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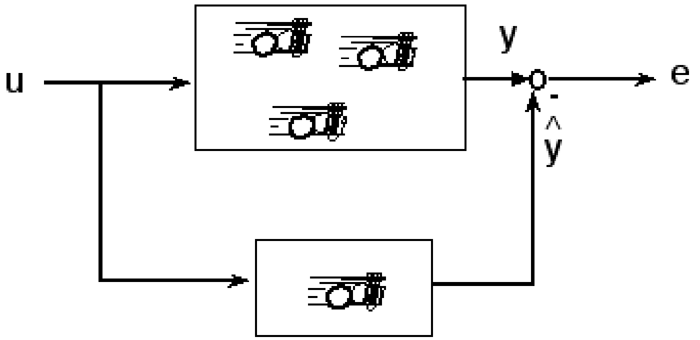
Distributed Intelligent Systems – W8

Multi-Level Modeling Methods Applied to Distributed Robotic Systems

Outline

- Multi-Level Modeling Methodology
  - Rationale
  - Theoretical background
  - Methodological framework



- Examples
  - Obstacle avoidance (linear)
  - Collaborative stick pulling (nonlinear)

Modeling Rationale, Choices, and Framework Overview

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Motivation for Modeling

- Understanding the interplay of the various elements of the system (e.g., robot features, robot numbers, environment, noise level)
- Having additional tools for designing and optimizing the distributed robotic system
- Delivering performance predictions for the ensemble in shorter time or before doing actual experiments
- Investigating experimental conditions difficult or impossible to reproduce in reality
- Formally analyzing system properties

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Modeling Choices

- Gray-box approach: to easily incorporate a priori information (e.g., # of agents, technological and environmental features)
- Probabilistic: to capture noisy interactions, noisy robotic components, stochastic control policies, and enable aggregation schemes towards abstraction
- Multi-level: to represent explicitly different design choices, trade off computational speed and faithfulness to reality, bridge mathematically tractable models and reality in an incremental way
- Bottom-up: start from the physical reality and increase the abstraction level until the highest abstraction level

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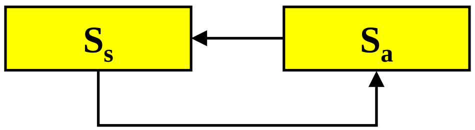
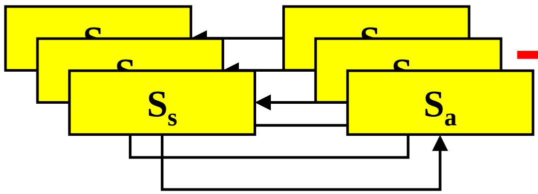
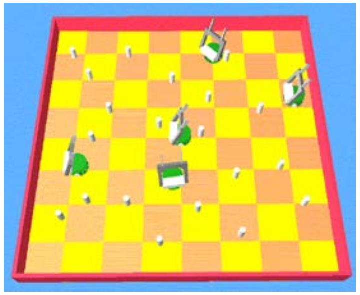
Multi-Level Modeling Methodology



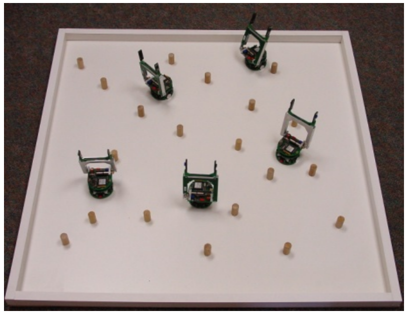
Macroscopic: representation of the whole swarm (typically a mathematical model)



Microscopic : multi-agent models, only relevant robot features captured, 1 agent = 1 robot



Submicroscopic: intra-robot (e.g., S&A, transceiver) and environment (e.g., physics) details reproduced faithfully



Target system (physical reality): information on controller, S&A, communication, morphology and environmental features

Experimental time

Abstraction

Common metrics

Multi-Level Implementation Choices for this Course

- Submicroscopic: Webots
- Microscopic: non spatial, state = behavior, exact model in terms of quantities (e.g., agent/state)
- Macroscopic: non spatial, mean field approach, Ordinary Differential Equation (ODE) approximation applies (e.g., average number agents/state)

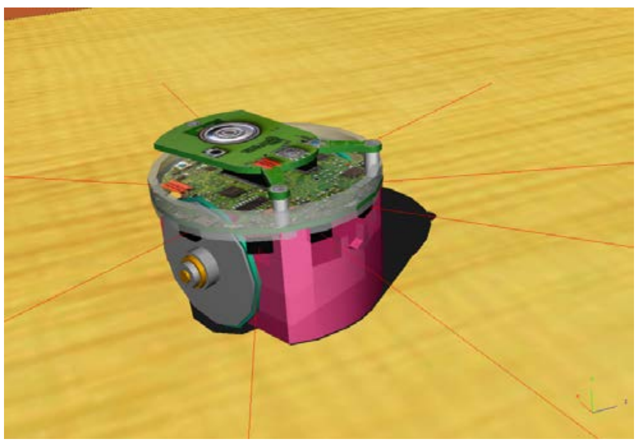
Experimental Invariant Features and Modeling Assumptions

Invariant Experimental Features

- Short-range (typically 1 robot diameter), crude (noisy, a few discrimination levels) proximity sensing
- Full mobility but limited navigation (no planning, no absolute localization)
- Limited use of long-range communication channels available on the platforms (only as a teammate sensor)
- Reactive, behavior-based control, with a few internal states
- No overcrowded arenas
- Multiple runs (typically 5+) for the same experimental parameters; randomized robot poses at the beginning

Modeling Assumptions: Semi-Markovian Properties

- Description for environment and multi-robot system using states
- The system future state is a function of the current state (and possibly of the amount of time spent in it)

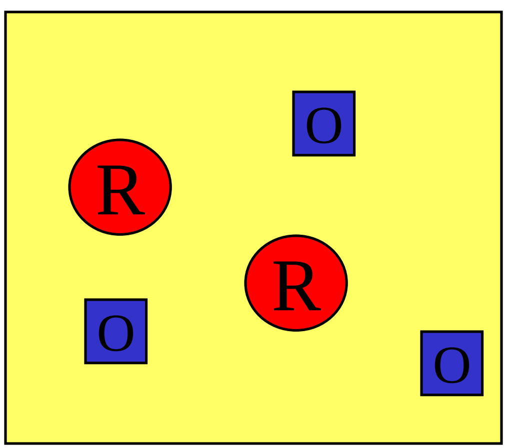


Submicroscopic
(pose, S&A state, etc.)

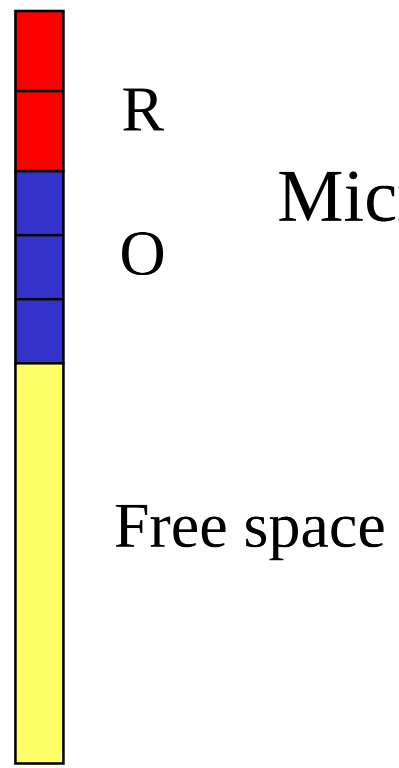
Microscopic/Macroscopic
(transition probabilities, state duration)

Modeling Assumptions: Spatiality

- nonspatial metrics for collective performance
- well-mixed system because of simple navigation, multiple randomized interactions in a convex environment, multiple runs with randomized initial conditions, no overcrowding (sparseness)

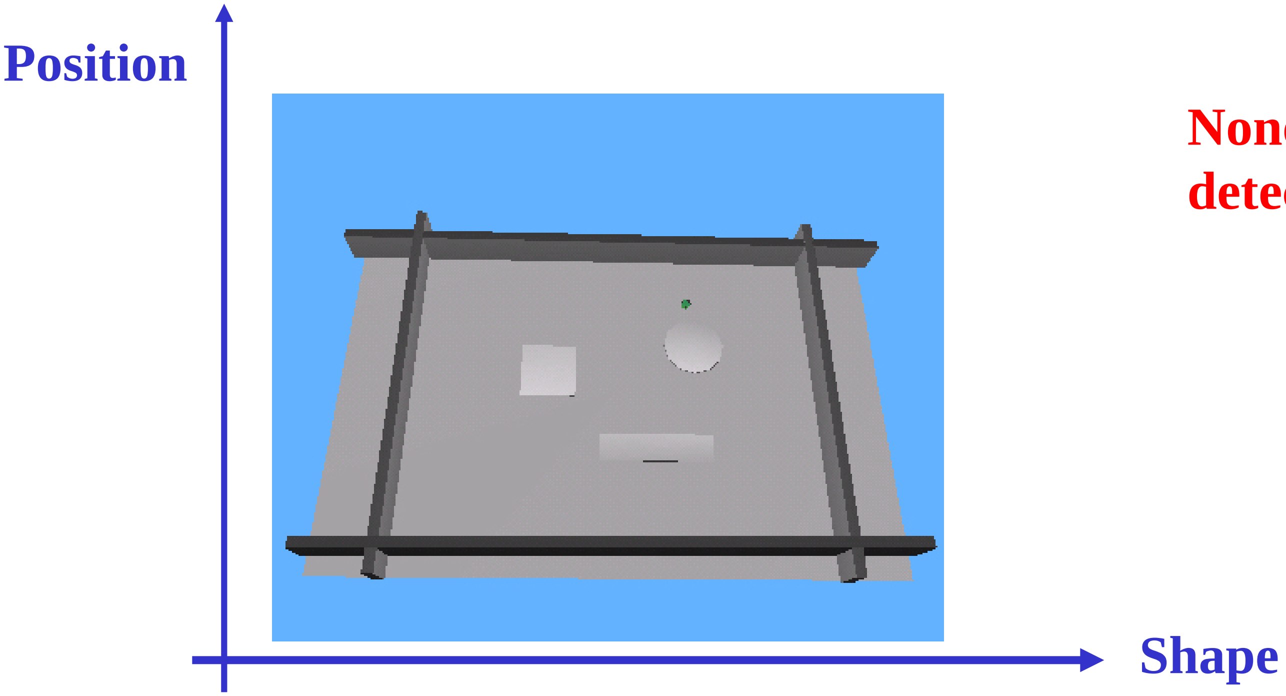


Submicroscopic: spatial



Micro/macroscopic: nonspatial

Experimental Validation of Spatiality Assumption

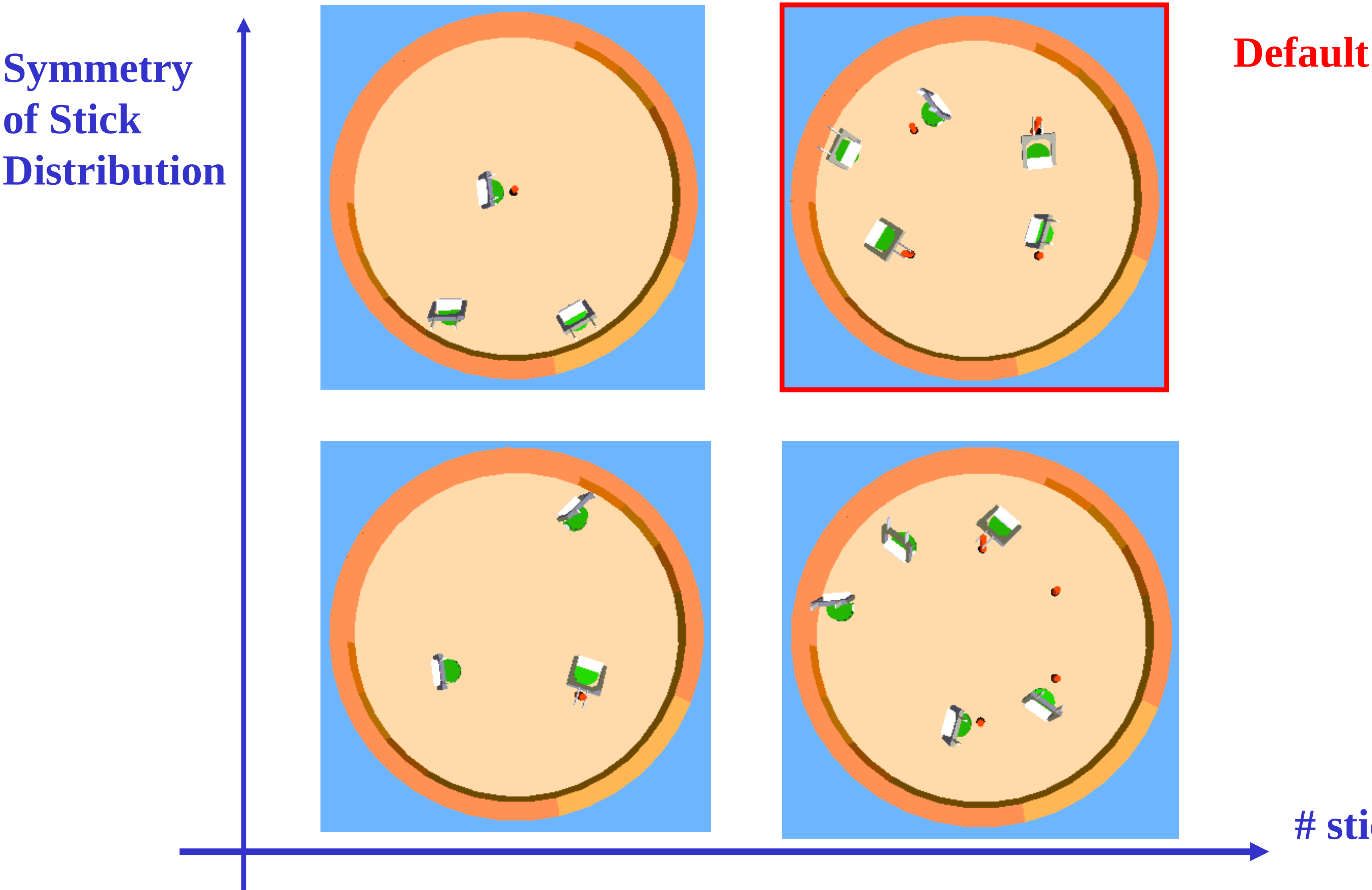


Nonembodied obstacles = detection surfaces

Numerical example (mean ± std dev, 3 locations, 100 h simulated time):

Size	Square	Rect.	Round	All shapes	Geometry
robot	0.31 ± 0.04	0.3 ± 0.03	0.32 ± 0.02	0.31 ± 0.03	0.31

Experimental Validation of Spatiality Assumption



Methodological Framework: Theoretical Background

Microscopic Level

p(n,t) = probability of an agent to be in the state *n* at time *t*

If Markov properties fulfilled:

$$\Delta p(n,t) = p(n,t+\Delta t) - p(n,t)$$

$$= \underbrace{\sum_{n'} p(n,t+\Delta t \mid n',t)\, p(n',t)}_{\text{inflow}} - \underbrace{\sum_{n'} p(n',t+\Delta t \mid n,t)\, p(n,t)}_{\text{outflow}}$$

Transition probability

Sum over all possible states n' the agent can be in

Probability the agent was in a given state n'

Macroscopic Level – Time-Continuous

Left and right side of the equation: averaging over the total number of agents, dividing by Δt, limit $\Delta t \rightarrow 0$; neglect distributions of the stochastic variables and assume homogeneous agents (mean field approach):

$$\frac{dN_n(t)}{dt} = \underbrace{\sum_{n'} W(n|n',t)N_{n'}(t)}_{\text{inflow}} - \underbrace{\sum_{n'} W(n'|n,t)N_n(t)}_{\text{outflow}}$$

Rate Equation
(time-continuous)

n, n' = states of the agents (all possible states at each instant)
N_n = average fraction (or mean number) of agents in state n at time t

$$W(n|n';t) = \lim_{\Delta t \rightarrow 0} \frac{p(n, t+\Delta t | n', t)}{\Delta t}$$

Transition rate

Macroscopic Level – Time-Discrete

Rate Equation (time-discrete):

$$N_n((k+1)T) = N_n(kT) + \sum_{n'} TW(n \mid n', kT) N_{n'}(kT) - \sum_{n'} TW(n' \mid n, kT) N_n(kT)$$

inflow — outflow

k = iteration index

T = time step, sampling interval

TW = transition probability per time step

Notation often simplified to:

$$N_n(k+1) = N_n(k) + \sum_{n'} P(n \mid n', k) N_{n'}(k) - \sum_{n'} P(n' \mid n, k) N_n(k)$$

T is specified in the text once of all, P is calculated from T*W or other calibration methods

Time Discretization: The Engineering Recipe

Time-discrete vs. time-continuous models:

1. Assess what's the time resolution needed for your system performance metrics (if time step chosen appropriately small, no impact on prediction accuracy in the type of experiments presented)

2. Choose whenever possible the most computationally efficient model: time-discrete less computationally expensive than emulation of continuity (e.g., Runge-Kutta, etc.)

3. Advantage of time-discrete models: a single common sampling rate can be defined among different modeling levels

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Methodological Framework: An Incremental Bottom-Up Recipe

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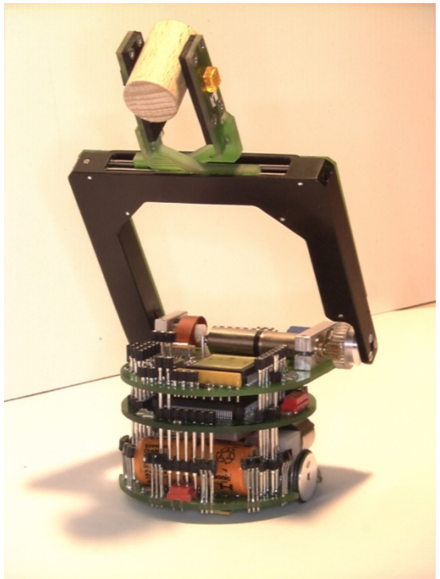
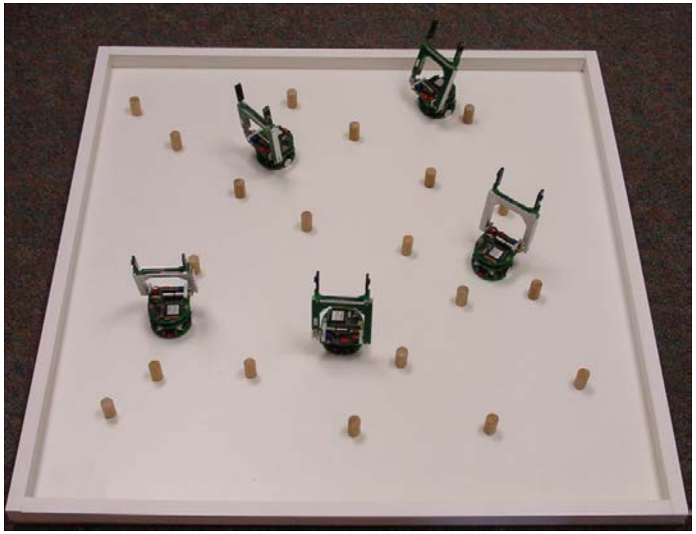
1. Target System & Task(s)

Perform basic design choices for the experimental set-up:

- Hardware and software for the robotic platform
- Environment in which robots operate
- Task(s) robots must accomplish

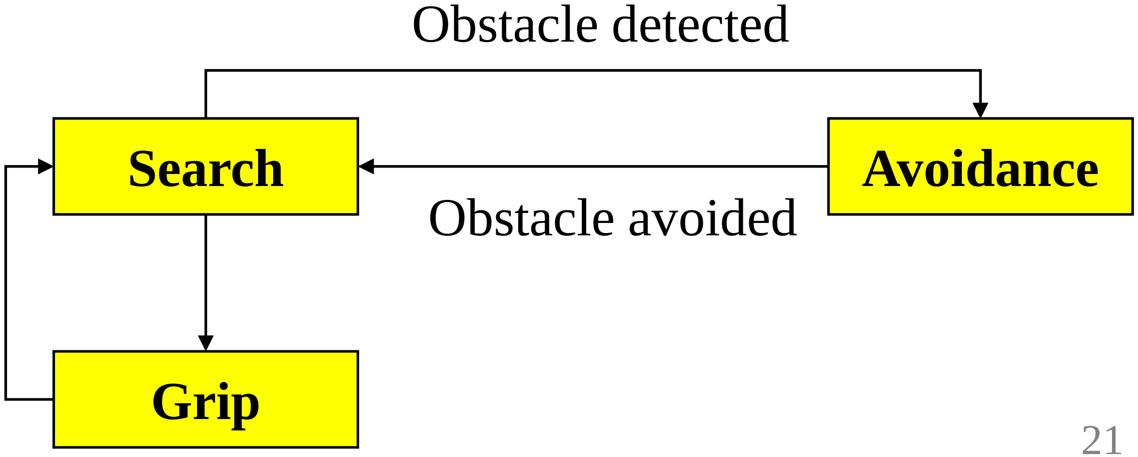




2. Metric(s) and State Space

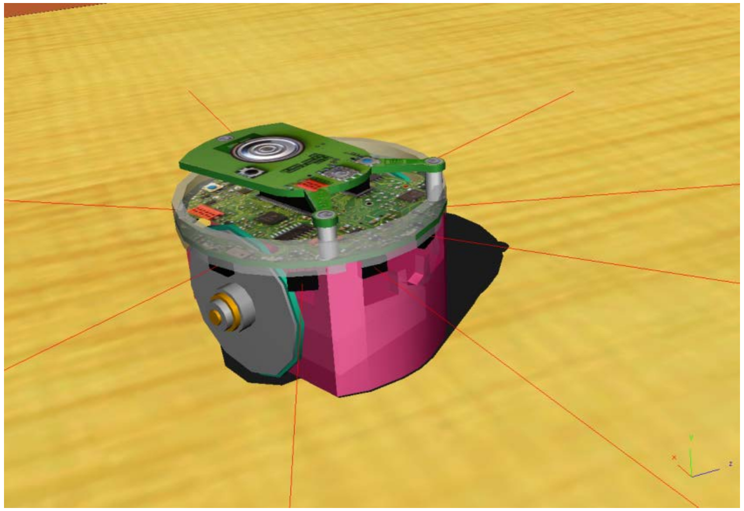
- Define system performance metric(s)
- Define state space (number of states, granularity)
- Performance metric(s) and state definitions well aligned!
- Exploit controller blueprint (if available) as additional source of information for defining the state space

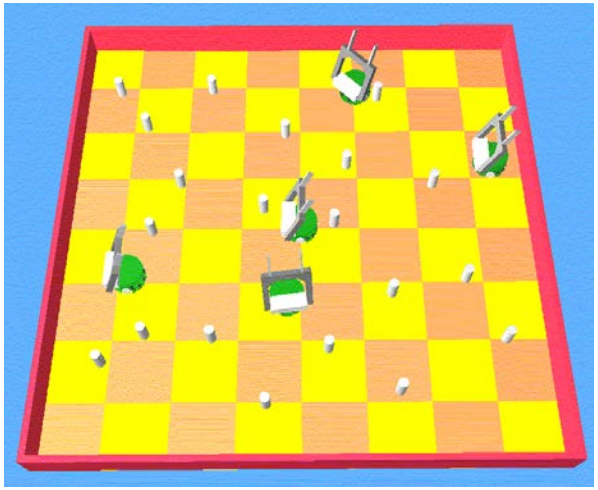
$$C(k) = p_{g2}N_s(k)N_g(k)$$



3. Submicroscopic Model

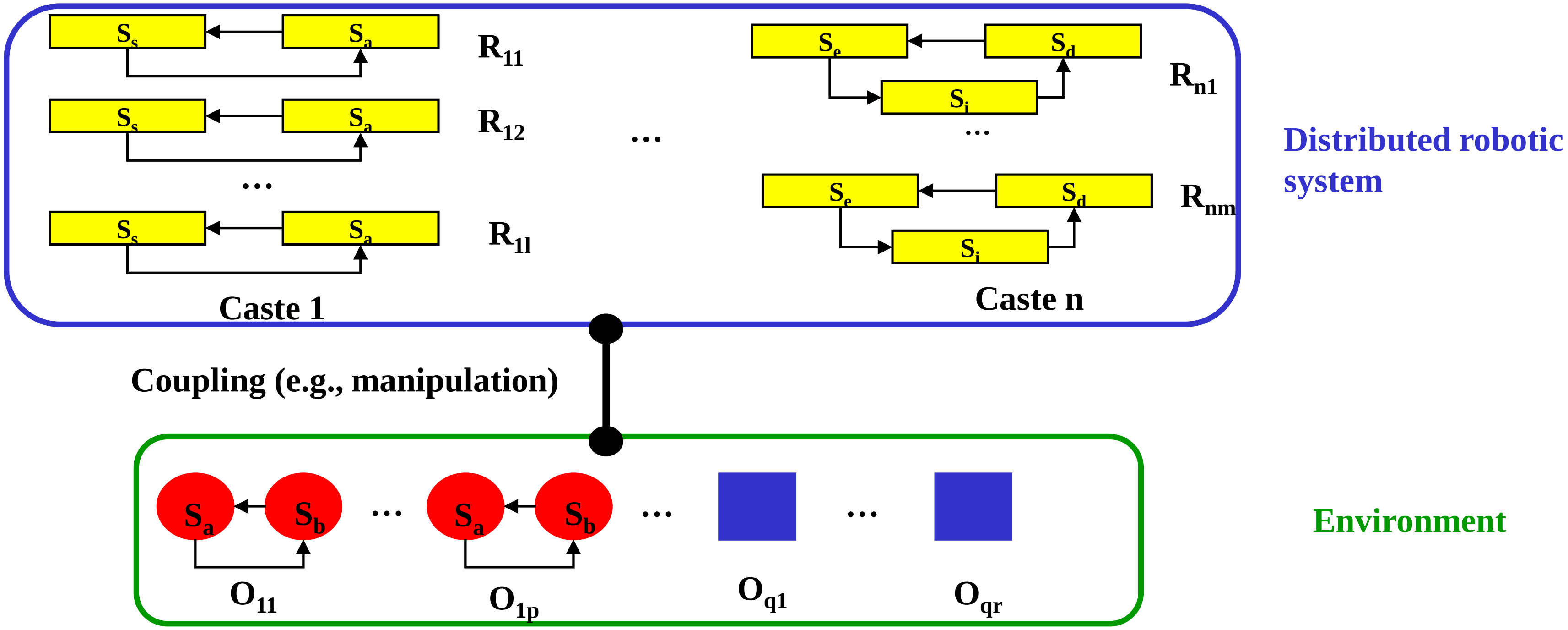
Implement faithfully your design choices in a submicroscopic model (in principle even running the same control code; libraries and APIs are usually provided in standard commercial or open-source simulators)





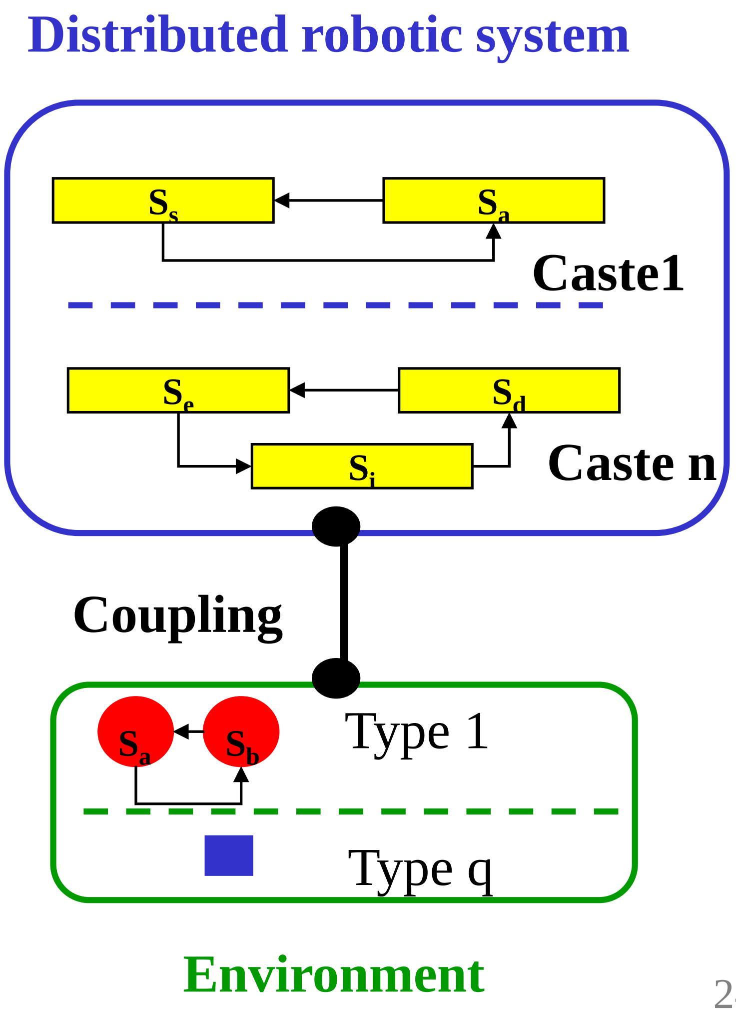
4. Microscopic Model

- Aggregate local interactions and reduce intra-robot details
- Maintain state space's structure as defined at Step 2
- Maintain individual representation (and exact discrete quantities) for each robotic node and environmental object of interest



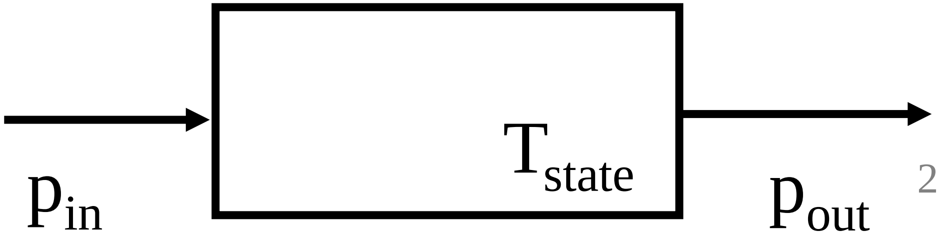
5. Macroscopic Model

- Aggregate individual nodes into one or multiple representations (castes) at collective level
- Maintain state space's structure as defined at Step 2
- Solve numerically or analytically the ODE system (mean field approach)
- Exploit conservation laws (e.g. # of robots in an enclosed arena) to simplify the representation of the dynamical system



6. Parameter Calibration

- Number of parameters is decreasing with the abstraction level
- Calibrate a given level based on the underlying one (e.g., submicroscopic with physical system; microscopic with submicroscopic, macroscopic with microscopic)
- Parametric (e.g., mean only, mean and variance) or non parametric (actual distribution recorded at the lower level) assumptions
- Various methods available
  - Ad hoc experiments [Correll & Martinoli, ISER 2004]
  - System identification techniques (e.g., constrained parameter fitting) [Correll & Martinoli, DARS 2006]
  - Statistical verification techniques (e.g., trajectory analysis) [Roduit et al., IROS 2007]
- Parameter example for micro- and macroscopic models:
  - State durations
  - State transition probabilities



State Durations & Discretization Interval

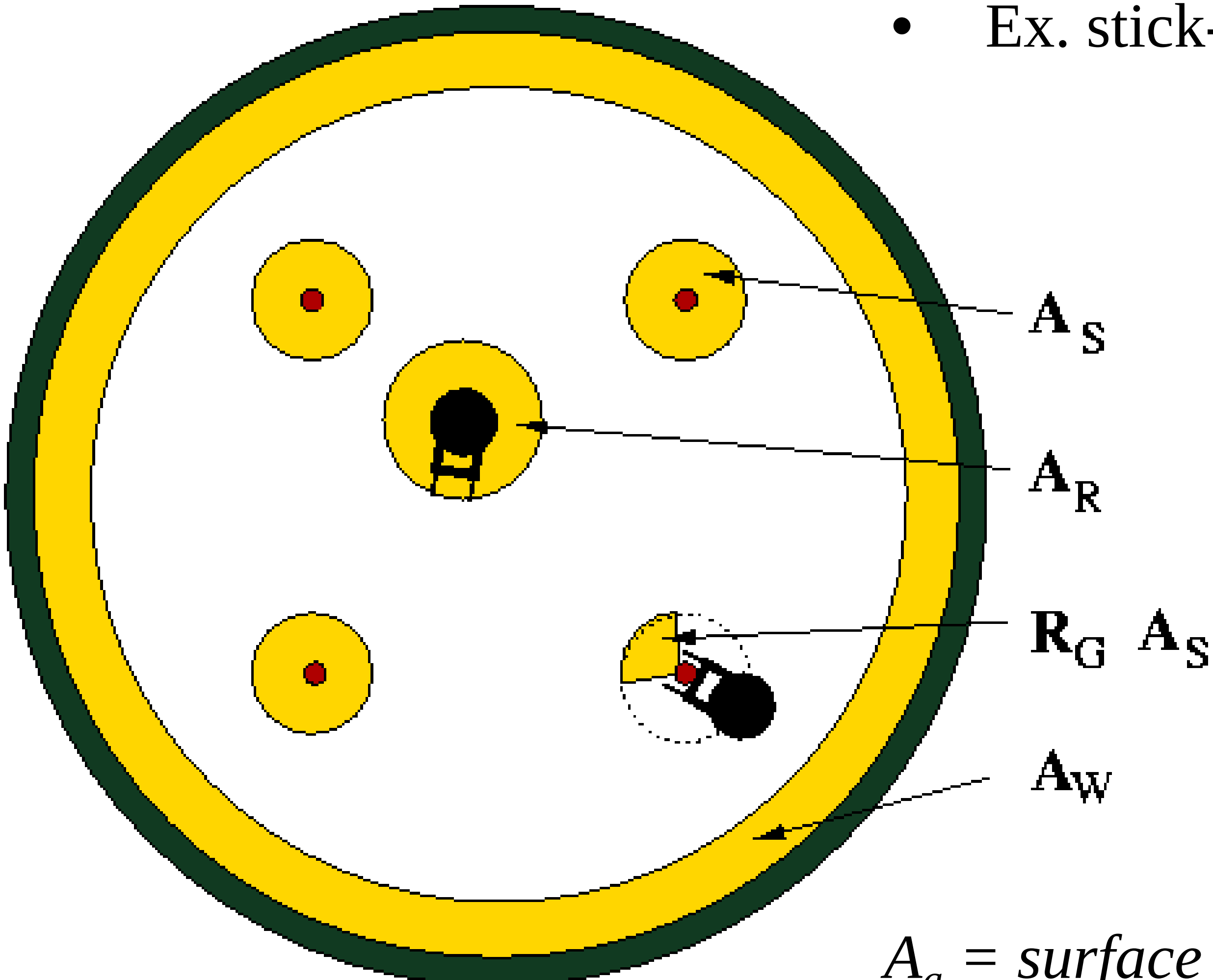
1. Measure all interaction times of interest in your system, i.e. those which might influence the system performance metrics.
   Note: often "delay states" can just summarize all what you need without getting into the details of what's going on within the state.

2. Consider only average values (we might consider also parameter distributions in the future, the modeling methodology does not prevent to do so)

3. For time-discrete systems: choose the **time step T** = GCF of all the durations measured (e.g., 3 s obstacle avoidance, 4 s object manipulation, T = 1 s) -> no rounding error.
   Note: more accuracy in parameter measuring means in this case more computational cost when simulating

State Transition Probabilities

- Geometric considerations
- Ad hoc calibration experiments
- Ex. stick-pulling experiment



$$p_s = A_s / A_a$$

$$p_r = A_r / A_a$$

$$p_R = p_r (N_0 - 1)$$

$$p_w = A_w / A_a$$

$$p_{g1} = p_s$$

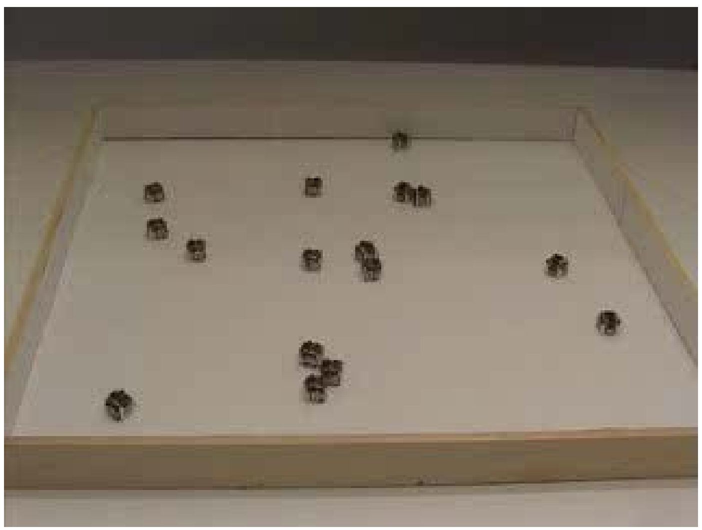
$$p_{g2} = R_g p_s$$

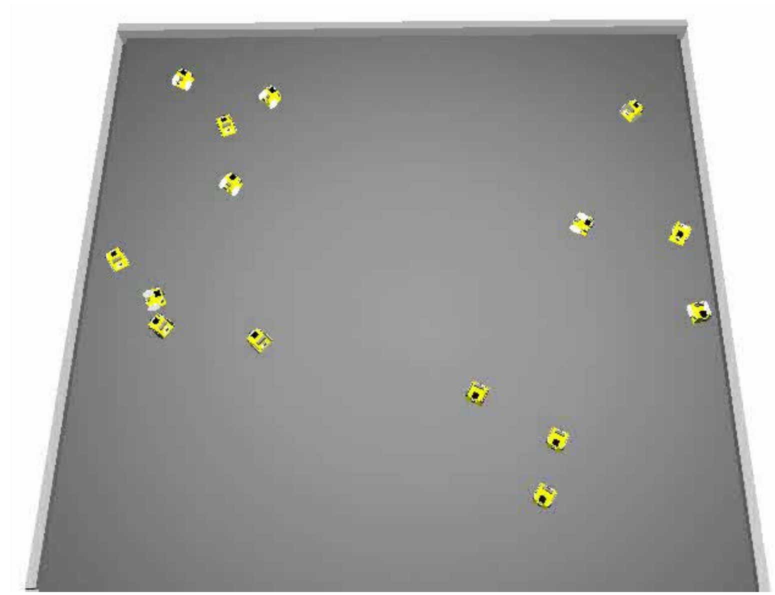
A_a = *surface of the whole arena*

Linear Example: Obstacle Avoidance

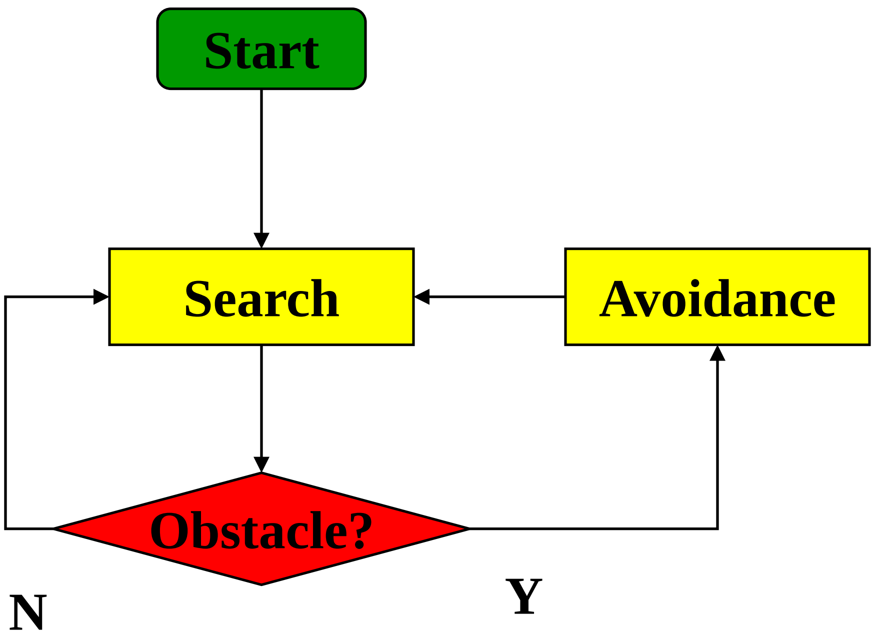
A Simple Linear Model

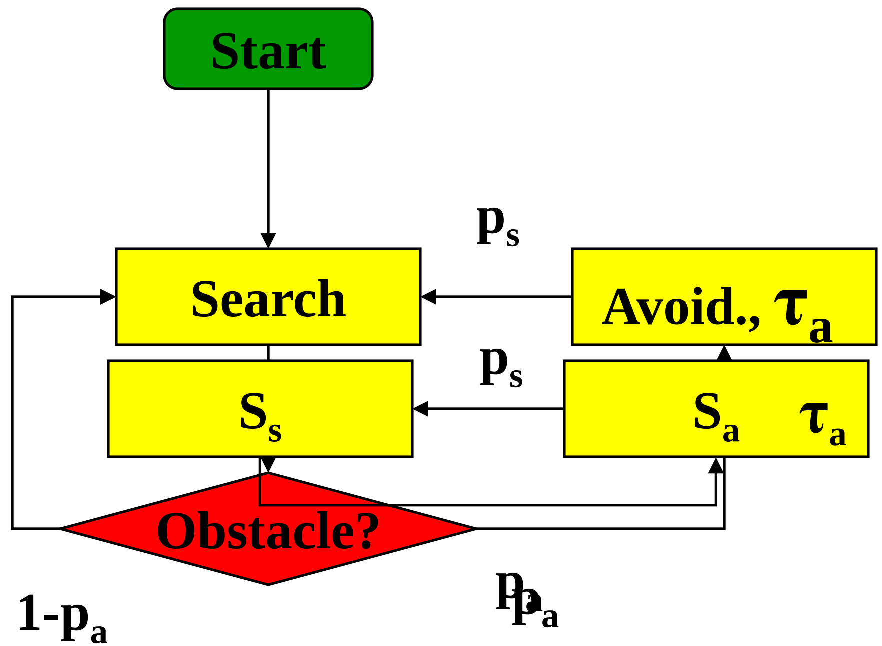
Example: search (moving forwards) and obstacle avoidance





A Simple Example





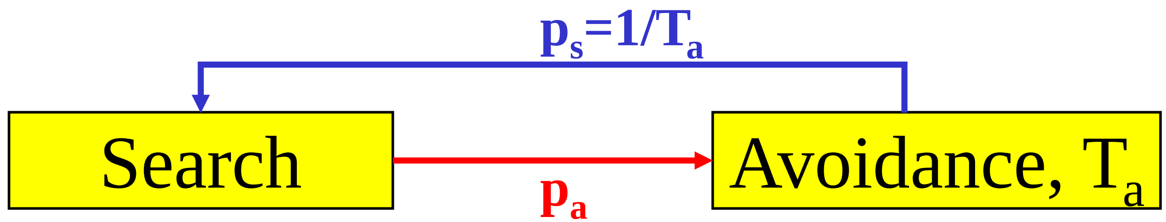
Deterministic robot's flowchart

Nonspatiality & microscopic characterization →

PFSM

Probabilistic agent's flowchart

Linear Model – Probabilistic Delay



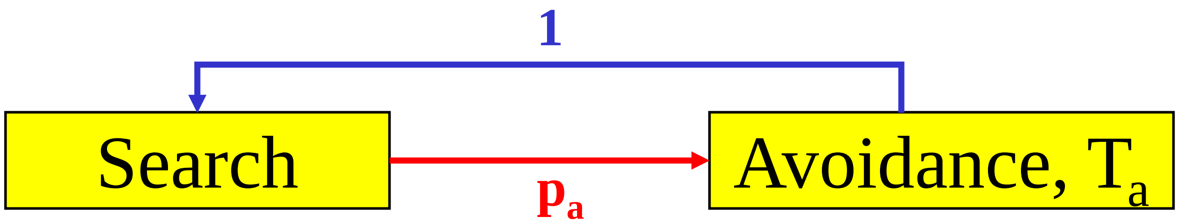
$$N_s(k+1) = N_s(k) - p_a N_s(k) + p_s N_a(k)$$

$$N_a(k+1) = N_0 - N_s(k+1)$$

$N_s(0) = N_0$; $N_a(0) = 0$

T_a = mean obstacle avoidance duration
p_a = probability of moving to obstacle av.
p_s = probability of resuming search
N_s = average # robots in search
N_a= average # robots in obstacle avoidance
N_0 = # robots used in the experiment
k = 0,1, … (iteration index)

Linear Model – Deterministic Delay



$$N_s(k+1) = N_s(k) - p_a N_s(k) + p_a N_s(k-T_a)$$

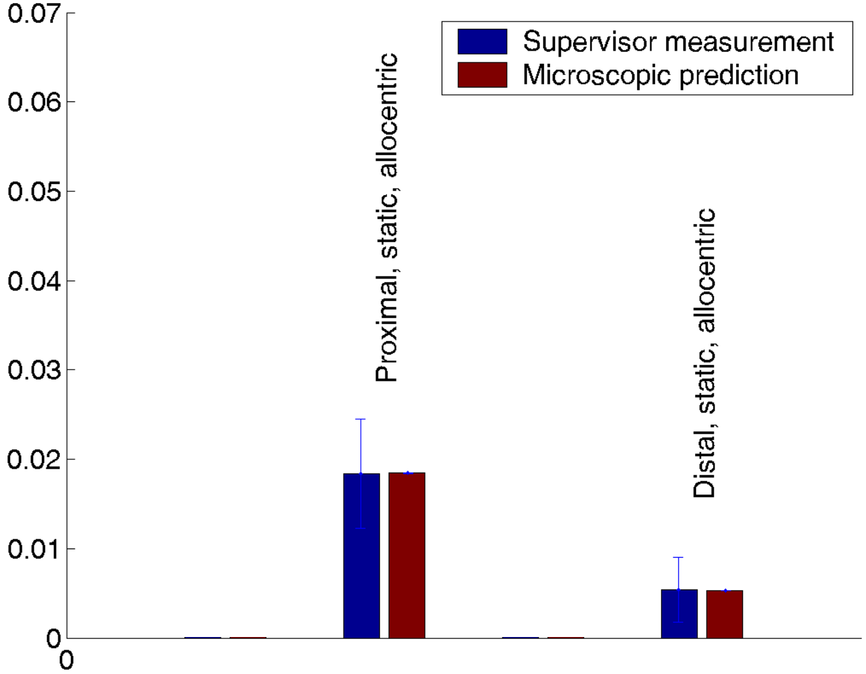
$$N_a(k+1) = N_0 - N_s(k+1)$$

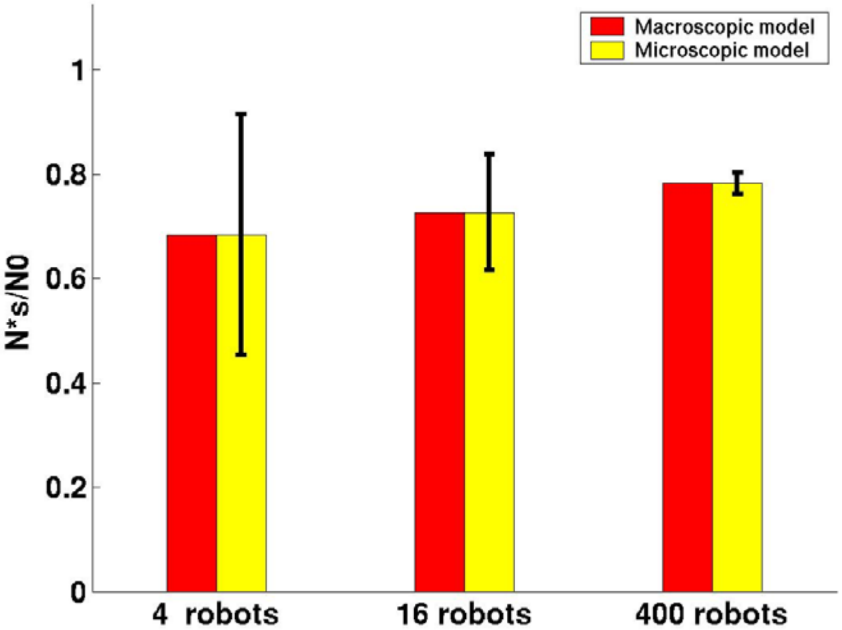
! $N_s(k) = N_a(k) = 0$ for all $k<0$!
$N_s(0) = N_0$; $N_a(0) = 0$

T_a = mean obstacle avoidance duration
p_a = probability moving to obstacle avoidance
N_s = average # robots in search
N_a = average # robots in obstacle avoidance
N_0 = # robots used in the experiment
k = 0,1, … (iteration index)

Linear Model – Sample Results

N_a*/N_0





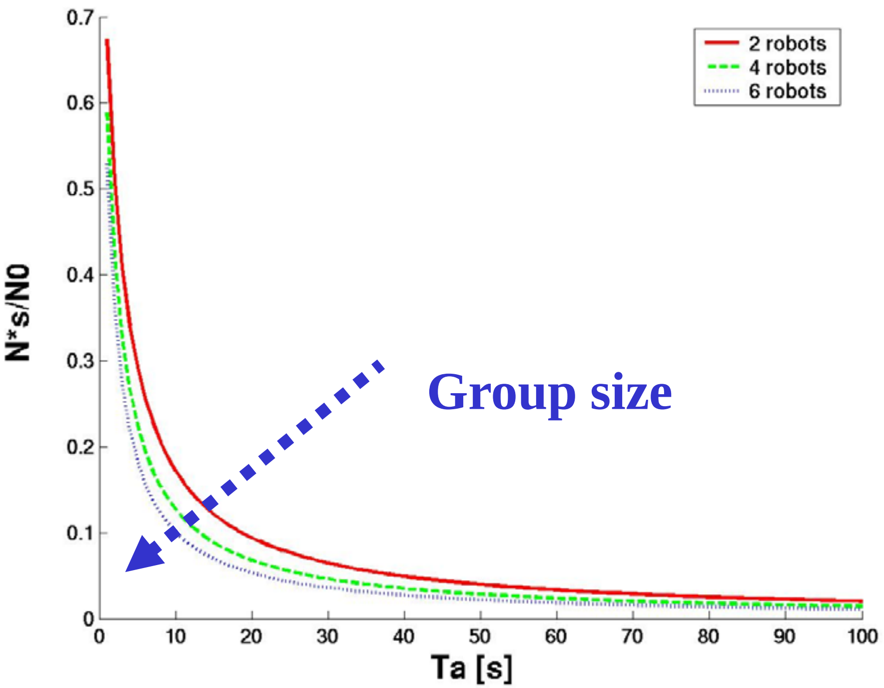
Submicro to micro comparison
(different controllers, steady state comparison)

Micro to macro comparison
(same robot density but wall surface become smaller with bigger arenas)

Steady State Analysis

- $N_n(k+1) = N_n(k)$ for all states n of the system $\rightarrow N_n^*$
- Note 1: equivalent to differential equation of $dN_n/dt = 0$
- Note 2: for time-delayed equations easier to perform the steady-state analysis in the Z-space but in t-space also ok (see IJRR-04)
- For our linear example (deterministic delay option):

$$N_s^* = \frac{N_0}{1 + p_a T_a} \qquad N_a^* = \frac{N_0 p_a T_a}{1 + p_a T_a}$$

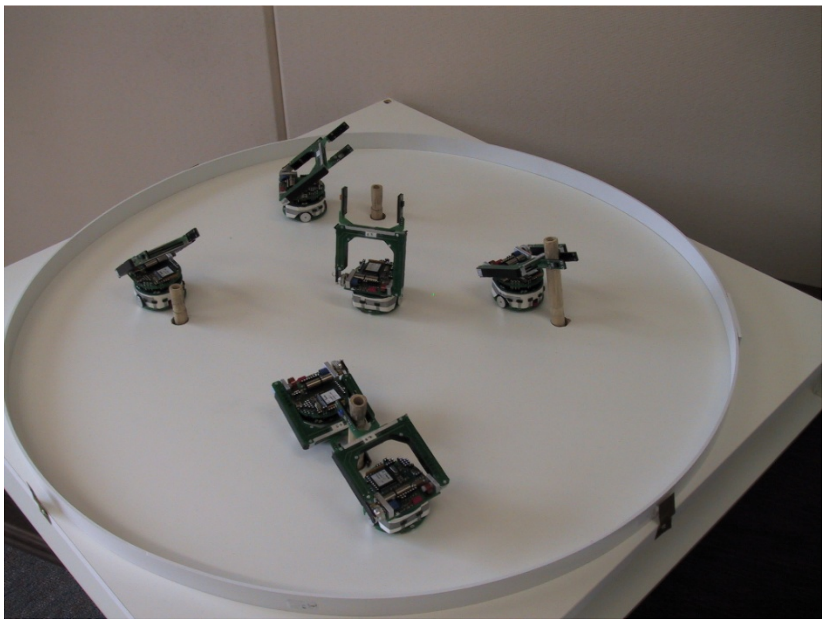


Ex.: normalized mean number of robots in search mode at steady state as a function of time for obstacle avoidance

Nonlinear Example – Collaborative Stick Pulling

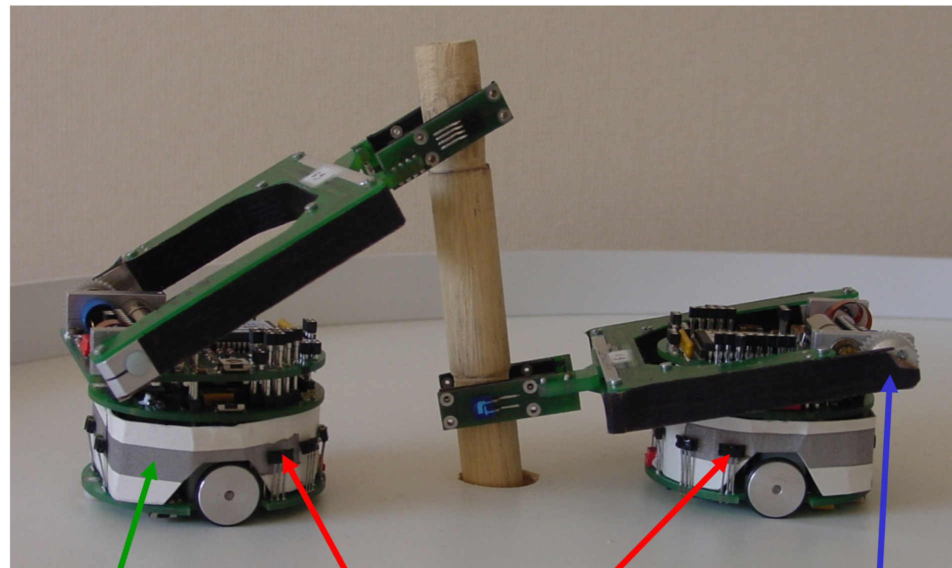
The Stick-Pulling Case Study

Physical Set-Up



- 2-6 robots
- 4 sticks
- 40 cm radius arena

Collaboration via indirect communication



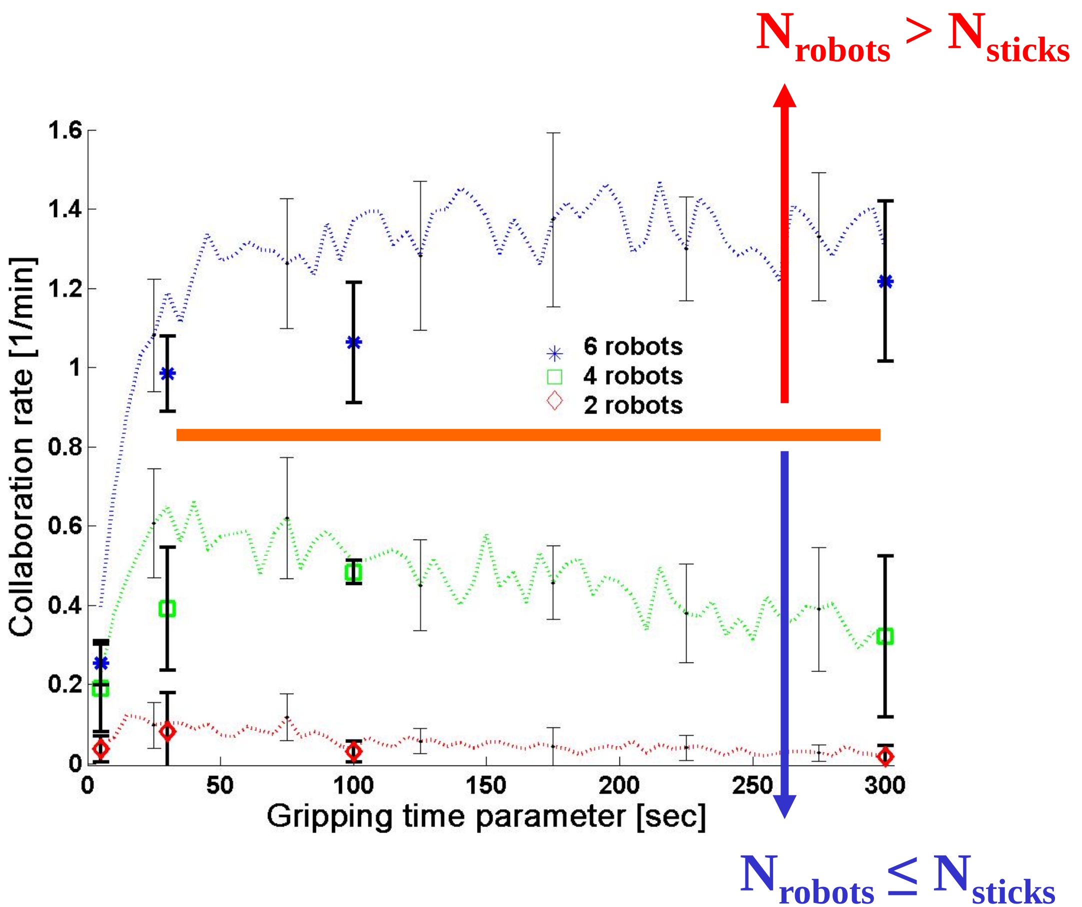
Systematic Experiments

Real robots

Submicroscopic model

- [Martinoli and Mondada, ISER, 1995]
- [Ijspeert et al., *AR*, 2001]

Results of Experiments and Submicroscopic Modeling



- Real robots (3 runs) and submicroscopic model (10 runs)
- System bifurcation as a function of #robots/#sticks

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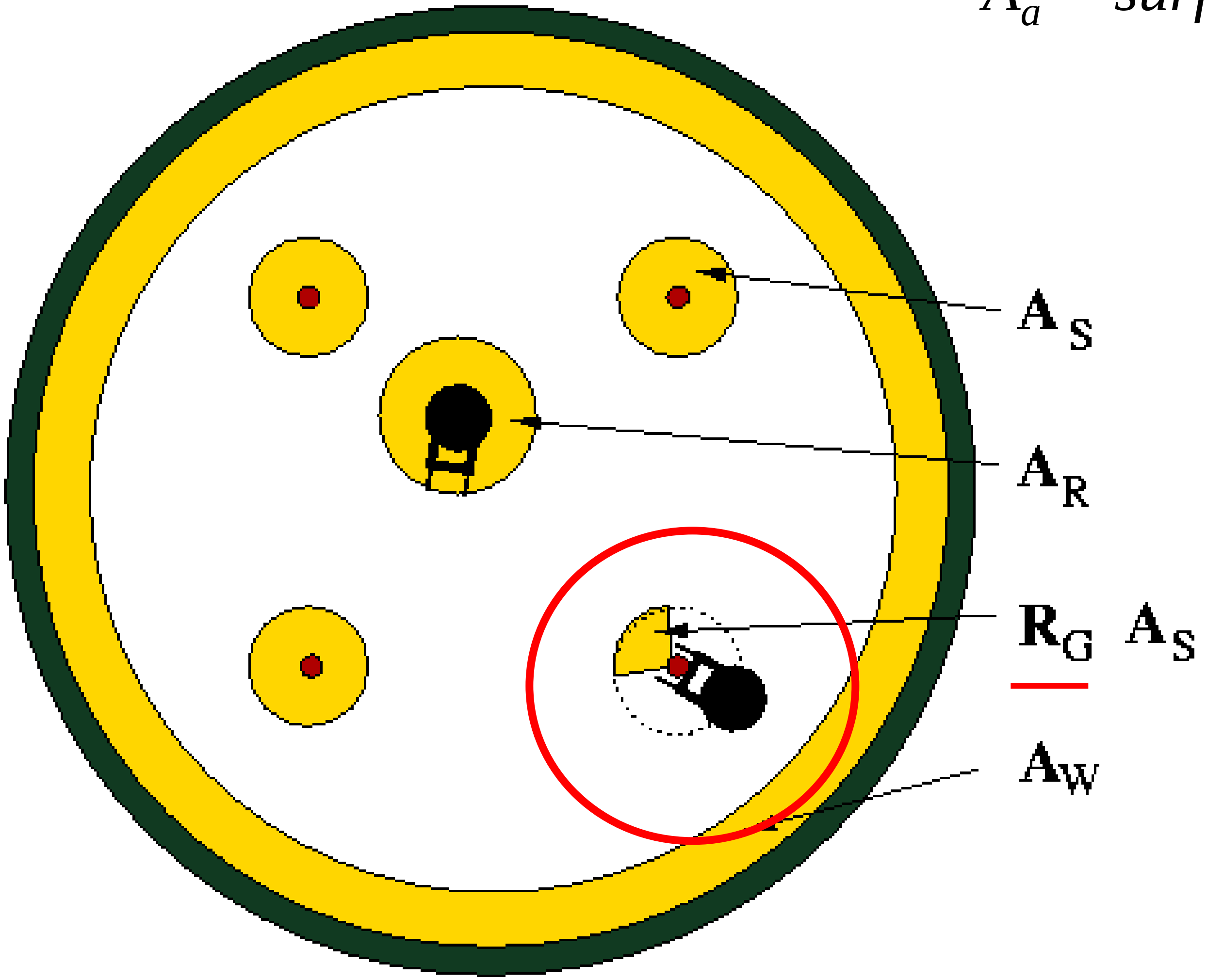
State Transition Probabilities

A_a = *surface of the whole arena*



$$p_s = A_s / A_a$$

$$p_r = A_r / A_a$$

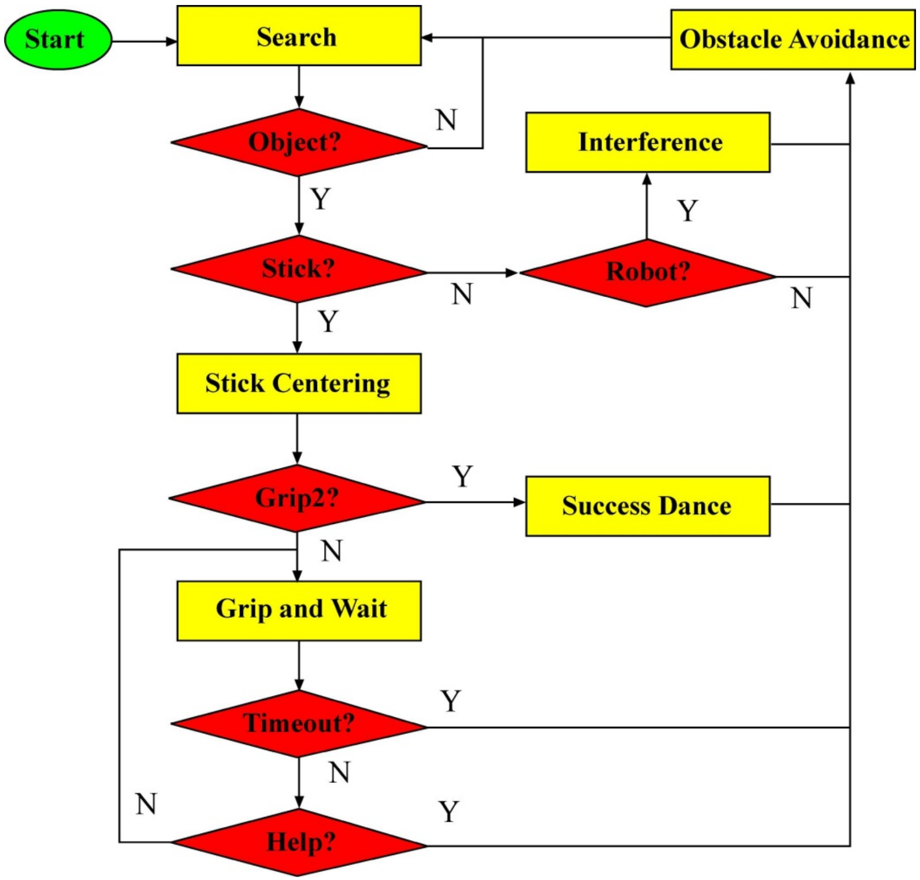
$$p_R = p_r (N_0 - 1)$$

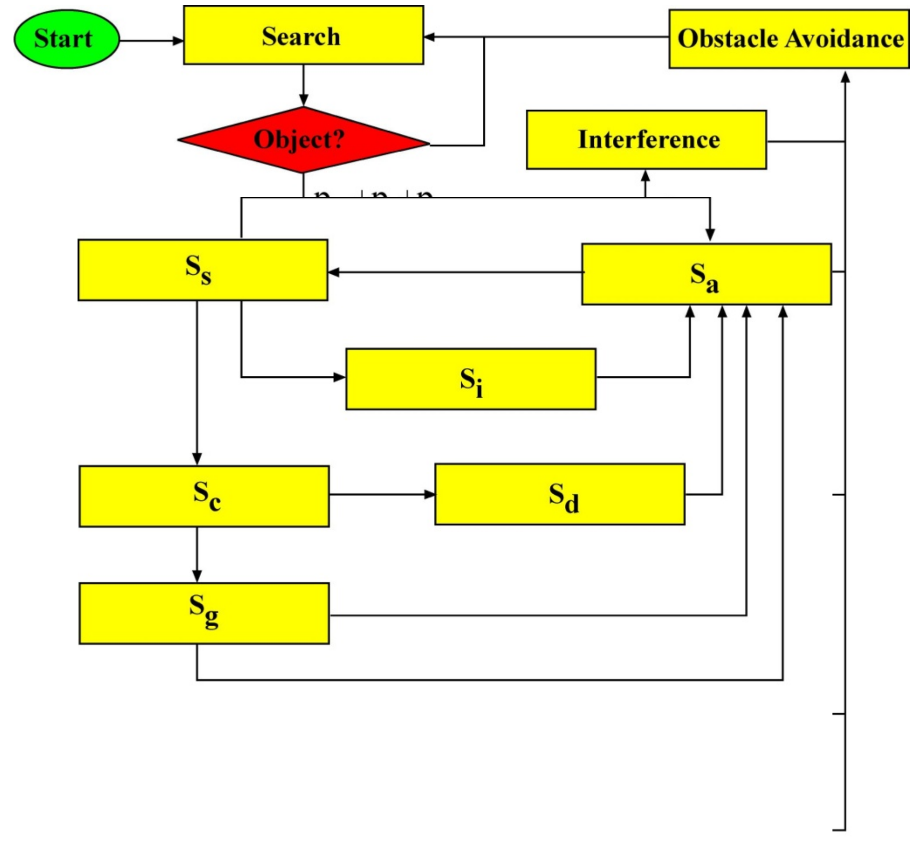
$$p_w = A_w / A_a$$

$$p_{g1} = p_s$$

$$p_{g2} = R_g p_s$$

From Reality to Abstraction





Deterministic robot’s flowchart

Nonspatiality & microscopic characterization

PFSM

Probabilistic agent’s flowchart

Full Macroscopic Model

For instance, for the average number of robots in searching mode:

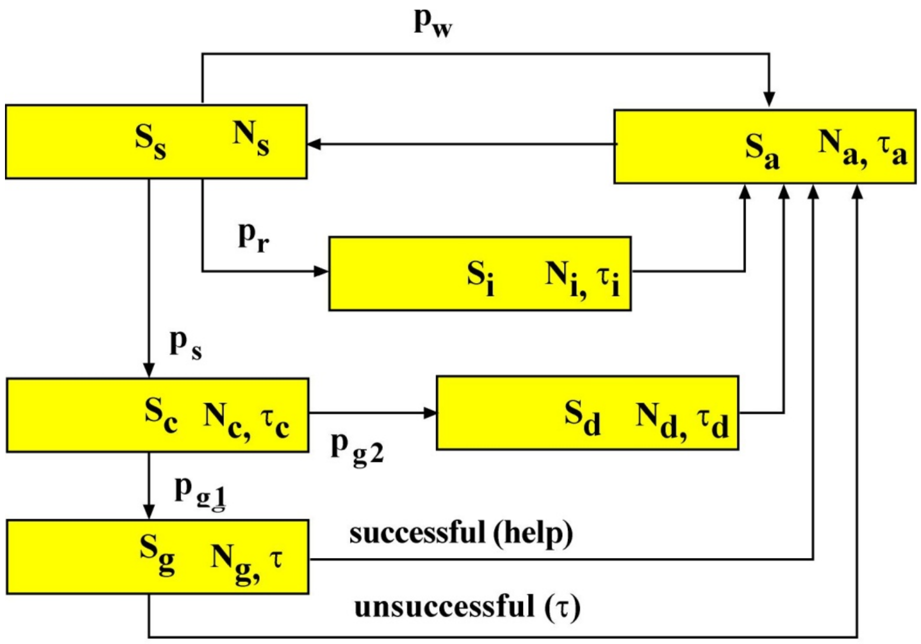
$$N_s(k+1) = N_s(k) - [\Delta_{g1}(k) + \Delta_{g2}(k) + p_w + p_R]N_s(k) + \Delta_{g1}(k - T_{cga})\Gamma(k;T_a)N_s(k - T_{cga})$$
$$+ \Delta_{g2}(k - T_{ca})N_s(k - T_{ca}) + \Delta_{g2}(k - T_{cda})N_s(k - T_{cda}) + p_w N_s(k - T_a) + p_R N_s(k - T_{ia})$$

with time-varying coefficients (nonlinear coupling):

$$\Delta_{g1}(k) = p_{g1}[M_0 - N_g(k) - N_d(k)]$$

$$\Delta_{g2}(k) = p_{g2}N_g(k)$$

$$\Gamma(k;T_{SL}) = \prod_{j=k-T_g-T_{SL}}^{k-T_{SL}}[1 - p_{g2}N_s(j)]$$



- 6 states: 5 DE + 1 cons. EQ
- $T_i, T_a, T_d, T_c \neq 0$; $T_{xyz} = T_x + T_y + T_z$
- T_{SL} = Shift Left duration
- [Martinoli et al., *IJRR*, 2004]

Swarm Performance Metric

Collaboration rate: # of sticks per time unit

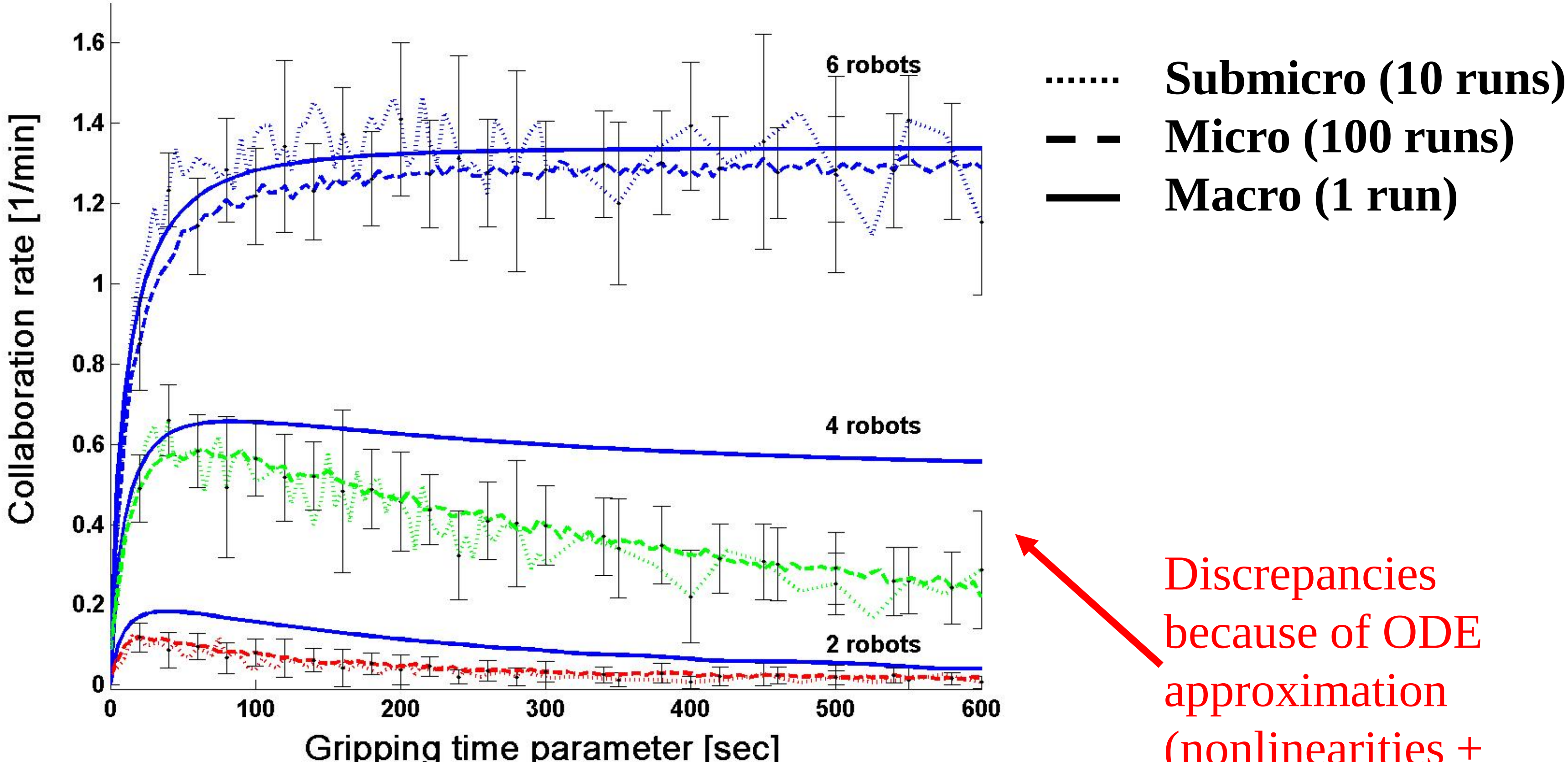
$$C(k) = p_{g2} N_s(k-T_{ca}) N_g(k-T_{ca})$$

: mean # of collaborations at iteration k

$$C_t(k) = \frac{\sum_{k=0}^{T_e} C(k)}{T_e}$$

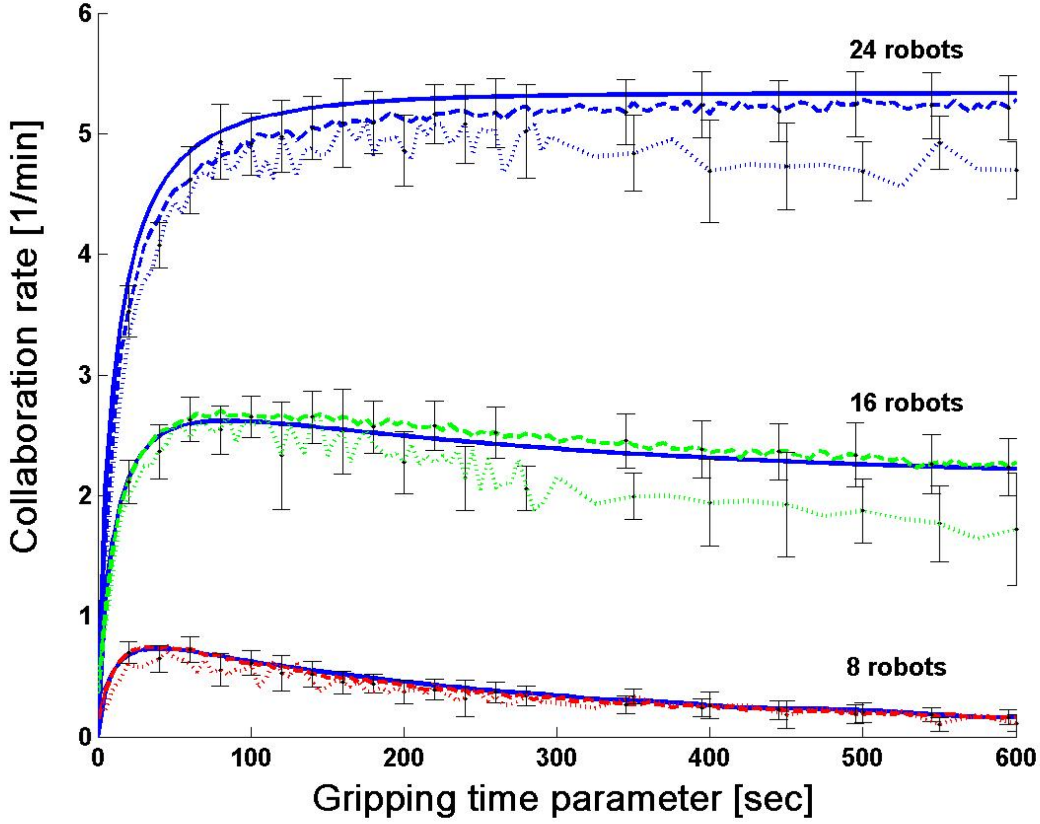
: mean collaboration rate over T_e

Results (Standard Arena)



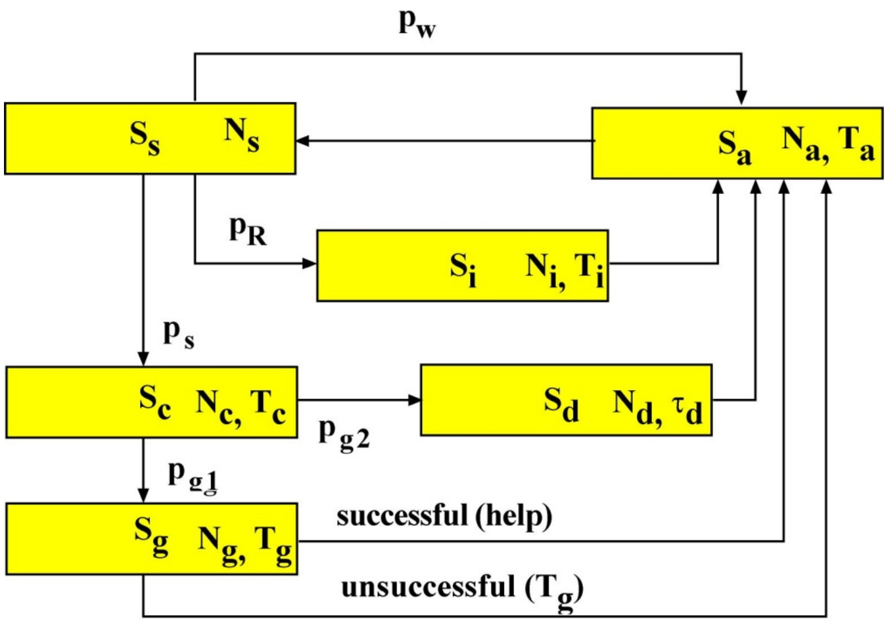
Discrepancies because of ODE approximation (nonlinearities + discrete exact vs. average quantities)

Results: 4 x #Sticks, #Robots and Arena Area



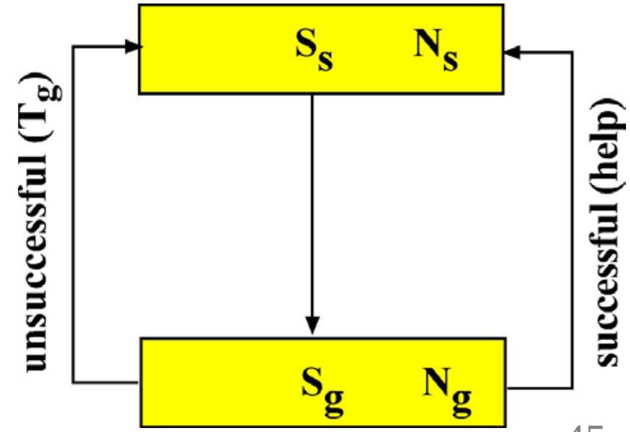
Submicro (10 runs)
Micro (100 runs)
Macro (1 run)

Reducing the Macroscopic Model



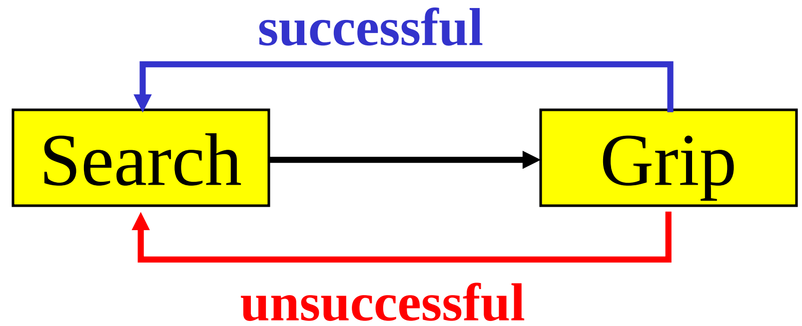
Goal: reach mathematical tractability

$T_i, T_a, T_d, T_c \ll T_g \rightarrow T_i = T_a = T_d = T_c = 0$



Reduced Macroscopic Model

Nonlinear coupling!



$$N_s(k+1) = N_s(k) - p_{g1}[M_0 - N_g(k)]N_s(k) + p_{g2}N_g(k)N_s(k) + p_{g1}[M_0 - N_g(k-T_g)]\Gamma(k;0)N_s(k-T_g)$$

$$N_g(k+1) = N_0 - N_s(k+1)$$

$$\Gamma(k;0) = \prod_{j=k-T_g}^{k}[1 - p_{g2}N_s(j)]$$

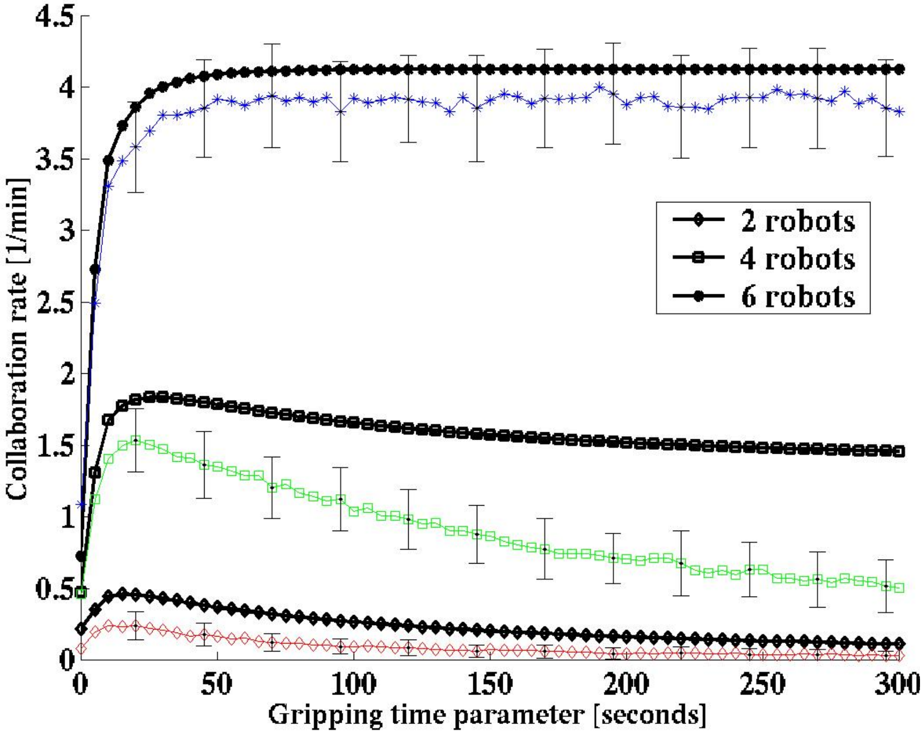
Initial conditions and causality

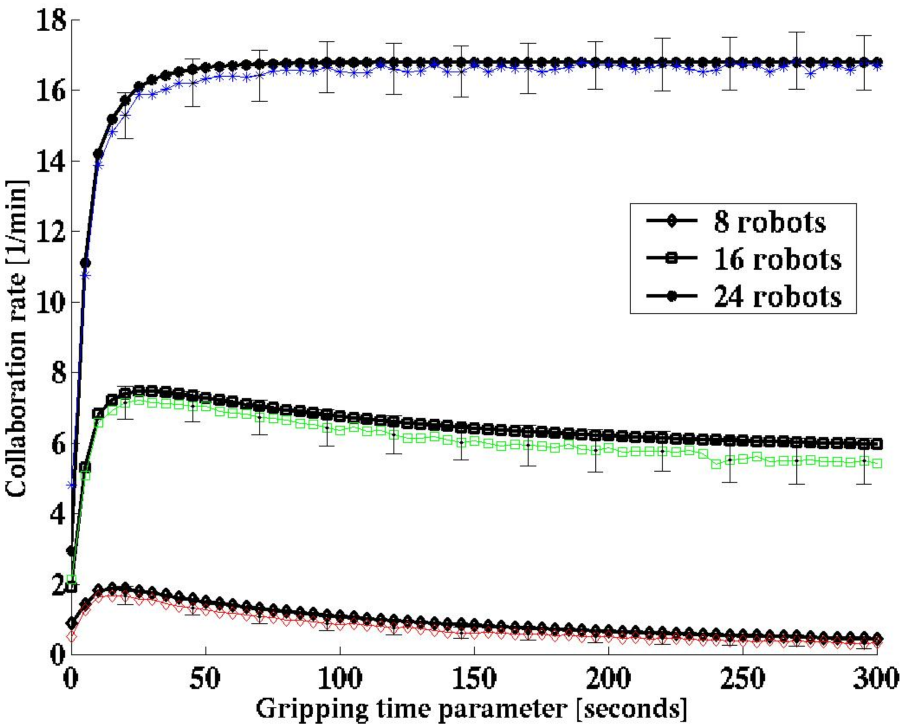
$N_s(0) = N_0$, $N_g(0) = 0$
$N_s(k) = N_g(k) = 0$ for all $k<0$

N_s = average # robots in searching mode
N_g = average # robots in gripping mode
N_0 = # robots used in the experiment
M_0 = # sticks used in the experiment
Γ = fraction of robots that abandon pulling
T_e = maximal number of iterations
$k = 0,1, \ldots T_e$ (iteration index)

Results Reduced Microscopic Model

- Microscopic (100 runs) and macroscopic models overlapped
- Only qualitatively agreement with submicroscopic/real robots results





- **4 robots, 4 sticks, R_a = 40 cm**
- **16 robots, 16 sticks, R_a = 80 cm**

Steady State Analysis (Reduced Macro Model)

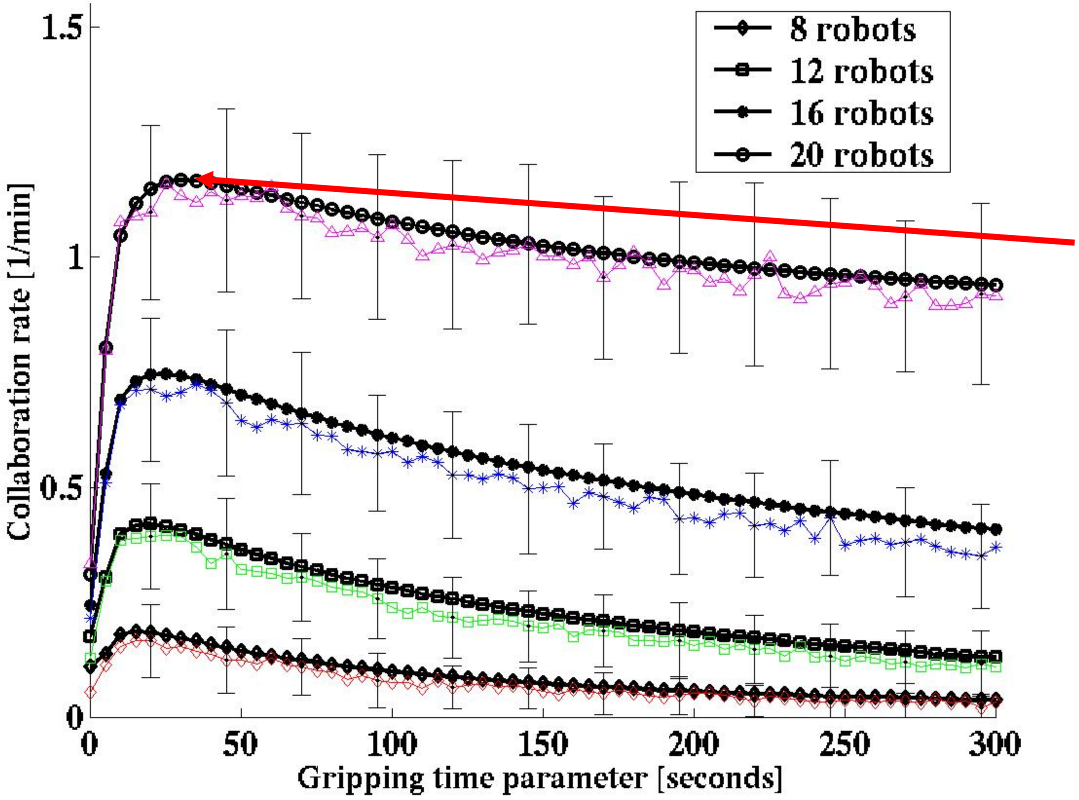
- Steady-state analysis [$N_n(k+1) = N_n(k)$] → It can be demonstrated that :

$$\exists \quad T_g^{opt} \quad for \quad \frac{N_0}{M_0} \leq \frac{2}{1+R_g}$$

with N_0 = number of robots and M_0= number of sticks,
$R_g \propto$ approaching angle for collaboration

- **Counterintuitive conclusion:** an optimal T_g can exist **also in scenarios with more robots than sticks** if the collaboration is **very difficult** (i.e. R_g very small)!

Analysis Verification (Micro and Macro Full Model)



20 robots and 16 sticks (optimal T_g)

Example: $\tilde{R}_g = \frac{1}{10} R_g$ (collaboration very difficult)

Optimal Gripping Time

- Steady-state analysis → T_g^{opt} can be computed analytically in the simplified model (numerically approximated value):

$$T_g^{opt} = \frac{1}{\ln(1 - p_{g1} R_g \frac{N_0}{2})} \ln \frac{1 - \frac{\beta}{2}(1 + R_g)}{1 - \frac{\beta}{2}} \quad for \quad \beta \leq \beta_c = \frac{2}{1 + R_g}$$

with $\beta = N_0/M_0$ = ratio robots-to-sticks

- T_g^{opt} can be computed numerically by integrating the full model ODEs or solving the full model steady-state equations

[Lerman et al, *Alife Journal*, 2001], [Martinoli et al, *IJRR*, 2004]

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Conclusion

Take Home Messages

- Three main levels of models: submicro, micro and macro
- Microscopic models use exact discrete quantities, macroscopic mean-field models use average quantities in terms of unit numbers
- Multi-level modeling allows for different approximations, accuracy/computation trade-offs
- If carefully designed, models allow also for system optimization and closing the loop between analysis and synthesis
- Methodological framework tested on multiple case studies (additional examples and open problems discussed next week)

Additional Literature – Week 8

Papers

- Prorok A., Correll N., and Martinoli A., “Multi-level Spatial Modeling for Stochastic Distributed Robotic Systems”. *Int. Journal of Robotics Research*, **30**(5): 574-589, 2011.
- Di Mario E., Mermoud G., Mastrangeli M., and Martinoli A. “A Trajectory-based Calibration Method for Stochastic Motion Models”. *Proc. of the 2011 IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, September 2011, San Francisco, U.S.A., pp. 4341-4347.
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- S. Berman, A. Halasz, M. A.Hsieh, and V. Kumar. “Optimal Stochastic Policies for Task Allocation in Swarms of Robots”, *Trans. on Robotics*, **25**(4): 927–937, 2009.
- M. A. Hsieh, A. Halasz, S. Berman, and V. Kumar. "Biologically Inspired Redistribution of a Swarm of Robots Among Multiple Sites”. *Swarm Intelligence*, **2** (2-4): 121–141, 2008.
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