoor Positioning Systems

- Overhead camera
- Infrared (IR) + RF technology
- Impulse Radio Ultra Wide Band (IR-UWB)
Overhead (Multi-)Camera Systems

• Tracking objects with one (or more) overhead cameras
• Absolute positions, available outside the robot/sensor
• Active, passive, or no markers
• Open-source software available (e.g., SwisTrack, developed at DISAL)
• Major issues: light, calibration

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<table>
<thead>
<tr>
<th></th>
<th></th>
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<tbody>
<tr>
<td>Accuracy</td>
<td>~ 1 cm (2D)</td>
</tr>
<tr>
<td>Update rate</td>
<td>~ 20 Hz</td>
</tr>
<tr>
<td># agents</td>
<td>~ 100</td>
</tr>
<tr>
<td>Area</td>
<td>~ 10 m²</td>
</tr>
</tbody>
</table>
6D Multi-Camera System

- 10-50 cameras
- mm accuracy
- 180 Hz update
- 2 ms latency
- 6D pose estimation
- 4-5 passive markers per object to be tracked needed
- 200 m³ arena

http://www.youtube.com/watch?v=hyGJBV1xnJI

http://www.youtube.com/watch?v=geqip_0Vjec
Infrared + Radio Technology

- **Principle:**
  - belt of IR emitters (LED) and receivers (photodiode)
  - IR LED used as antennas; modulated light (carrier 10.7 MHz), RF chip behind
  - Range: measurement of the Received Signal Strength Intensity (RSSI)
  - Bearing: signal correlation over multiple receivers
  - Measure range & bearing can be coupled with standard RF channel (e.g. 802.11) for heading assessment
  - Can also be used for 20 kbit/s IR communication
  - Robot ID communicated with the IR channel (ad hoc protocol)

\[\text{\cite{Pugh et al., IEEE Trans. on Mechatronics, 2009}}\]
Infrared + Radio Technology

Performance summary:
• Range: 3.5 m
• Update frequency 25 Hz with 10 neighboring robots (or 250 Hz with 2)
• Accuracy range: <7% (MAX), generally decrease 1/d
• Accuracy bearing: < 9º (RMS)
• LOS method
• Possible extension in 3D, larger range (but more power) and better bearing accuracy with more photodiodes (e.g. Bergbreiter, PhD UCB 2008, dedicated ASIC, up to 15 m, 256 photodiodes, single emitter with conic lense)
IR-UWB System - Principles

- Impulse Radio Ultra-Wide Band
- Based on time-of-flight (TDOA, Time Difference of Arrival)
- UWB tags (emitters, a few cm, low-power) and multiple synchronized receivers
- Emitters can be unsynchronized but then dealing with interferences not trivial (e.g., Ubisense system synchronized)
- Absolute positions available on the receiving system
- Positioning information can be fed back to robots using a standard narrow-band channel
- 6 - 8 GHz central frequency
- Very large bandwidth (>0.5GHz) → high material penetrability
- Fine time resolution → high theoretical ranging accuracy (order of cm)
IR-UWB System – Performances

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>Accuracy</td>
<td>15 cm (3D)</td>
</tr>
<tr>
<td>Update rate</td>
<td>34 Hz / tag</td>
</tr>
<tr>
<td># agents</td>
<td>~ 10000</td>
</tr>
<tr>
<td>Area</td>
<td>~ 1000 m²</td>
</tr>
</tbody>
</table>

Ex. State-of-art system (e.g., Ubisense 7000 Series, Compact Tag)

- Degraded accuracy performance if
  - Inter-emitter interferences
  - Non-Line-of-Sight (NLOS) bias
  - Multi-path
Selected Outdoor Positioning Techniques

- GPS
- Differential GPS (dGPS)
Global Positioning System

Note: the first and still most prominent example of GNSS systems (Global Navigation Satellite Systems)
Global Positioning System

• Initially 24 satellites (including three spares), 32 as of December 2012, orbiting the earth every 12 hours at a height of 20.190 km.

• Satellites synchronize their transmission (location + time stamp) so that signals are broadcasted at the same time (ground stations updating + atomic clocks on satellites)

• Location of any GPS receiver is determined through a time of flight measurement (ns accuracy!)

• Real time update of the exact location of the satellites:
  - monitoring the satellites from a number of widely distributed ground stations
  - a master station analyses all the measurements and transmits the actual position to each of the satellites

• Exact measurement of the time of flight
  – the receiver correlates a pseudocode with the same code coming from the satellite
  – the delay time for best correlation represents the time of flight.
  – quartz clock on the GPS receivers are not very precise
  – the range measurement with (at least) four satellites allows to identify the three values (x, y, z) for the position and the clock correction ΔT

• Recent commercial GPS receiver devices allows position accuracies down to a few meters with best satellite visibility conditions.

• 200-300 ms latency, so max 5 Hz GPS updates
**dGPS**

**Differential GPS**

- Measured: $x \ y \ z$
- Delta: $x \ y \ z$
- True: $x \ y \ z$

Corrections applied after survey

- True: $x \ y \ z$
- Measured: $x \ y \ z$
- Delta: $x \ y \ z$
Odometry
"Using proprioceptive sensory data influenced by the movement of actuators to estimate change in pose over time"

- **Start:** initial position
- **Actuators:**
  - Legs
  - Wheels
  - Propeller
- **Sensors (proprioceptive):**
  - Wheel encoders (DC motors), step counters (stepper motors)
  - Inertial measurement units, accelerometers
  - Nervous systems, neural chains
- **Idea:** navigating a room with the light turned off
Example:

- Example: Cataglyphis desert ant
- Excellent study by Prof. R. Wehner (University of Zuerich, Emeritus)
- Individual foraging strategy
- Underlying mechanisms
  - Internal compass (polarization of sun light)
  - Dead-reckoning (path integration on neural chains for leg control; note: in robotics typically using also heading sensors)
  - Local search (around 1-2 m from the nest)
- Extremely accurate navigation: averaged error of a few tens of cm over 500 m path!
More examples

• Human in the dark
  • Very bad odometry sensors
  • \( d_{\text{Odometry}} = O(1/m) \)

• (Nuclear) Submarine
  • Very good odometry sensors
  • \( d_{\text{Odometry}} = O(1/10^3 \text{ km}) \)

• Navigation system in tunnel uses dead reckoning based on
  • Last velocity as measured by GPS
  • Car’s odometer, compass
Odometry using Wheel Encoders
Optical Encoders

- Measure displacement (or speed) of the wheels
- Principle: mechanical light chopper consisting of photo-barriers (pair of light emitter and optical receiver) + pattern on a disc anchored to the motor shaft
- Quadrature encoder: 90° placement of 2 complete photo-barriers, 4x increase resolution + direction of movement
- Integrate wheel movements to get an estimate of the position -> odometry
- Typical resolutions: 64 - 2048 increments per revolution.
Pose (Position and Orientation) of a Differential-Drive Robot

\[
\begin{bmatrix}
    x_I \\
    y_I \\
    \theta
\end{bmatrix}
= \begin{bmatrix}
    x_R \\
    y_R \\
    \theta
\end{bmatrix}
= R(\theta) \begin{bmatrix}
    x_I \\
    y_I \\
    \theta
\end{bmatrix}
\]

Orthogonal Rotation Matrix

\[
R(\theta) = \begin{bmatrix}
    \cos \theta & \sin \theta & 0 \\
    -\sin \theta & \cos \theta & 0 \\
    0 & 0 & 1
\end{bmatrix}
\]

From *Introduction to Autonomous Mobile Robots*, Siegwart R. and Nourbakhsh I. R.
Absolute and Relative Motion of a Differential-Drive Robot

\[ \dot{\xi}_R = R(\theta)\dot{\xi}_I \]

\[
\begin{bmatrix}
\dot{x}_R \\
\dot{y}_R \\
\dot{\theta}_R
\end{bmatrix}
=
\begin{bmatrix}
0 & 1 & 0 \\
-1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
\dot{x}_I \\
\dot{y}_I \\
\dot{\theta}_I
\end{bmatrix}
=
\begin{bmatrix}
\dot{y}_I \\
-\dot{x}_I \\
\dot{\theta}_I
\end{bmatrix}
\]

Ex. \( \theta = \pi/2 \)
Forward Kinematic Model

How does the robot move given the wheel speeds and geometry?

- Assumption: no wheel slip (rolling mode only)!
- In miniature robots no major dynamic effects due to low mass-to-power ratio

\[ v(t) = f(l, r, \theta, \dot{\phi}_1, \dot{\phi}_2) \]

\[ \dot{x} \quad \dot{y} \quad \dot{\theta} \]

\[ \dot{\phi}_i = \text{wheel } i \text{ speed} \]
Recap ME/PHY Fundamentals

\[ v = \omega r = \dot{\varphi} r \]

- \( v \) = tangential speed
- \( \omega \) = rotational speed
- \( r \) = rotation radius
- \( \varphi \) = rotation angle
- \( C \) = rotation center
- \( P \) = peripheral point
- \( P' \) = contact point at time \( t \)

**Rolling!**
Forward Kinematic Model

Linear speed = average wheel speed 1 and 2:

\[ \nu = \frac{r \dot{\phi}_1}{2} + \frac{r \dot{\phi}_2}{2} \]

Rotational speed = sum of rotation speeds (wheel 1 clockwise, wheel 2 counterclockwise):

\[ \omega = \frac{r \dot{\phi}_1}{2l} + \frac{-r \dot{\phi}_2}{2l} \]

Idea: linear superposition of individual wheel contributions
Forward Kinematic Model

1. \( \dot{x}_R = v = \frac{r \dot{\phi}_1}{2} + \frac{r \dot{\phi}_2}{2} \)

2. \( \dot{y}_R = 0 \)

3. \( \dot{\theta}_R = \omega = \frac{r \dot{\phi}_1}{2l} + \frac{-r \dot{\phi}_2}{2l} \)

4. \( \dot{\xi}_I = R^{-1}(\theta) \dot{\xi}_R \)

\[ \dot{\xi}_I = \begin{bmatrix} \cos \theta & -\sin \theta & 0 \\ \sin \theta & \cos \theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{r \dot{\phi}_1}{2} + \frac{r \dot{\phi}_2}{2} \\ \frac{r \dot{\phi}_1}{2l} + \frac{-r \dot{\phi}_2}{2l} \end{bmatrix} \]
Odometry

- Given our absolute pose over time, we can calculate the robot pose after some time $t$ through integration.
- Given the kinematic forward model, and assuming no slip on both wheels:

$$\xi_I(T) = \xi_{I0} + \int_0^T \xi_I dt = \xi_{I0} + \int_0^T R^{-1}(\theta) \dot{\xi}_R dt$$

- Given an initial pose $\xi_{I0}$, after time $T$, the pose of the vehicle will be $\xi_I(T)$.
- $\xi_I(T)$ computable with wheel speed 1, wheel speed 2, and parameters $r$ and $l$. 
Localization Uncertainties in Odometry
Deterministic Error Sources

- Limited encoder resolution
- Wheel misalignment and small differences in wheel diameter

From Week 7 lecture: Can be fixed by calibration
Non-Deterministic Error Sources

• From Week 7 lecture: no deterministic prediction possible → we have to describe them probabilistically

• Example: accelerometer-based odometry
MEMS-Accelerometer

Microelectromechanical systems
- Sampling frequency typically a function of the application and of the accelerometer characteristics

Table 2. Operating Characteristics

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Symbol</th>
<th>Min</th>
<th>Typ</th>
<th>Max</th>
<th>Unit</th>
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<tbody>
<tr>
<td>Operating Range</td>
<td>$V_{DD}$</td>
<td>2.2</td>
<td>3.3</td>
<td>3.6</td>
<td>V</td>
</tr>
<tr>
<td>Supply Voltage</td>
<td>$I_{DD}$</td>
<td>—</td>
<td>500</td>
<td>800</td>
<td>μA</td>
</tr>
<tr>
<td>Supply Current</td>
<td>$I_{DD}$</td>
<td>—</td>
<td>3</td>
<td>10</td>
<td>μA</td>
</tr>
<tr>
<td>Supply Current at Sleep Mode</td>
<td>$T_A$</td>
<td>—20</td>
<td>—</td>
<td>+85</td>
<td>°C</td>
</tr>
<tr>
<td>Acceleration Range, X-Axis, Y-Axis, Z-Axis</td>
<td>$g_{rs}$</td>
<td>—</td>
<td>±1.5</td>
<td>—</td>
<td>g</td>
</tr>
<tr>
<td>$g_{rs}$</td>
<td>—</td>
<td>±2</td>
<td>—</td>
<td>g</td>
<td></td>
</tr>
<tr>
<td>$g_{rs}$</td>
<td>—</td>
<td>±4</td>
<td>—</td>
<td>g</td>
<td></td>
</tr>
<tr>
<td>$g_{rs}$</td>
<td>—</td>
<td>±8</td>
<td>—</td>
<td>g</td>
<td></td>
</tr>
</tbody>
</table>

Output Signal

Zero g ($T_A = 25^\circ C, V_{DD} = 3.3 V$)$^{(3)}$

Zero g | $V_{OFF}$ | 1.485 | 1.65 | 1.815 | V |

Sensitivity ($T_A = 25^\circ C, V_{DD} = 3.3 V$)

1.5g | $S_{1.5g}$ | 740 | 800 | 800 | mV/g |

2g | $S_{2g}$ | 555 | 600 | 645 | mV/g |

4g | $S_{4g}$ | 277.5 | 300 | 322.5 | mV/g |

6g | $S_{6g}$ | 185 | 200 | 215 | mV/g |

Sensitivity | $S_{T_A}$ | — | ±3 | — | %/°C |

Bandwidth Response

$XY$ | $f_{3dB}$ | — | 350 | — | Hz |

$Z$ | $f_{3dB}$ | — | 150 | — | Hz |

The MMA7260Q low cost capacitive micromachined accelerometer features signal conditioning, a 14-bit I2C interface, temperature compensation and a Selectable switch for the selection among 8 sensitivities. Z-zero offset full scale span and filter cut-off (4-8 times) is available. It requires no external bias. Includes a Sleep mode that makes it ideal for handheld battery powered electronics.

Features

- Selectable Sensitivity (1.5g, 2g, 4g)
- Low Current Consumption (150μA)
- Sleep Mode 0μA
- Low Voltage Operation: 2.2-V to 3.6-V
- 5000 g x 1400 g x 250 g
- ±250°/°C sensitivity
- Fast Turn On Time
- ±0.5% g long term stability
- Integral Signal Conditioning with Low Pass Filter
- Noise Design, High Shock Sensitivity
- Pin-Free Terminations
- Environmentally Preferred Package
- Low Cost
Odometry in 1D

\[ X_0 = 0 \]

\[ t=0 \]
Odometry in 1D

\[ X_1 = X_0 + \int_{0}^{1} \int_{0}^{1} a(t) dt \, dt \]

\( t=1 \)
Odometry in 1D

\[ X_1 = X_0 + a_1 \Delta t^2 \]
Odometry in 1D

\[
\begin{align*}
\hat{X}_1 &= X_0 + (a_1 + \ddot{a}_1) \Delta t^2 \\
X_1 &= X_0 + a_1 \Delta t^2
\end{align*}
\]
Odometry in 1D

\[
\hat{X}_2 = \hat{X}_1 + (a_2 + \ddot{a}_2) \Delta t^2 \\
X_2 = X_1 + a_2 \Delta t^2
\]
Odometry in 1D

\[
\hat{X}_3 = \hat{X}_2 + (a_3 + \ddot{a}_3) \Delta t^2
\]

\[
X_3 = X_2 + a_3 \Delta t^2
\]
Odometry in 1D

\[ \hat{X}_4 = \hat{X}_3 + (a_4 + \ddot{a}_4) \Delta t^2 \]

\[ X_4 = X_3 + a_4 \Delta t^2 \]
Odometry in 1D

\[ \hat{X}_5 = \hat{X}_4 + (a_5 + \ddot{a}_5) \Delta t^2 \]

\[ X_5 = X_4 + a_5 \Delta t^2 \]
1D Odometry: Error Modeling

- Error happens!
- Odometry error is cumulative.
  → grows without bound
- We need to be aware of it.
  → We need to model odometry error.
  → We need to model sensor error.
- Acceleration is random variable $A$ drawn from “mean-free” Gaussian ("Normal") distribution.
  → Position $X$ is random variable with Gaussian distribution.
1D Odometry with Gaussian Uncertainty

![Graph showing estimated and true positions at t=0](image)
1D Odometry with Gaussian Uncertainty

\[ t=1 \]

\[ \mu = 1 \]

\[ \sigma = 0.2 \]
1D Odometry with Gaussian Uncertainty

\[ \mu = 2 \]
\[ \sigma = 0.4 \]

estimated position
true position

\[ t = 2 \]
1D Odometry with Gaussian Uncertainty

\[ \mu = 3 \]
\[ \sigma = 0.6 \]
1D Odometry with Gaussian Uncertainty

\[ \mu = 4 \]
\[ \sigma = 0.8 \]

\( t = 4 \)

Estimated position
True position
1D Odometry with Gaussian Uncertainty

\[ \mu = 5 \]
\[ \sigma = 1 \]
Nondeterministic Error Sources in 2D Odometry using Wheel Encoders

- Variation of the contact point of the wheel
- Unequal floor contact (e.g., wheel slip, nonplanar surface)

- Wheels cannot be assumed to roll perfectly
- Measured encoder values do not perfectly reflect the actual motion
- Pose error is cumulative and incrementally increases
- Probabilistic modeling for assessing quantitatively the error
Odometric Error Types and Visualization

- Range error: sum of the wheel movements
- Turn error: difference of wheel motion
- Drift error: difference between wheel errors lead to heading error

Example for a differential-wheel vehicle: Gaussian assumption; ellipses shows $3\sigma$ bound

[From *Introduction to Autonomous Mobile Robots*, Siegwart R. and Nourbakhsh I. R.]
Robot Localization based on Proprioceptive and Exteroceptive Sensors
Features

- Odometry based position error grows without bound.
- Use relative measurement to features ("landmarks") to reduce position uncertainty

**Feature:**
- Uniquely identifiable
- Position is known
- We can obtain relative measurements between robot and feature (usually angle or range).

**Examples:**
- Doors, walls, corners, hand rails
- Buildings, trees, lanes
- GPS satellites
Automatic Feature Extraction

• High level features:
  • Doors, persons

• Simple visual features:
  • Edges (Canny Edge Detector 1983)
  • Corner (Harris Corner Detector 1988)
  • Scale Invariant Feature Transformation (2004)

• Simple geometric features
  • Lines
  • Corners

• “Binary” feature
Feature-based navigation

\[ t = 5 \]

\[ \mu = 5; \sigma = 1 \]
Feature-based navigation

$t=5$

$\mu = 5; \sigma = 1$
Feature-based navigation

\[ t = 5 \]

\[ \mu = 5; \sigma = 1 \]

\[ r = 3.2m \]
Feature-based navigation

- Estimated position
- True position
- Range-based position

$t = 5$

$\mu = 5; \sigma = 1$

$\mu = 5.8; \sigma = 1.2$

$r = 3.2m$
Feature-based navigation

\[ t=5 \]

\[ \mu = 5; \sigma = 1 \]

\[ \mu = 5.8; \sigma = 1.2 \]

\[ r = 3.2m \]
Sensor fusion

- Given:
  - Position estimate $X \leftarrow N(\mu=5;\sigma=1)$
  - Range estimate $R \leftarrow N(\mu=3.2;\sigma=1.2)$

What is the best estimate AFTER incorporating $r$?

→ Kalman Filter

- Requires:
  - Gaussian noise distribution for all measurements
  - Linear motion and measurement model
  - …

Feature-based navigation

\[ t = 5 \]

\[ \mu = 5; \sigma = 1 \]

\[ \mu = 5.8; \sigma = 1.2 \]

\[ r = 3.2m \]
Feature-based navigation

\[ t = 5 \]

\[ \mu = 5.5; \sigma = 0.6 \]

\[ \mu = 5; \sigma = 1 \]

\[ \mu = 5.8; \sigma = 1.2 \]

\[ r = 3.2m \]
Feature-based navigation

\[ t = 5 \]

\[ \mu = 5.5; \sigma = 0.6 \]
Feature-based navigation

$t=6$

$\mu = 6.5; \sigma = 0.8$
Feature-based navigation

$t = 5$

$\mu = 5.5; \sigma = 0.6$
Feature-based navigation

Belief representation through Gaussian distribution

• Advantages:
  • Compact (only mean and variance required)
  • Continuous
  • Powerful tools (Kalman Filter)

• Disadvantages:
  • Requires Gaussian noise assumption
  • Uni-modal
  • Cannot represent ignorance ("kidnapped robot problem")
Feature-based navigation

![Graph showing estimated and true positions at time t=5]
Feature-based navigation

![Graph showing estimated position, true position, and range-based position with a range of 3.2m at t=5]
Feature-based navigation

\[ t = 5 \]

- estimated position
- true position
- range-based position

\[ r = 3.2m \]
Feature-based navigation

$t=5$

- estimated position
- true position
- range-based position

$r = 4.2m$

$r = 3.2m$
Feature-based navigation

![Graph showing estimated and true positions with range-based positions at t=5 and r=4.2m, r=3.2m]
Feature-based navigation
Feature-based navigation

$t=0$
Feature-based navigation

\[ \forall \text{particles } s_i : s_i^{t=1} = s_i^{t=0} + a + \tilde{a}(N(\mu = 0, \sigma = 0.2)) \]
Feature-based navigation

\[ t = 2 \]

\[ \mu = 2 \]
\[ \sigma = 0.4 \]

\[ \forall \text{particles } s_i : s_i^{t=2} = s_i^{t=1} + a + \tilde{\alpha}(\mathcal{N}(\mu = 0, \sigma = 0.2)) \]
Feature-based navigation

\[ \forall \text{particles } s_i : s_i^{t=3} = s_i^{t=2} + a + \tilde{a}(\mathcal{N}(\mu = 0, \sigma = 0.2)) \]

\[ \mu = 3, \quad \sigma = 0.6 \]

\[ t = 3 \]
Feature-based navigation

\[ \forall \text{particles } s_i: s_i^{t+1} = s_i^t + a + \tilde{a}(N(\mu = 0, \sigma = 0.2)) \]

\[ \mu = 4 \]
\[ \sigma = 0.8 \]

\( t = 4 \)

Estimated position
True position

\( X \text{[m]} \)
Feature-based navigation

\[ \forall \text{particles } s_i: \quad s_i^{t=5} = s_i^{t=4} + a + \tilde{a}(\mathcal{N}(\mu = 0, \sigma = 0.2)) \]
Feature-based navigation

\[ \forall \text{ particles } s_i: \quad s_i^{t=5} = s_i^{t=4} + a + \tilde{a}(N(\mu = 0, \sigma = 0.2)) \]
Feature-based navigation

∀ particles $s_i$: $s_i^{t=5} = s_i^{t=4} + a + \tilde{a}(\mathcal{N}(\mu = 0, \sigma = 0.2))$
Feature-based navigation

∀ particles \( s_i \): \( s_i^{t=5} = s_i^{t=4} + a + \tilde{a}(N(\mu = 0, \sigma = 0.2)) \)
Feature-based navigation

\forall particles s_i: s_i^{t=5} = s_i^{t=4} + a + \tilde{a}(N(\mu = 0, \sigma = 0.2))
Feature-based navigation

∀ particles $s_i$: $s_i^{t=5} = s_i^{t=4} + a + \tilde{a}(\mathcal{N}(\mu = 0, \sigma = 0.2))$
Feature-based navigation

∀ particles $s_i$: $s_i^{t=5} = s_i^{t=4} + a + \tilde{a}(N(\mu = 0, \sigma = 0.2))$
Feature-based navigation

$t = 5$

- estimated position
- true position
- range-based position

$r = 4.2m$

$r = 3.2m$
Feature-based navigation

$t=5$

estimated position
true position
range-based position
Feature-based navigation

Belief representation through particle distribution

• Advantages:
  • Can model arbitrary beliefs
  • No assumptions on noise characteristic

• Disadvantages:
  • No unique solution
  • Not continuous
  • Computationally expensive
  • Tuning required
Feature-based navigation
Feature-based navigation
Conclusion
Take Home Messages

• There are several localization techniques for indoor and outdoor systems
• Each of the localization methods/positioning system has advantage and drawbacks
• Odometry is an absolute positioning method using only proprioceptive sensors but affected by a cumulative error
• Localization error can be modeled and estimated
• The error propagation methods/filtering techniques are tools applicable to a large variety of noisy problems, not just localization and navigation
• Feature-based navigation is a way to compensate odometry limitations
Additional Literature – Week 11

Books

• Weston J. and Titterton D, “Strapdown Inertial Navigation”, IET, 2005
