Lab 5

School of Architecture, Civil and Environmental Engineering

EPFL, SS 2015-2016

http://disal.epfl.ch/teaching/signals_instruments_systems/
Lab 5 Outline

• Concepts:
  – Convolution
  – Sampling
  – Aliasing
  – Reconstruction

• Tools:
  – Matlab
Part 1: Continuous Convolution

\[ (f * g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) \, d\tau \]

- For each value of \( t \):
  1. Flip (reflect) \( g \)
  2. Shift \( g \) by \( t \)
  3. Multiply \( f \) and \( g \)
  4. Integrate over \( \tau \)

- Note that the result does not depend on \( \tau \)!

\[ \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) \, d\tau \]
Part 2: Discrete Convolution

\[(f \ast g)(t) = \int_{-\infty}^{\infty} f(\tau) \cdot g(t - \tau) d\tau\]

\[(f \ast g)(n) = \sum_{m=-\infty}^{\infty} f(m) \cdot g(n - m)\]

- Similar to the continuous version
- The integral becomes an infinite sum
- Matlab, operating on a computer, can only emulate continuity and therefore use the discrete version with an adjustable discretization level in time and amplitude
Example of Discrete Convolution

\[ (f * g)[n] \]

\[ f[m] \]
\[ f[m] \]
\[ g[m] \]
\[ g[-m] \]
Part 3: Sampling and reconstruction

Signal generation

Filtering

Sampling

Reconstruction

- \( f(t) = \sum_i A_i \sin(2\pi f_i t) \)
- frequencies
- amplitudes

- Not in this lab

- Sampling frequency

- Linear interpolation
- Whittaker-Shannon
Reconstruction: Linear vs. Wittaker-Shannon

Graphic Illustration of Time-Domain Interpolation

- Original CT signal $x(t)$
- After sampling
- After passing the LPF $x_r(t)$

Interpolation Methods

- Bandlimited Interpolation
- Zero-Order Hold
- First-Order Hold — Linear interpolation

Mathematical Representation:

$$\rho(t) = \sum_{n=-\infty}^{+\infty} \delta(t - nT)$$