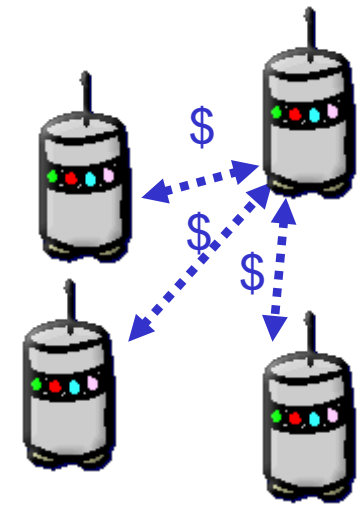


Distributed Intelligent Systems – W6

Division of Labor: Threshold-Based and Market-Based Algorithms

Outline

- Division of labor in natural systems
- Threshold-based algorithms
- Market-based algorithms
- Threshold-based and market-based algorithms applied to multi-robot coordination
- Comparison between threshold-based and market-based algorithms for multi-robot coordination



Division of Labor in Natural Systems : Motivation and Overview in Social Insects

The Division of Labor and its Control

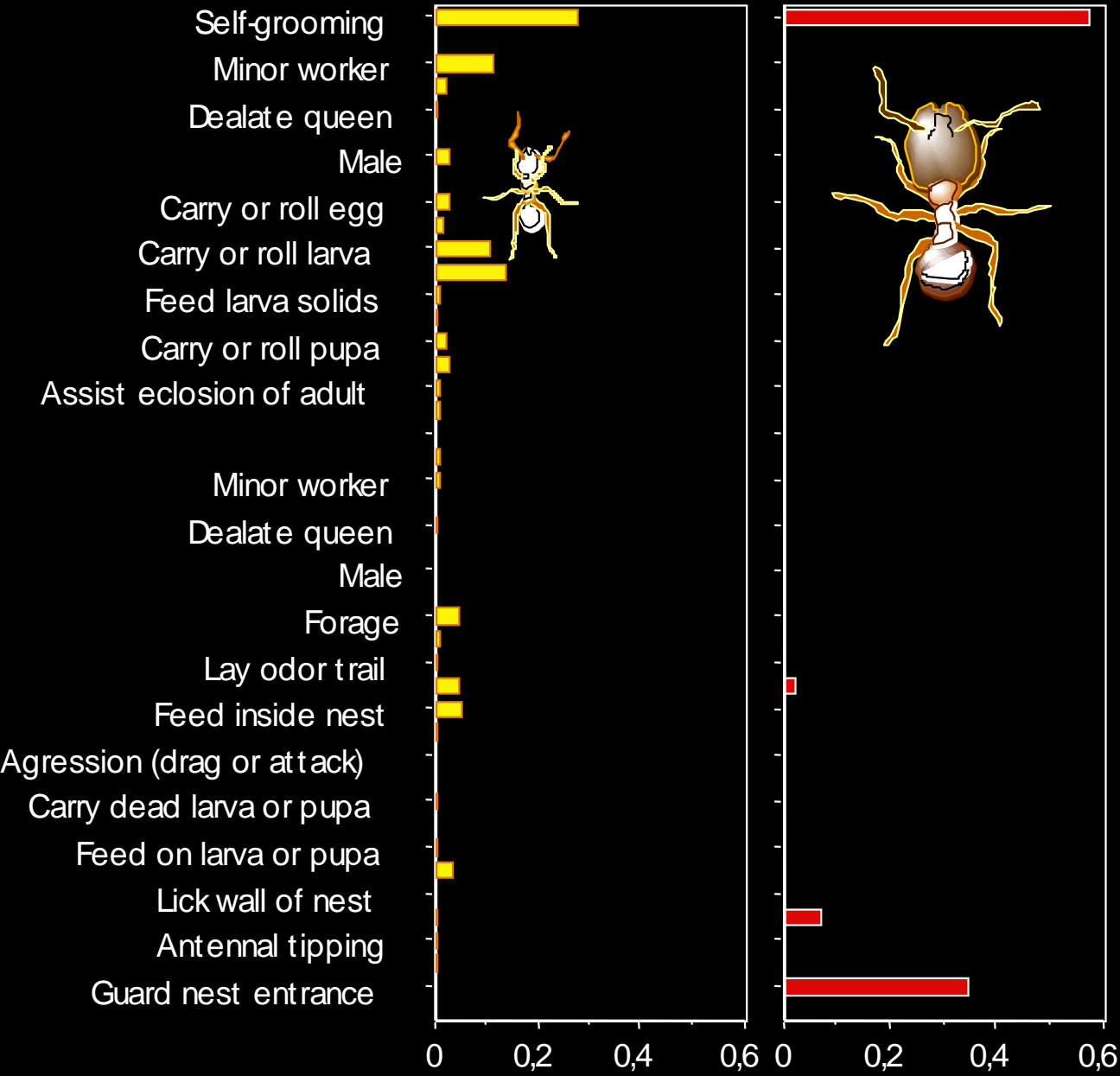
Talking about division of labor means that there is a **potential redundancy** in the role/contribution of individuals and **coordination** in role/task allocation helps in decreasing redundancy and increasing efficiency as a group

The control of task allocation

- The most obvious sign of the division of labor is the existence of **castes**.
- We distinguish between three kinds of castes: **physical**, **temporal** (temporal polyethism) and **behavioral**.
- The individuals belonging to different castes are usually **specialized** for the performance of a series of precise tasks.

Physical Castes (Wilson, E. O., 1976)

■ Minor ■ Major

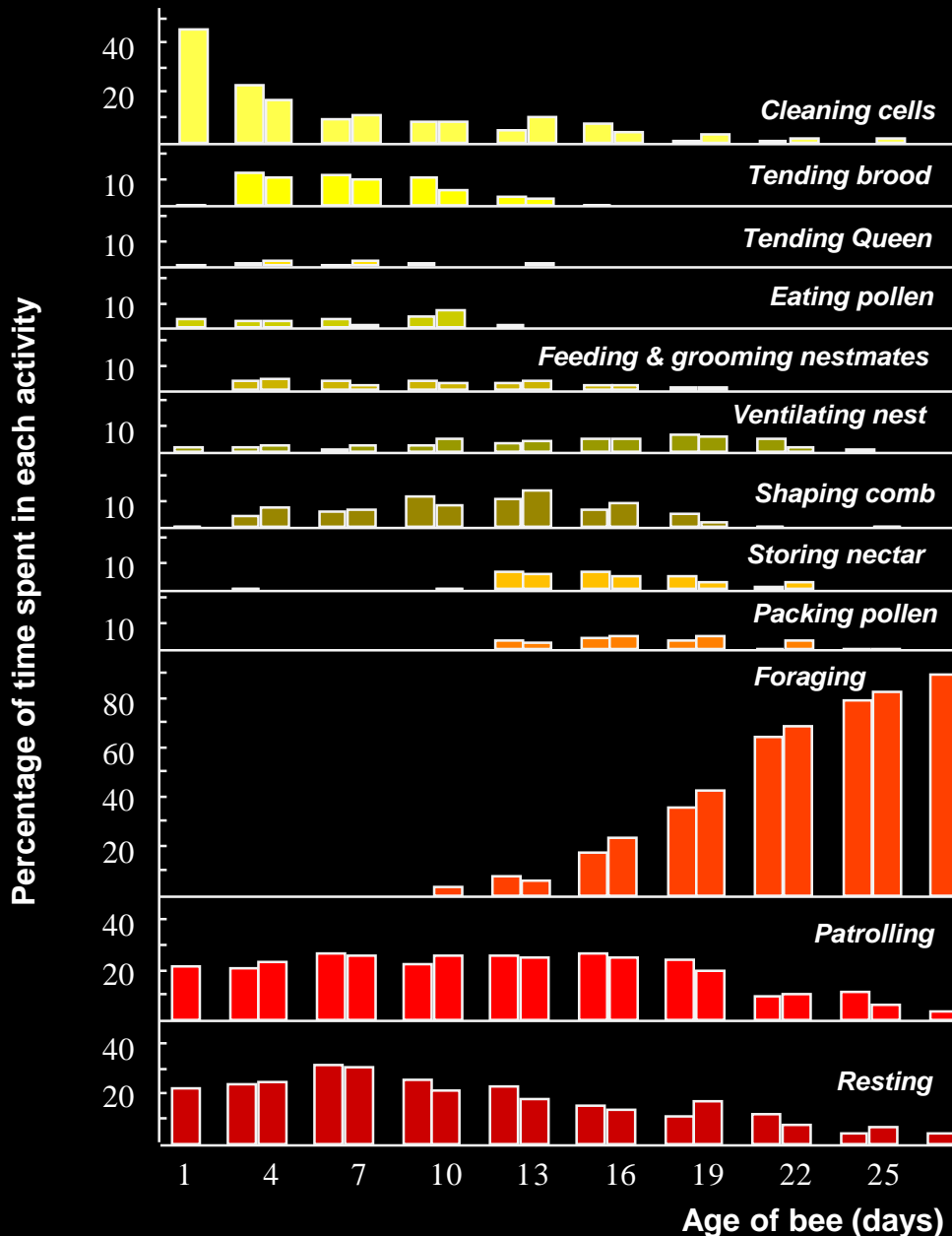


Behavioral repertoires of majors and minors

In *Pheidole guilelmimuellerei* the minors show **ten times as many different basic behaviors** as the majors

Average fraction of time spent in a given activity/behavior

Temporal Polyethism

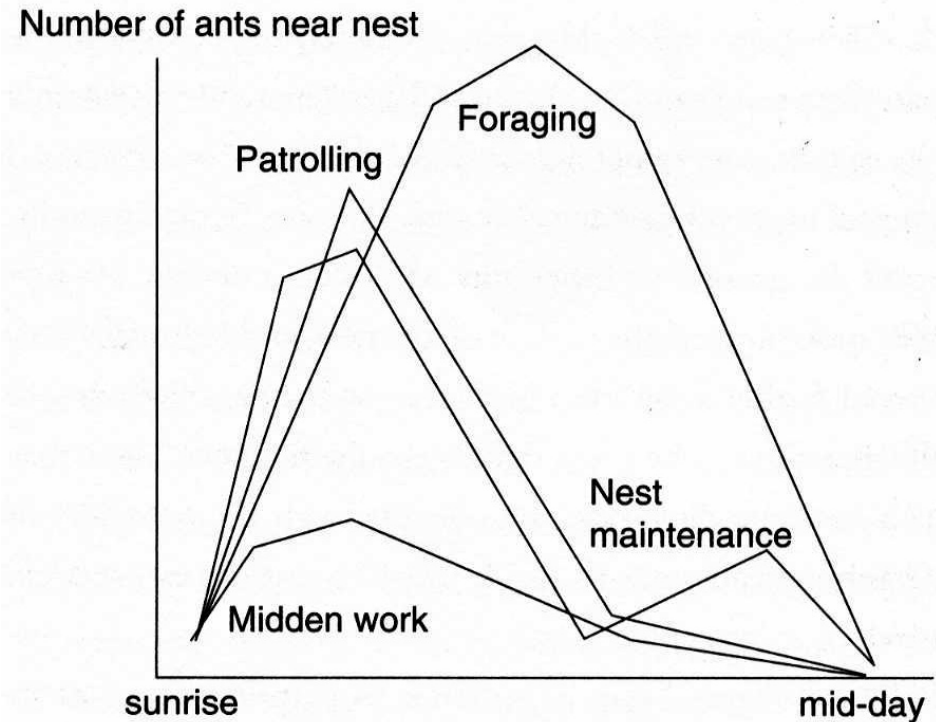


Behavioral changes in worker bees as a function of age

Young individuals work on internal tasks (brood care and nest maintenance).
Older individuals forage for food and defend the nest.

Behavioral Castes

Allocation of the daily activities in a colony of desert harvester ants (Portal, AZ)



From D. Gordon, "Ants at Work", 1999

The Division of Labor and its Control

Flexibility of social roles

- The number of individuals performing different tasks and the nature of the tasks to be done **are subject to constant change** in the course of the life of a colony.
- The proportions of workers performing the different tasks **varies** in response to internal or environmental perturbations.
- This is **true under certain conditions** even when **hard** morphological differences exist or **irreversible** aging processes take place.

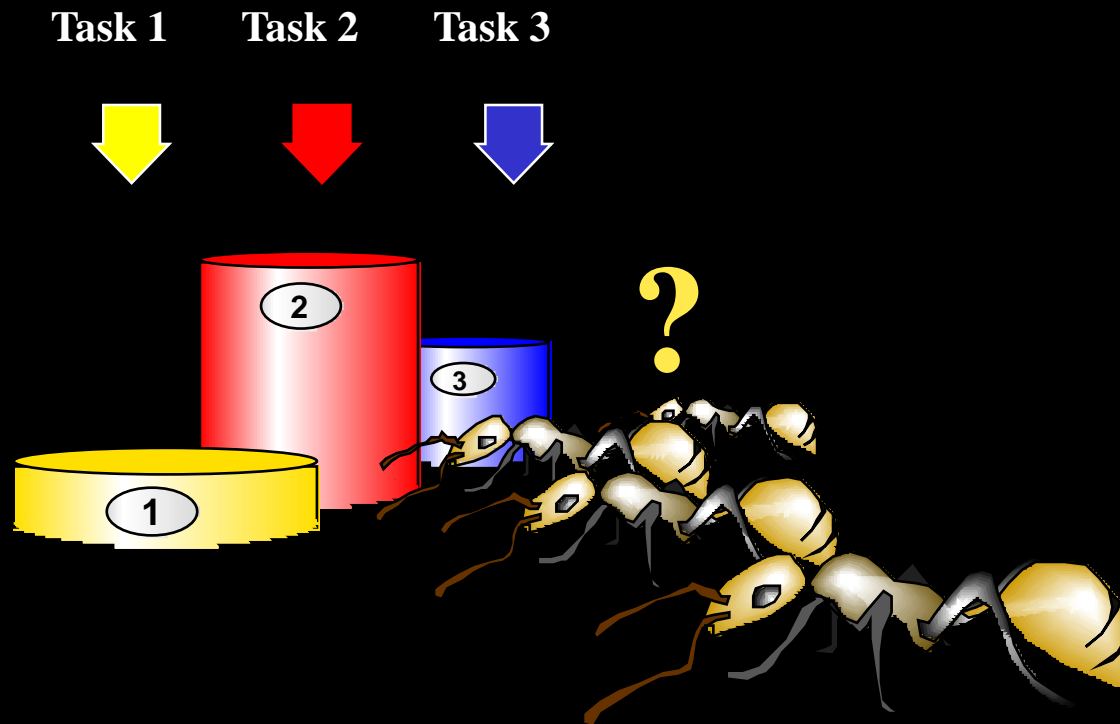
→ The division of labor in social insects must be **flexible**.

→ A **threshold-based behavioral response** appears to be a biologically plausible mechanism for achieving such flexibility

How is flexibility implemented
at the level of the individual?

The Division of Labor and the Flexibility of Social Roles

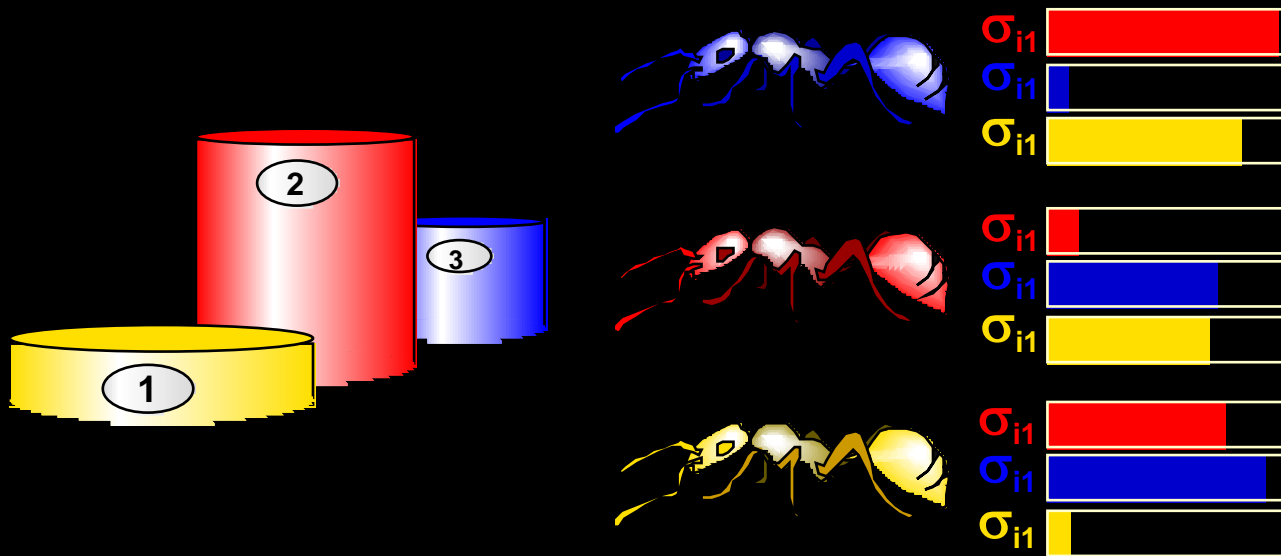
The control of task allocation



How is dynamic task allocation achieved?

The Division of Labor and the Flexibility of Social Roles

The control of task allocation explained with a fixed-threshold model



The lower the threshold, the lower can be the stimulus for achieving a given response; respectively, the lower the threshold, the higher will be the response of an individual for a given stimulus.

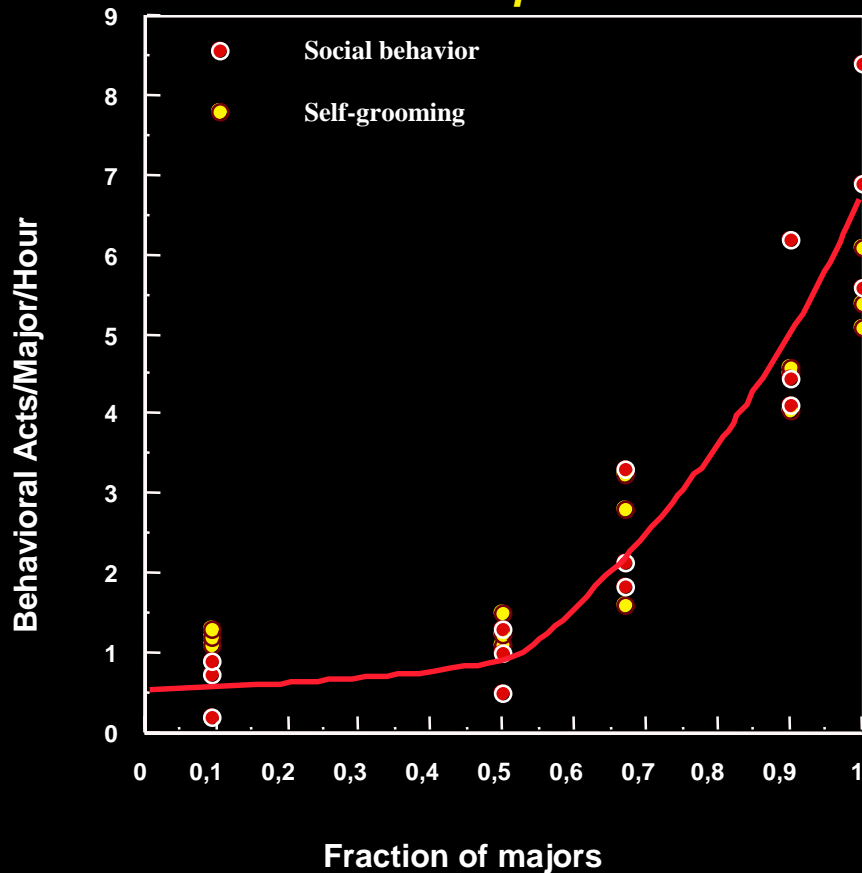
An Example of Control of the Division of Labor Implying the Existence of Fixed Response Thresholds

Pheidole dentata

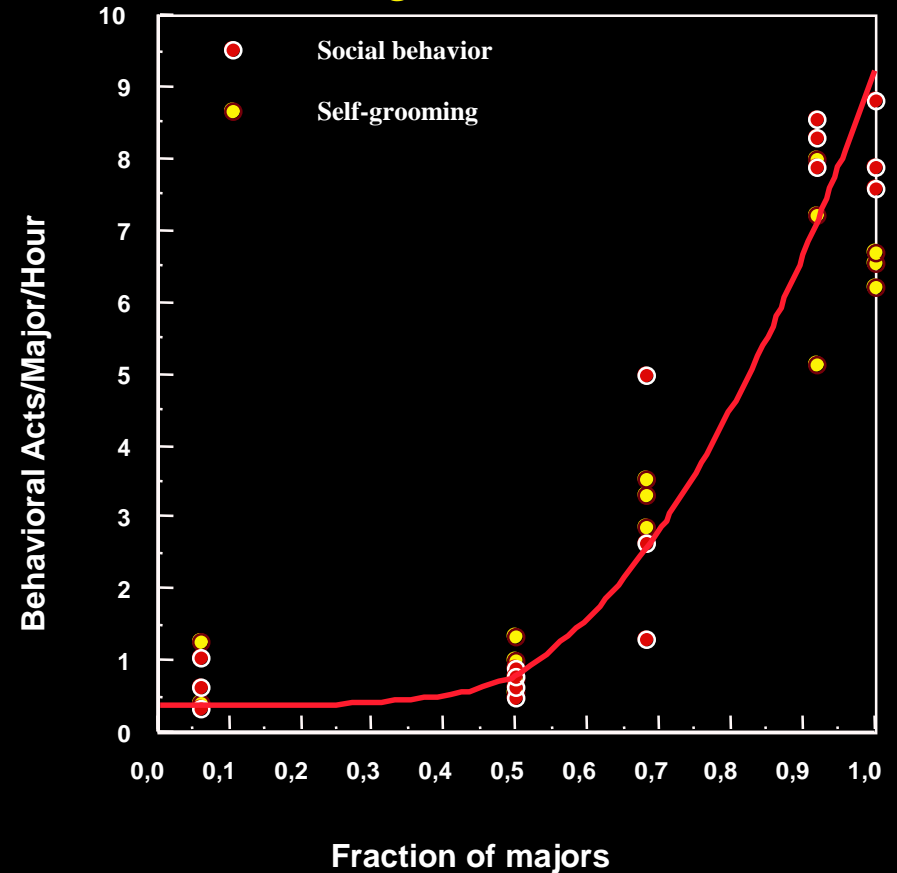


The idea of a response threshold

Pheidole pubiventris



P. guilelmimuelleri



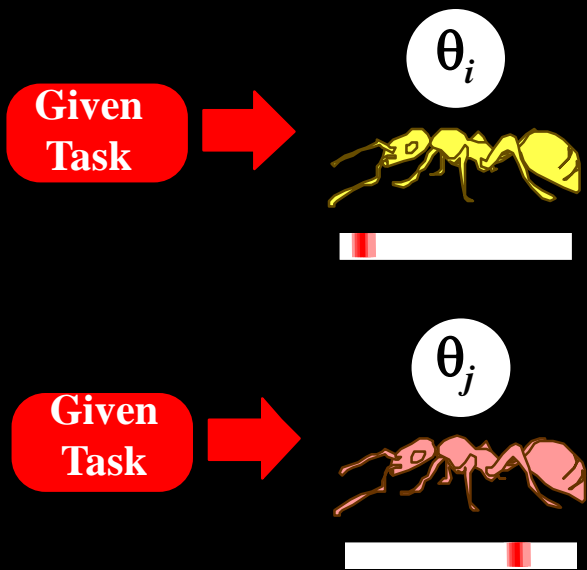
A Macroscopic Model based on Fixed Thresholds for 1 Task and 2 Castes

Using a Model for Explaining Wilson's Results and more ...

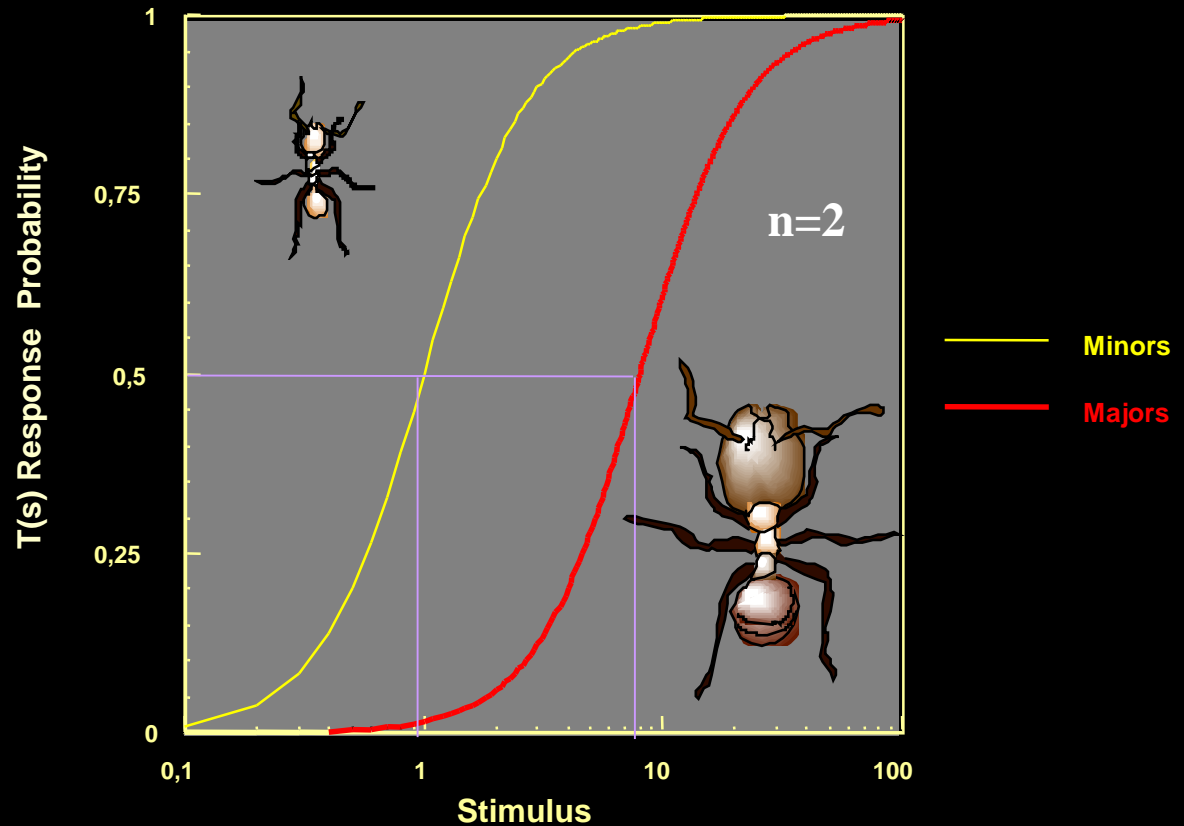
- **Macroscopic** model, **continuous** time
- Response to stimuli based on **fixed** thresholds
- No detailed microscopic mechanisms for stimuli perception and answer incorporated in the model
- Assumptions: **nonspatial** model, i.e. equiprobable exposure of individuals to the stimuli associated with the task and stimulus homogeneously distributed over space
- **1 task** (e.g., behavioral acts), **2 castes** (e.g., minor and major)

The Division of Labor and its Control

An example of a response threshold

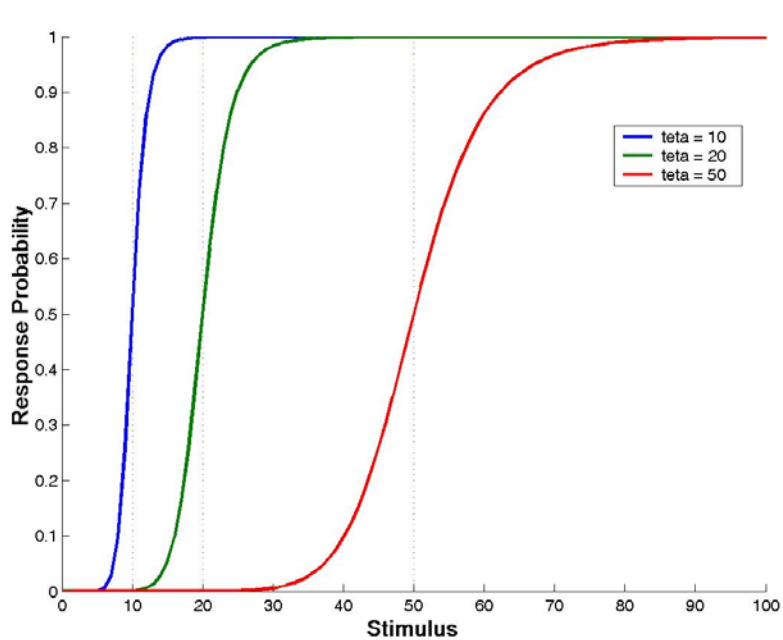


$$T_{\theta_i}(s) = \frac{s^n}{s^n + \theta_i^n}$$

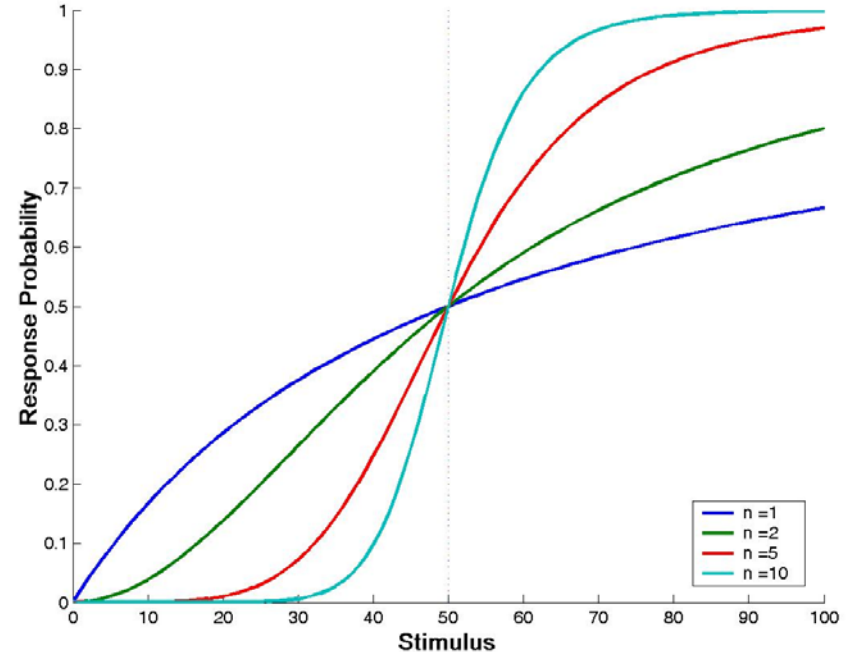


s : intensity of the stimulus associated with the task θ_{minors} θ_{majors}
 θ_i : response threshold of caste i to the task
 n : nonlinearity parameter, e.g. $n = 2$

Properties and Parameters of the Threshold-Based Response Function



Different θ for $n = 10$



Different n for $\theta = 50$

Note on the Aggregation of Variables in Macroscopic Models

Concerned Variable	Mainly in the lecture slides	Mainly in the handouts
n_{ij}, N_i	caste i = set of individuals with the same threshold	1 individual = 1 caste
X_{ij}	Average number of active individuals in caste i carrying out task j	Average time spent by an individual i carrying out task j

Note 1: depending on the significance of the variable some model parameter might slightly change their meaning (per unit step, per individual, etc.)

Note 2: at the macroscopic level we do not deal with details, just need statistical meaningful state variables; self-organization has among its ingredients multiple interactions; that can be achieved with a lot of individuals or a few individuals interacting a lot over time and repeated runs

Note 3: see also week 8 lecture slides and Lerman et al, SAB 2004 paper

Fixed Threshold Model with 1 Task and 2 Castes

Two castes of individuals in the colony (physical, behavioral, or age-based castes); each individual can perform the task or doing nothing. Note n_i and N_i swapped in comparison to your handout!

N_1 = number of individuals of type 1

N_2 = number of individuals of type 2

n_1 = mean number individuals of type 1 engaged in carrying out the task

n_2 = mean number of individuals of type 2 engaged in carrying out the task

$x_1 = n_1 / N_1$: fraction of individuals of type 1 engaged in carrying out the task (or fraction of time/number of actions spent in carrying out the task)

$x_2 = n_2 / N_2$: fraction of individuals of type 2 engaged in carrying out the task (or fraction of time/number of actions spent in carrying out the task)

$1-x_1$: fraction of individuals of type 1 not performing the task (or fraction of time/number of actions spent in not carrying out the task)

$1-x_2$: fraction of individuals of type 2 not performing the task (or fraction of time/number of actions spent in not carrying out the task)

$N_1 + N_2 = N$ = total number of individuals in the colony

Fixed Threshold Model with 1 Task and 2 Castes

Dynamics of the fraction of active individuals in each caste:

$$\partial_t x_1 = \frac{s^2}{s^2 + \theta_1^2} (1-x_1) - r_a x_1$$

$$\partial_t x_2 = \frac{s^2}{s^2 + \theta_2^2} (1-x_2) - r_a x_2$$

Note: $\partial_t x = \frac{dx}{dt}$

- s : intensity of the stimuli associated with the task
- θ_i : response threshold of caste i to the task
- r_a : rate of task abandoning (i.e. probability per infinitesimal time interval that an active individual abandons the task on which it is engaged for moving to idle)
- $1/r_a$: average time spent by an individual performing a task before abandoning it (probabilistic delay model, see Week 8 lecture)

Fixed Threshold Model with 1 Task and 2 Castes

Dynamics of demand associated with the task

$$\partial_t s = \delta - \frac{\alpha}{N}(n_1 + n_2)$$

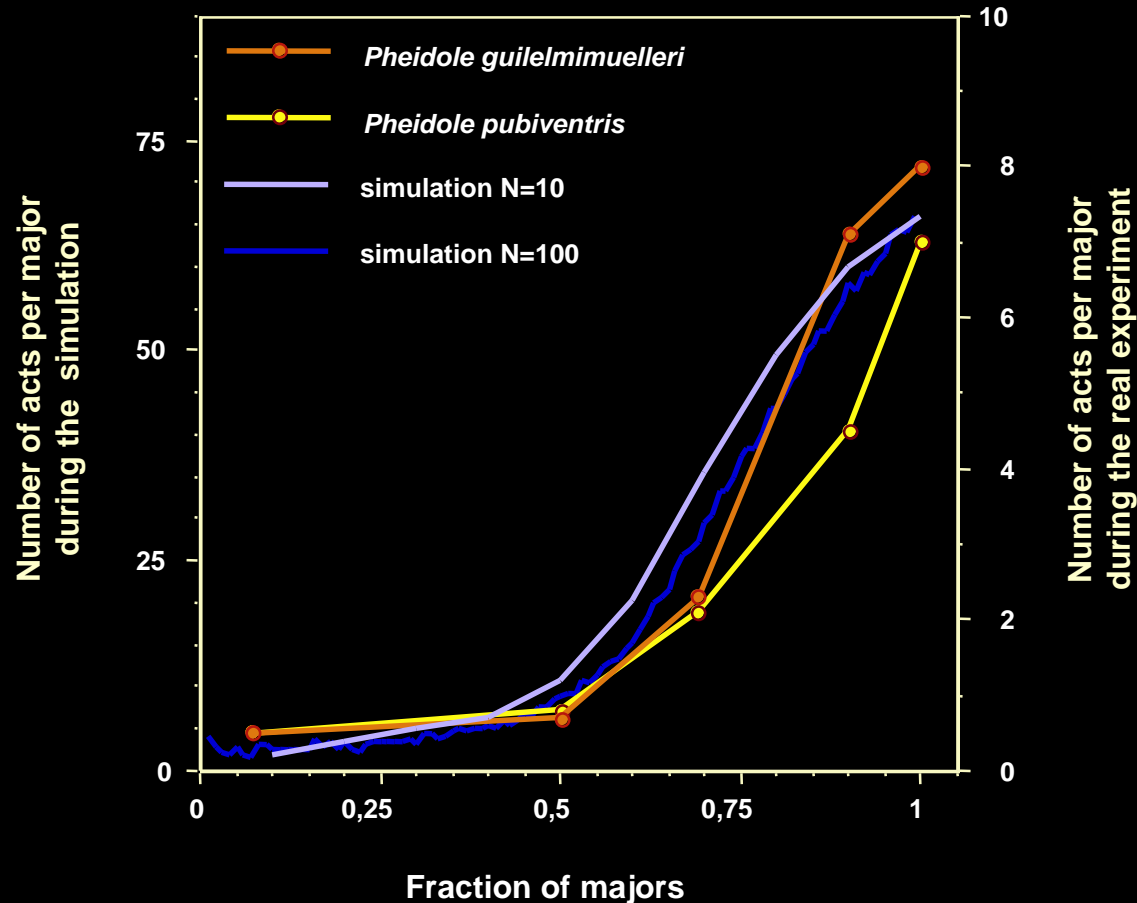
- δ : rate of stimulus increase (demand common to both castes)
- α : normalized effectiveness rate for the individual contribution on the task (in this case an active individual of type 1 contribute in the same way as an active individual of type 2 when performing the task)

Fixed Threshold Model with 1 Task and 2 Castes

Comparison between the macroscopic model and experimental results

Parameters
of the simulation

$N = 10, 100$
 $\theta_1 = 8, \theta_2 = 1$
 $\alpha = 3$
 $\delta = 1$
 $r_a = 0.2$



Variable Threshold Model for Controlling the Division of Labor

Polist Wasps: an Interesting Example



© Guy Theraulaz

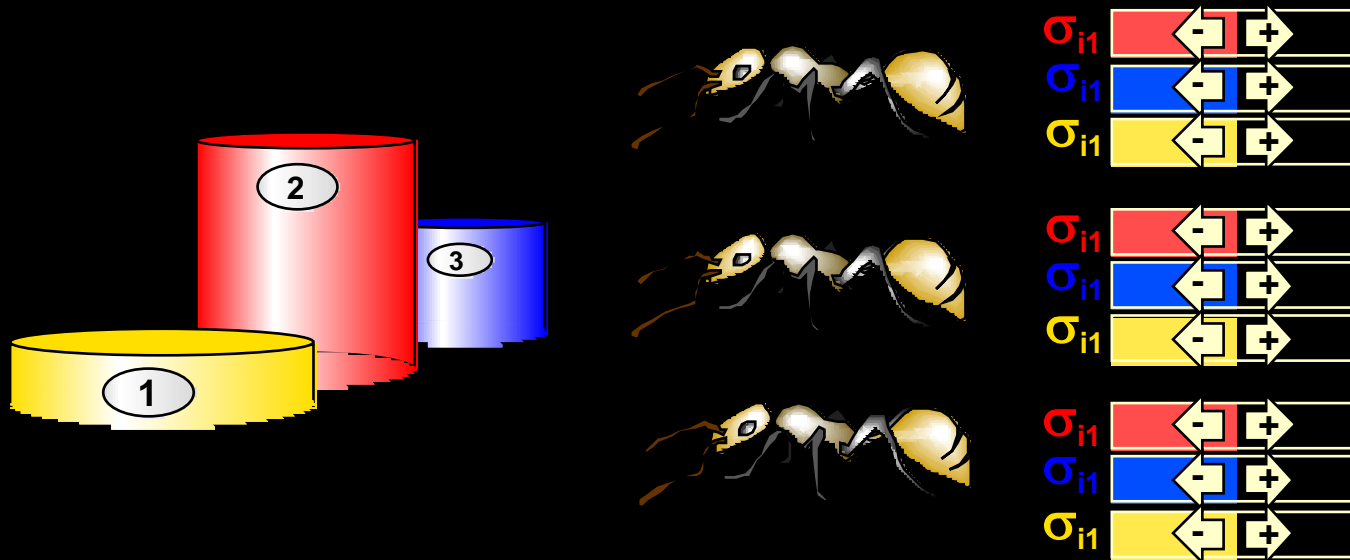
Origins of the Division of Labor in the Polist Wasps

Polists : primitive eusocial species

- Colonies usually contain only a small number of individuals (ca 20).
- These species do not show **morphological differences between castes in the adult stages**, nor any control or physiological determination of the role an individual will play in the colony as an adult.
- Individual behavior is **very flexible**; all individuals are able to perform the whole range of tasks which determine the survival of the colony.
- The integration and coordination of individual activities is achieved through the **interactions** which occur between the members of the colony, and between the members of the colony and the local environment.
- **Specialists** can be created from generalists

The Division of Labor and the Flexibility of Social Roles

The control of task allocation explained with a variable-threshold model



The lower the threshold, the lower can be the stimulus for achieving a given response; respectively, the lower the threshold, the higher will be the response of an individual for a given stimulus.

Variable Threshold Model with 1 Task and m Castes

Individual behavioral algorithm

$\theta_i \rightarrow \theta_i - \xi$ when i performs the task

$\theta_i \rightarrow \theta_i + \varphi$ when i does not perform the task

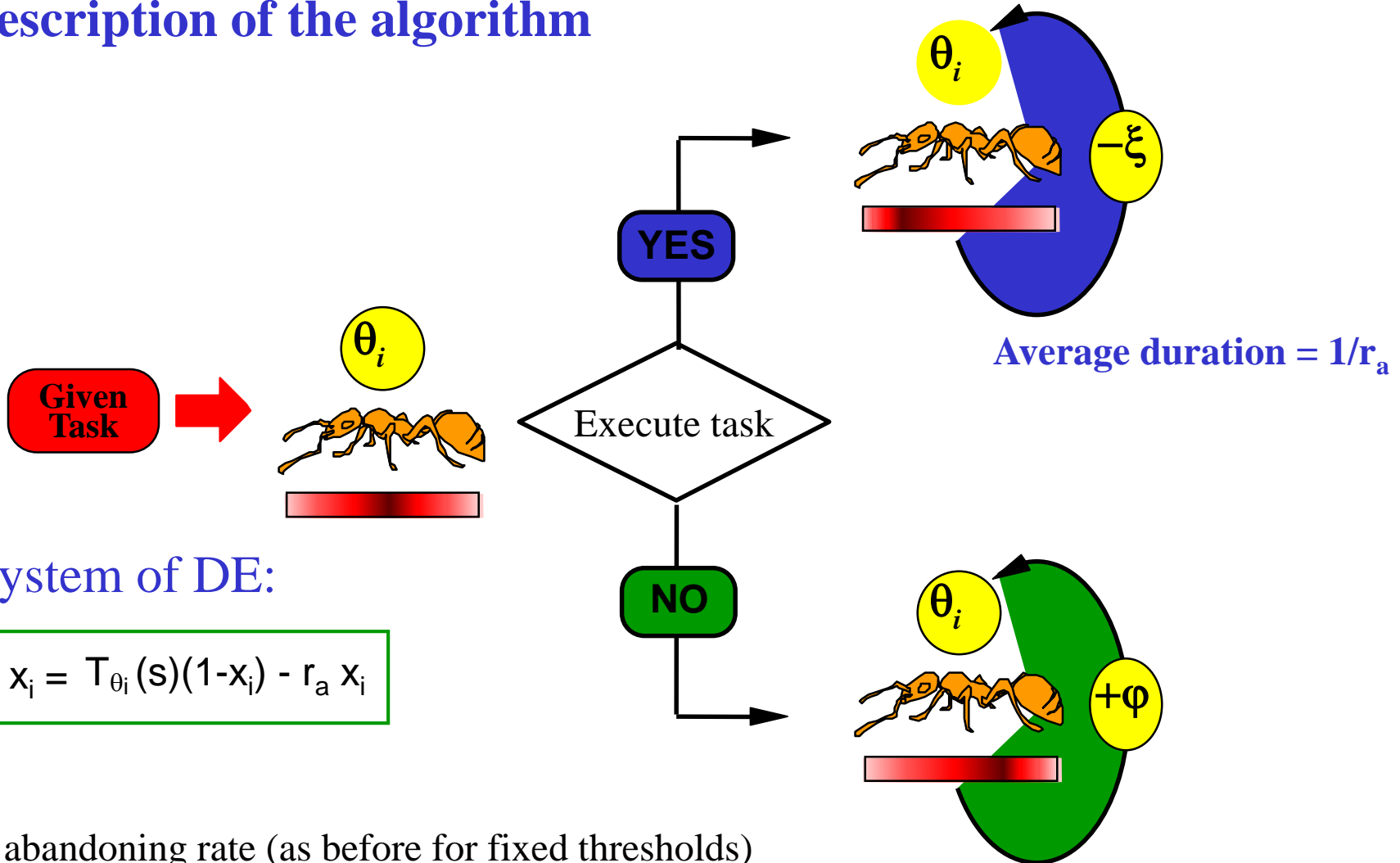
$$T_{\theta_i}(s) = \frac{s^2}{s^2 + \theta_i^2}$$

Parameters of individuals

- i** : caste index [1 ...m]
- s** : intensity of stimuli associated with the task
- θ_i** : response threshold of individual i to the task: $\theta_i \in [\theta_{\min}, \theta_{\max}]$
- ξ** : incremental learning parameter (the threshold of an individual carrying out a task is reduced by ξ)
- φ** : incremental forgetting parameter (the threshold of an individual not carrying out a task increases by φ)

Variable Threshold Model with 1 Task and m Castes

Description of the algorithm



System of DE:

$$\partial_t x_i = T_{\theta_i}(s)(1-x_i) - r_a x_i$$

r_a : abandoning rate (as before for fixed thresholds)

Variable Threshold Model with 1 Task and m Castes

Example of dynamics of the overall stimulus (demand) associated with a task

$$\partial_t s = \delta - \frac{\alpha}{N} \left(\sum_{i=1}^m n_i \right)$$

δ : rate of stimulus increase

α : normalized effectiveness rate for the individual contribution to the task (in this case all the active individual belonging to different castes contribute in the same way)

n_i : number of active individual belonging to caste i

Example with 6 Castes ($\theta_1 \dots \theta_6$)

Model Parameters

$\theta_i = [1 \dots 1000]$, initial random distribution

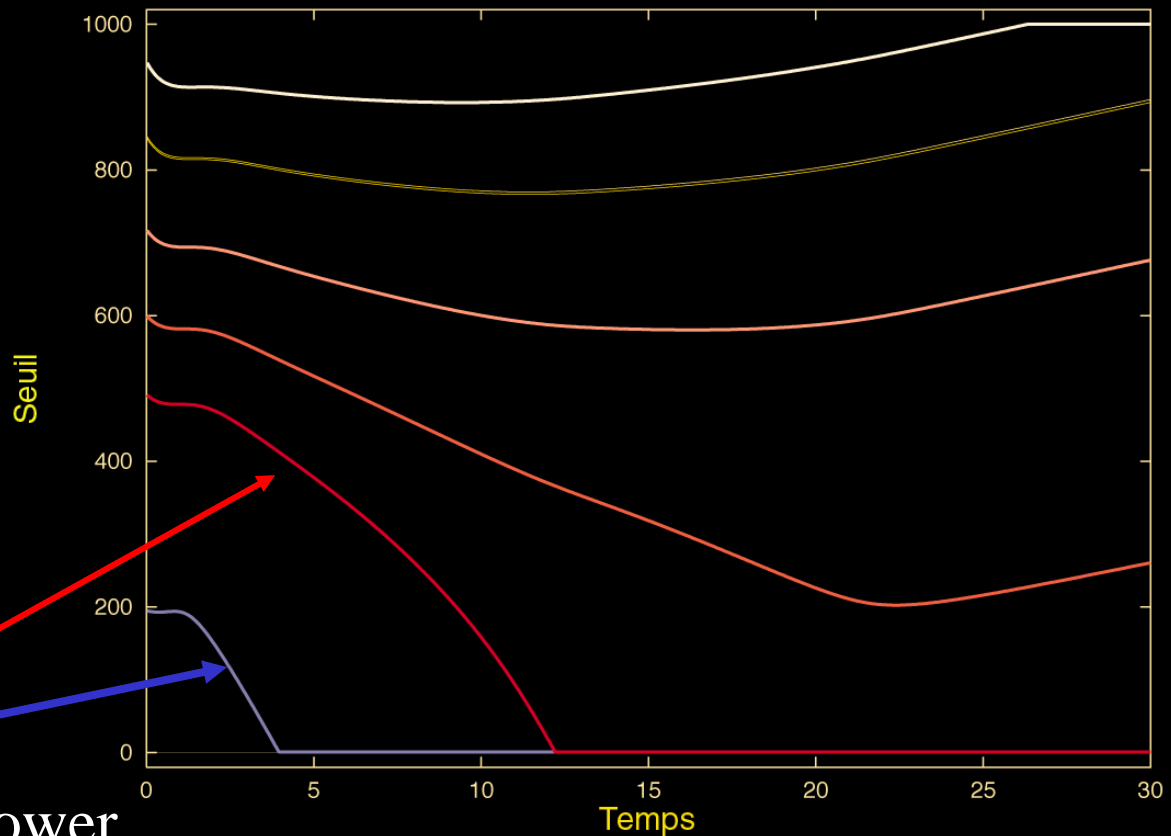
$\alpha = 3$

$\delta = 1$

$r_a = 0.2$

$\xi = 10$

$\varphi = 1$

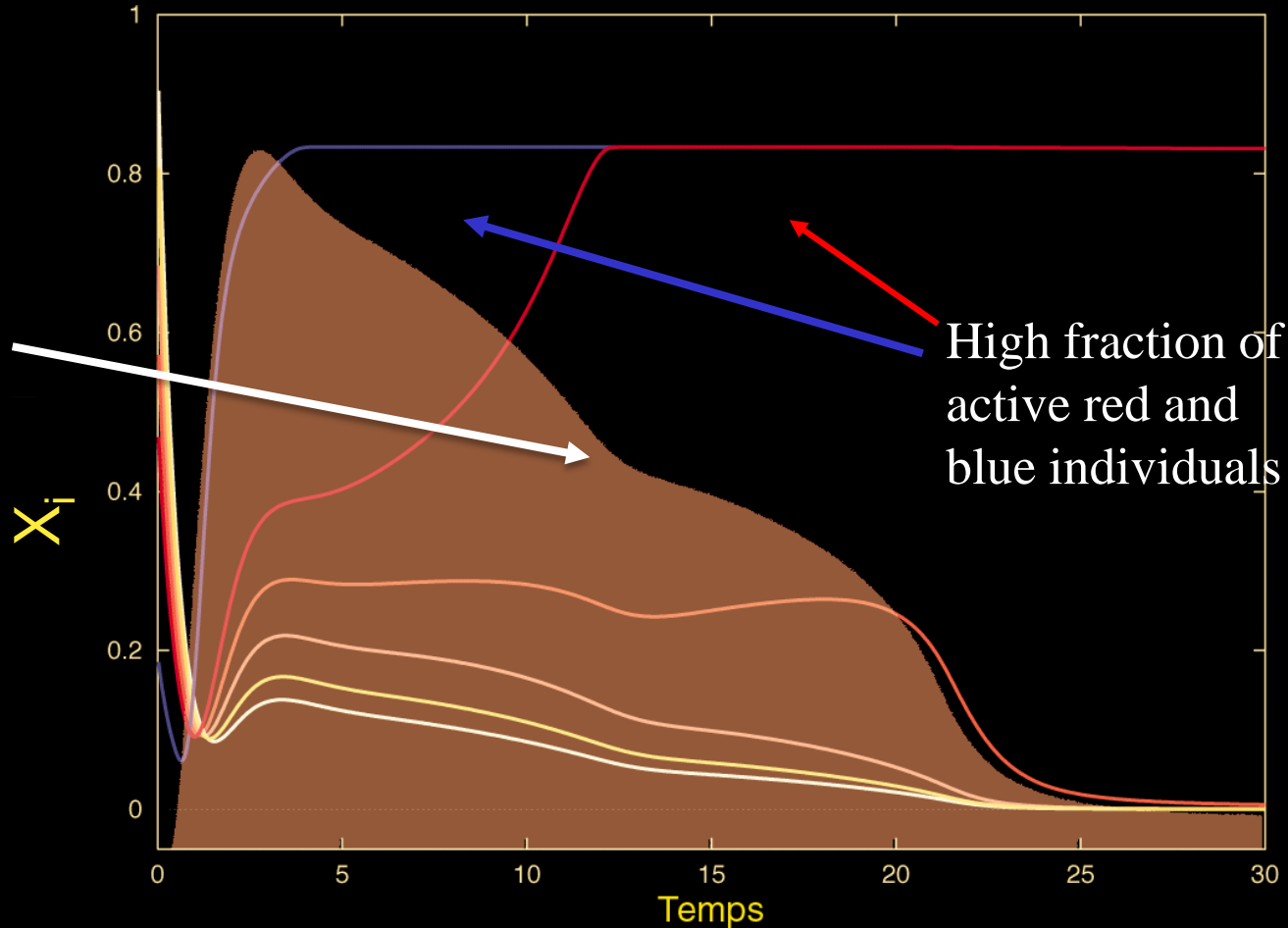


Red and blue caste lower the threshold -> specialists (note started from low thresholds)

Thresholds' evolution

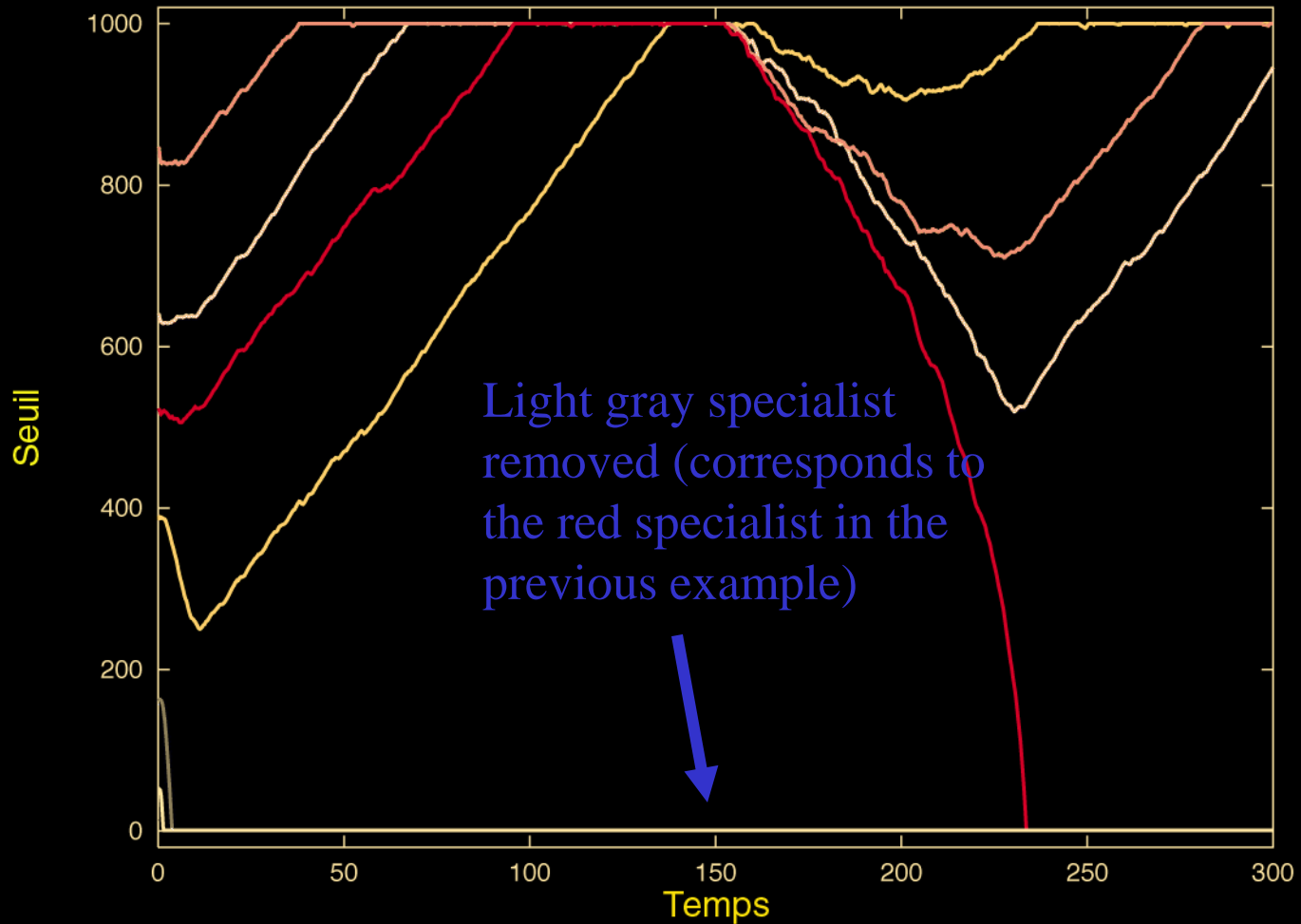
Example with 6 castes ($\theta_1 \dots \theta_6$)

Demand evolution



Evolution of the fraction of active individuals in each caste with overlapped demand

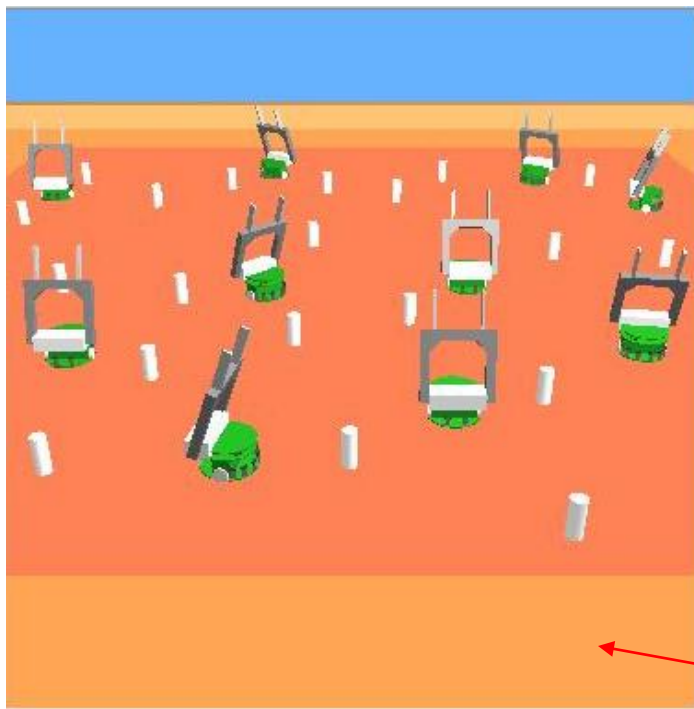
Example with 6 Castes and one Caste Removed at $t = 150$



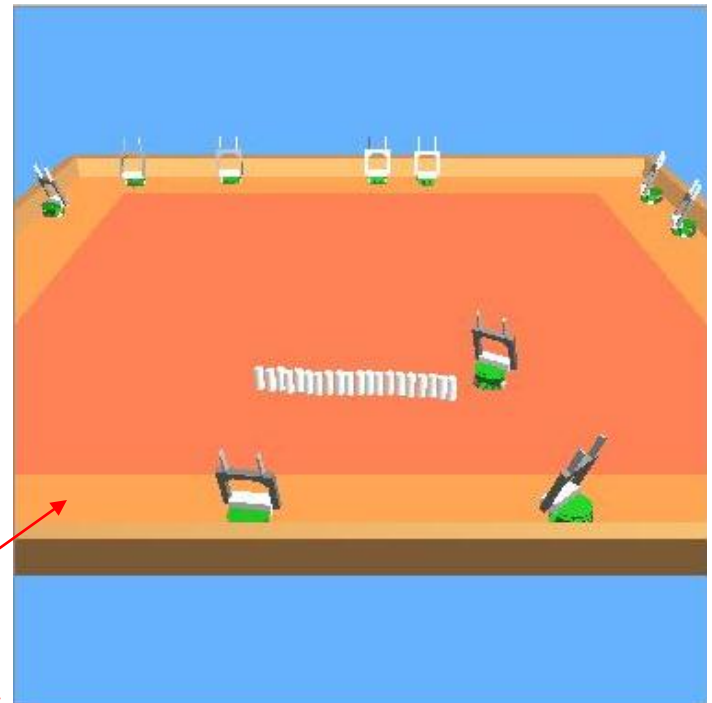
Examples of Threshold-Based Multi-Robot Coordination

Threshold-Based Control of Aggregation (Agassounon and Martinoli, 2002)

Special type of aggregation: linear structure assembly; only seeds at the tips of a cluster can be manipulated



Initial situation

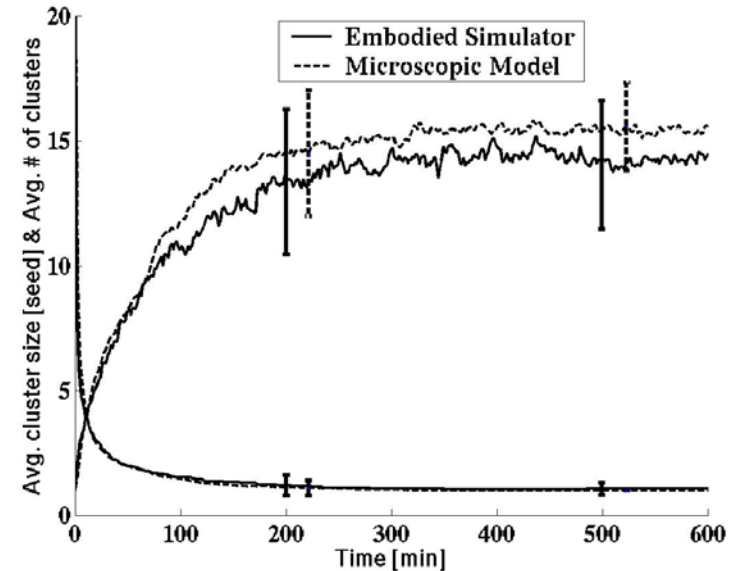


Final situation

Parking lot

Motivations for Regulating the Aggregation Activity

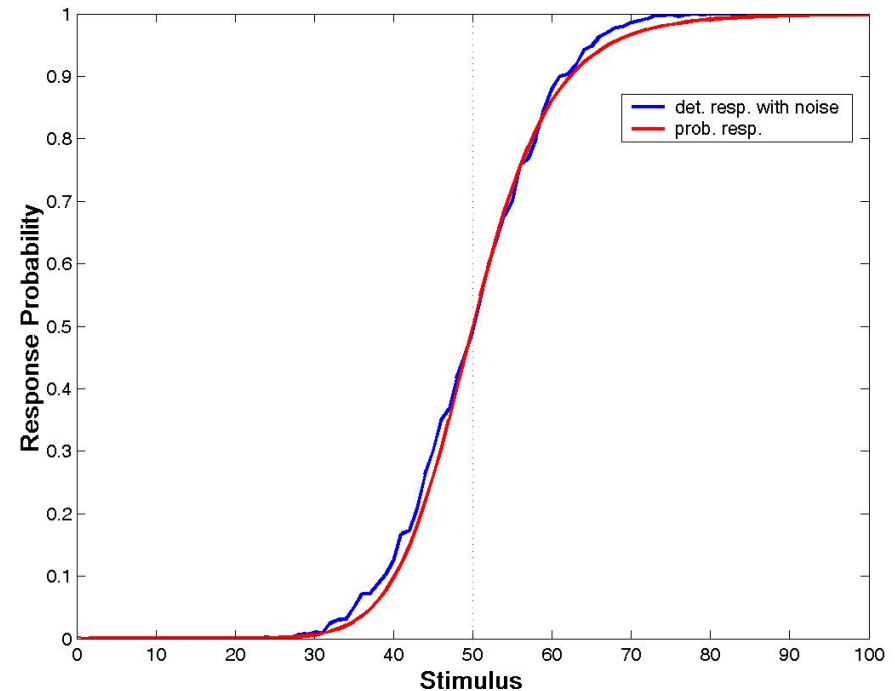
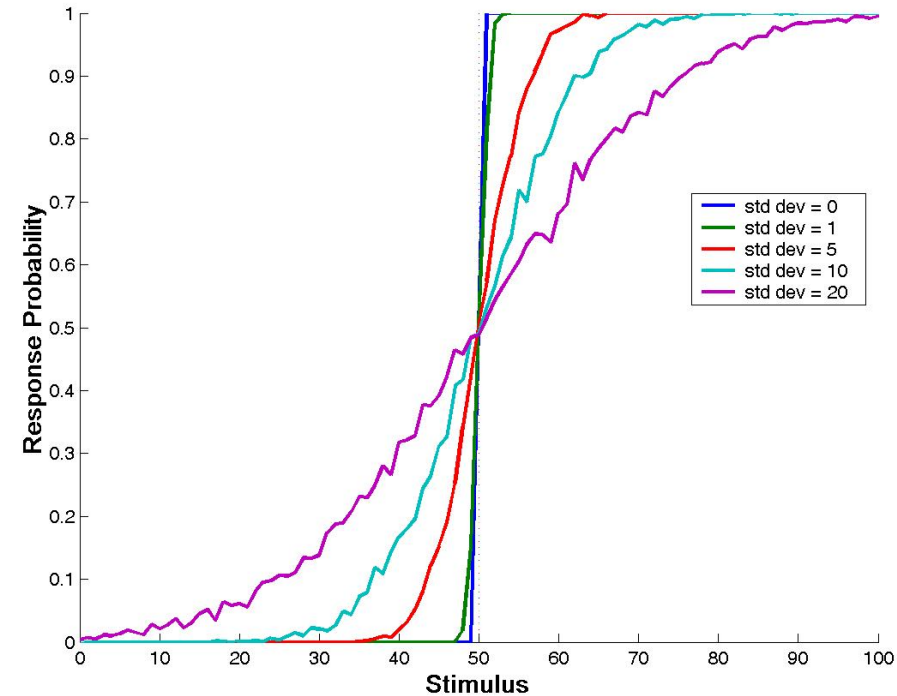
- **Evolution of manipulation sites:** number of manipulation sites decreases with progress of aggregation → more competition (interference) among robots for the same manipulation sites
- **End criterion:** a power-efficient building system should stop working when the task is accomplished
- **Increasing the final cluster size:** at the end all the seeds should belong to the single cluster (only those on the ground count for the aggregation metrics)



Without activity regulation: 10 active agents all the time

[Agassounon and Martinoli, AAMAS 2002]

Deterministic vs. Probabilistic Responses



Deterministic response for $\theta = 50$ and various levels of Gaussian noise (1000 runs)

Deterministic with noise ($\sigma = 9$) vs. probabilistic response ($n = 10$) for $\theta = 50$ (1000 runs)

Implementation of the Threshold-Based Algorithm

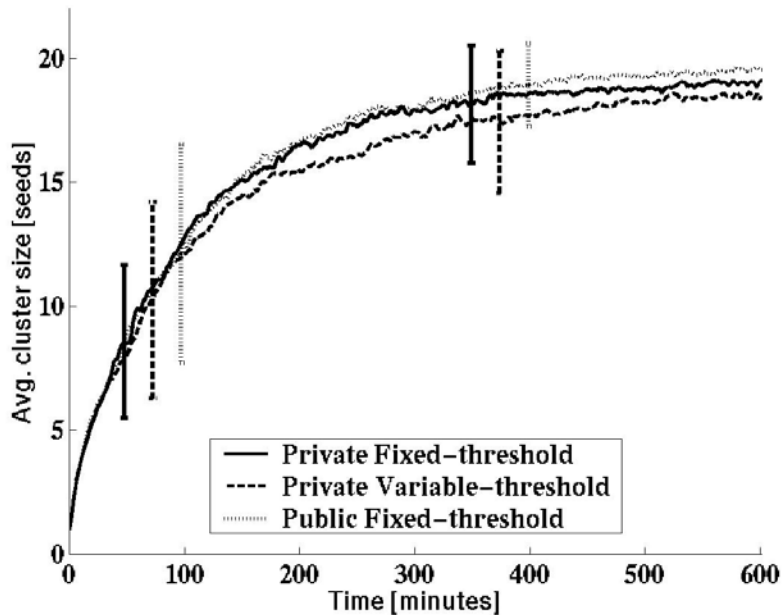
- **1 task**: aggregation \rightarrow 1 threshold per robot
- **Fully distributed algorithm** (demand evaluation and worker allocation)
- Key choices for minimizing complexity and maximizing interchangeability:
 - Robots have **the same capabilities**, no reason to have different threshold
 - **Probabilistic response** even with a single threshold will suffice to regulate the activity; not all the robots stop at the same time, when one drop the work, direct influence on aggregation demand
 - How do we implement: **deterministic response + noise = probabilistic response** \rightarrow exploit local perception based on on-board sensors as noise generator!
- **Chosen stimulus: time needed to find a seed to manipulate**; the larger the time, the lower the stimulus associated with the aggregation demand
- **Special case of demand evolution**: it does not increase automatically but stay constant if nothing is done. Initial condition: $s(0) = S_0$ and $\delta = 0$ (instead of $s(0) = 0$ $\delta > 0$ as in the previous examples) \rightarrow **switching mechanism asymmetric**: active \rightarrow idle possible; idle \rightarrow active not implemented

Note: the system performance can be improved with better demand estimation using networking (public strategies) and variable thresholds.

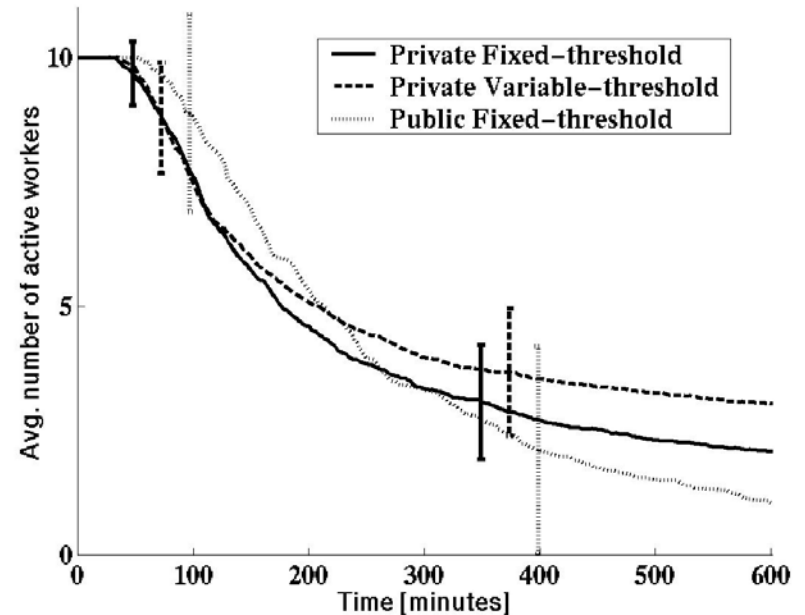
[Agassounon and Martinoli, AAMAS 2002]

Performance with Threshold-Based Activity Regulation

20 seeds, 10 robots, 80 x 80 cm



Average cluster size



Number of active robots

Market-Based Algorithms

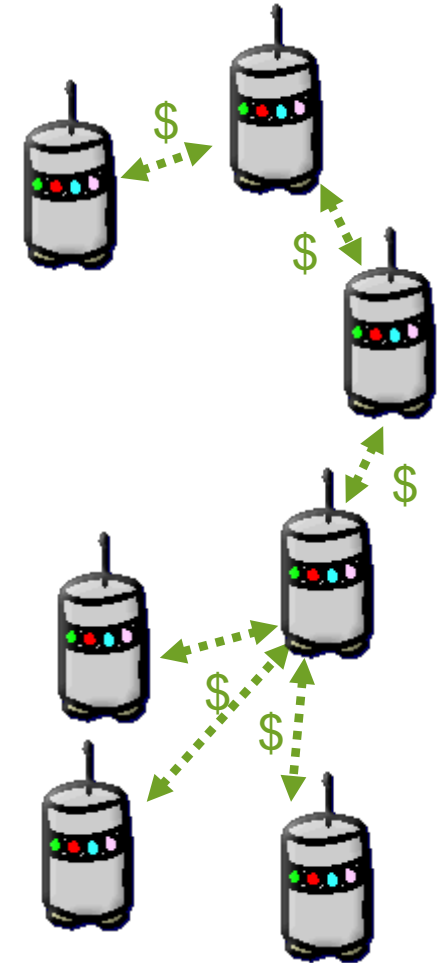
Market-based Coordination

Robots model market economy:

- Resources, tasks have precise worth
- They are traded over the market
- Accomplish task \rightarrow paid revenue
- Consume resources \rightarrow incurs cost
- GOAL: maximize profit (= revenue - cost)

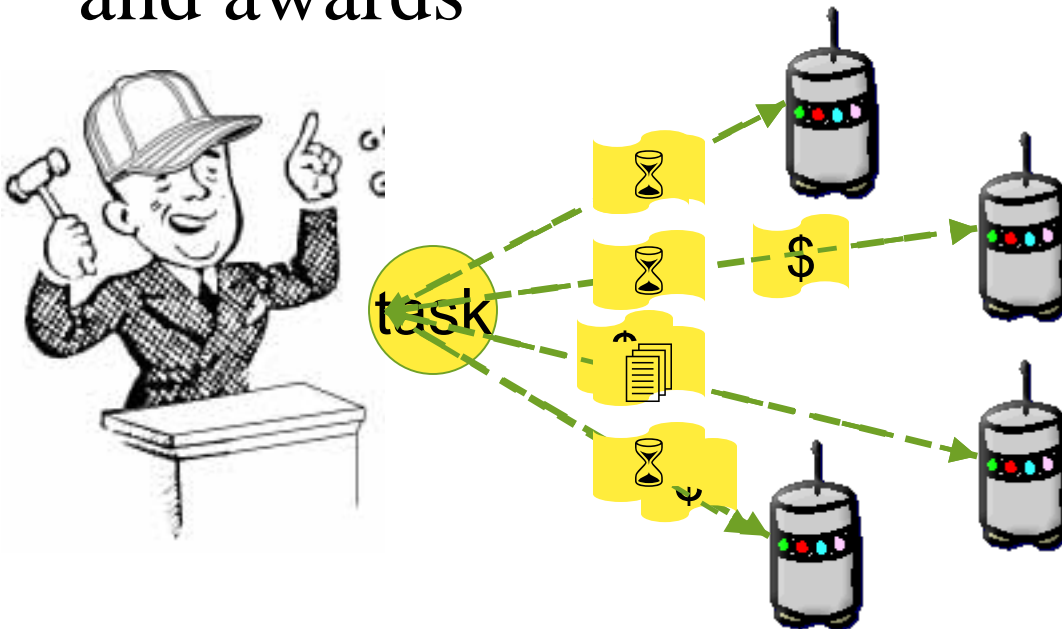


pursuit of individual profit \rightarrow good global solution

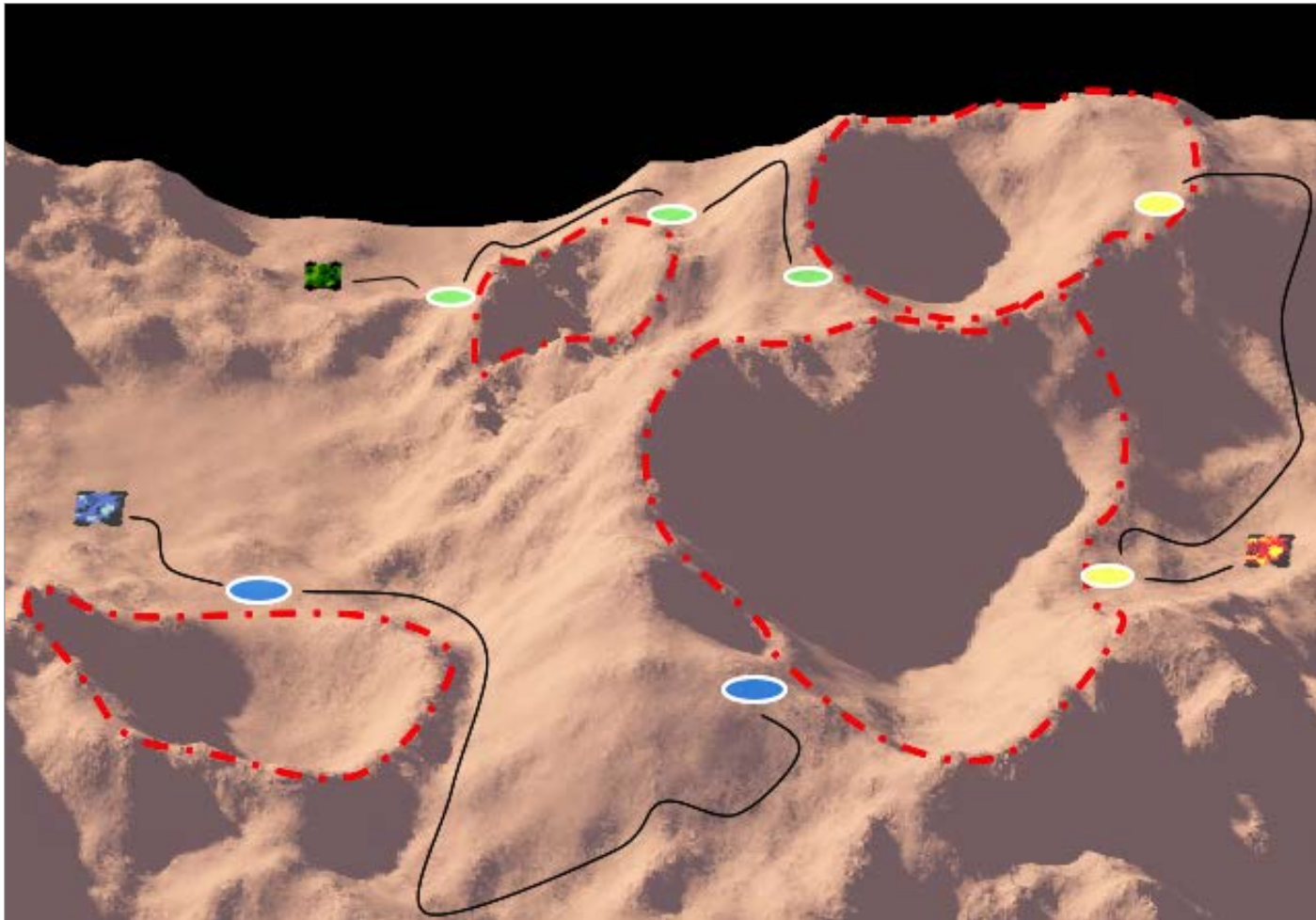


Allocation uses Auctions

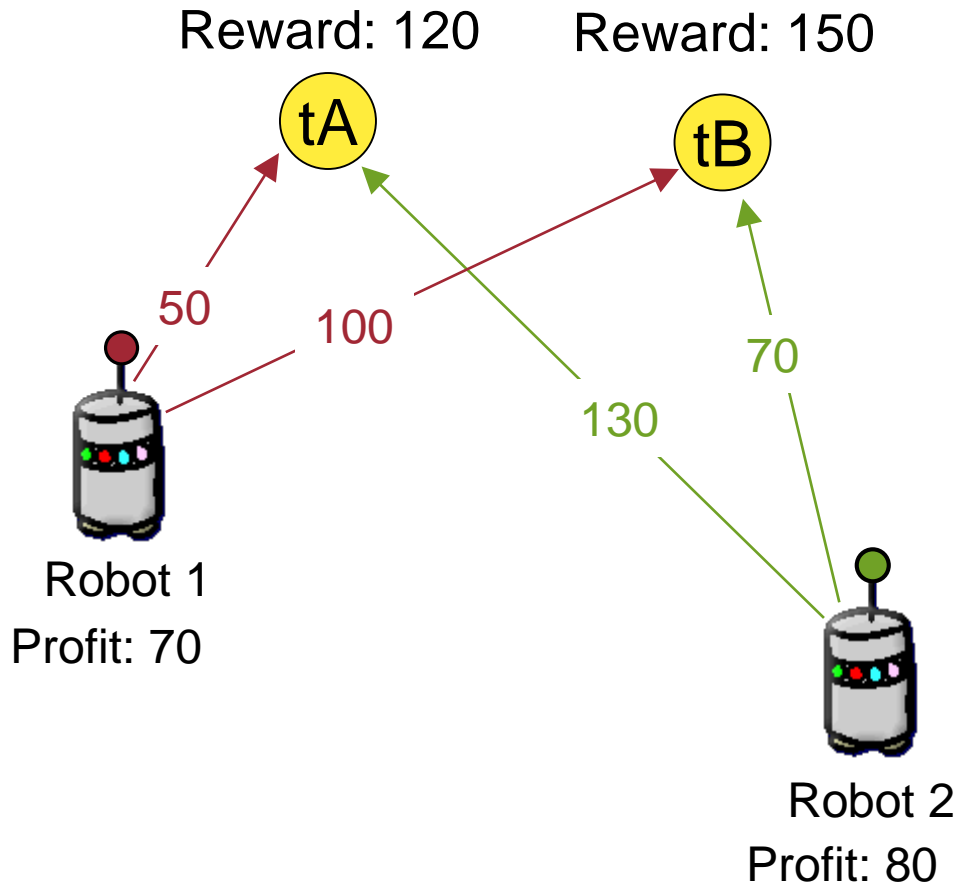
- Task(s) offered in a task announcement
- Robots bid (often marginal cost/utility)
- Auctioneer (can be a robot) clears auction and awards



Distributed Sensing Mission



A Simple Example



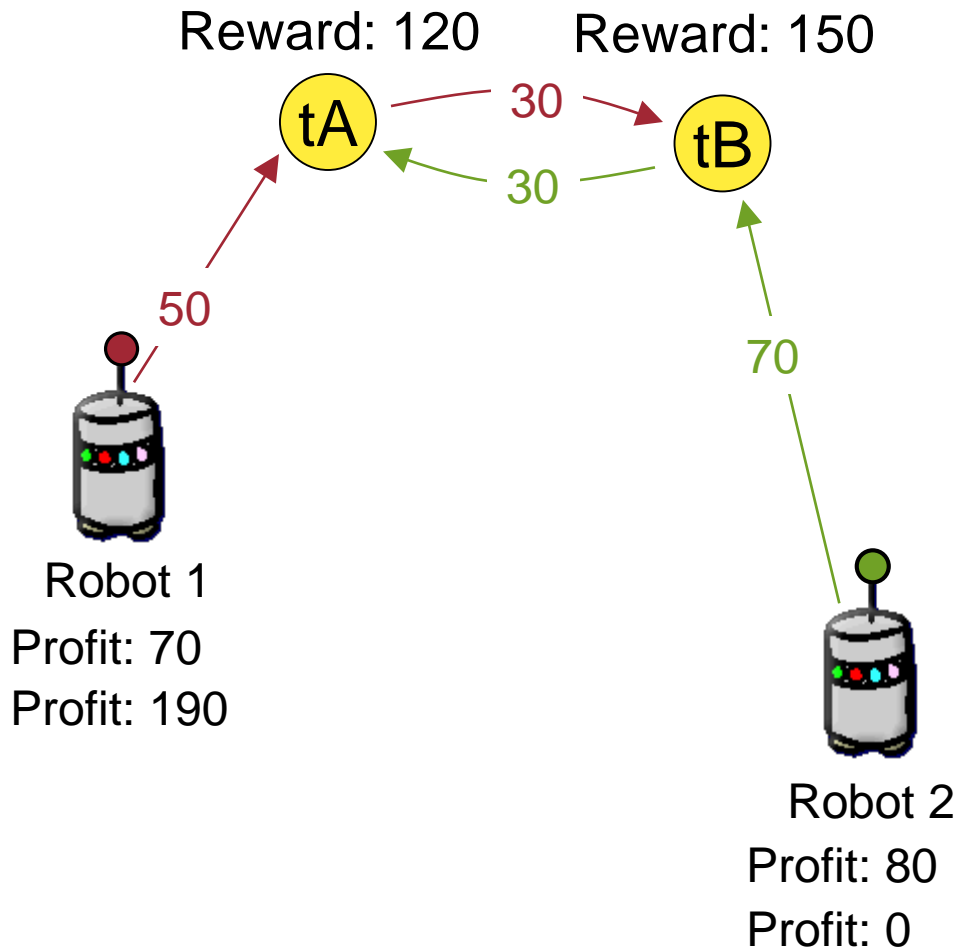
Bids Placed for Tasks

	tA	tB
<i>Robot 1</i>	50	100
<i>Robot 2</i>	X	70

System cost: $50+70=120$

No bid since min bid > reward
(called also **reserve price**)

A Simple Example



Bids Placed for Tasks

	<i>tA</i>	<i>tB</i>	<i>tAtB</i>
<i>Robot 1</i>	50	100	80
<i>Robot 2</i>	X	70	100

System cost: $50+70=120$

System cost: $50+30=80$

Formally

- Team has:
 - decomposable objective (e.g. visit a set of locations)
 - limited set of resources (e.g. time, energy, sensors)
- A global objective function C ($C: \text{Allocation}_i \rightarrow \mathbb{R}^+$)
- A local utility (or cost) function U_j ($U_j: \text{Actions}_j \rightarrow \mathbb{R}^+$)
- A mapping M for credit-assignment ($M: U \leftrightarrow C$)
- Market mechanism for redistribution (e.g. auction)

Auctions and Bidding

Types of Auctions

- Single task
- Combinatorial $\{1, 12, 13, \dots, 1234\}$: 2^T
- Multitask $\{12, 34\}$: 2



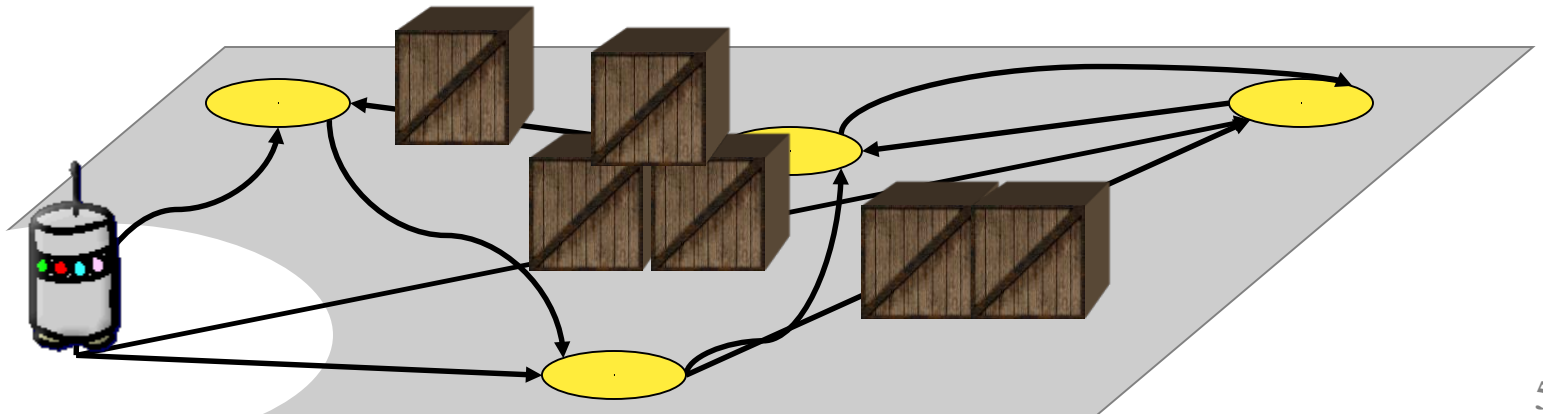
Costs v. Utilities

- Cost-based auction:
 - Minimize global cost function
 - Lowest bidder wins
- Utility-based auction:
 - quality, value - cost:
 - more/less important tasks
 - more/less capable robots
 - Maximize global utility
 - Highest bidder wins
- Capture multiple factors: quality, time, energy, etc.
 - How to combine units of quality and cost?
 - Weighted sum: α quality + β time + γ energy ...
 - How to find weights? 1 unit of quality = ? units of time

Computation Complexity and Uncertainty

Task Valuation

- Valuation functions may
 - rely on accurate information
 - require solving complex problems (e.g. tsp)
- Solution: heuristics and approximations
- Problem: inaccurate bids \Rightarrow poor allocation



Uncertainty

- Uncertainty in
 - local state
 - teammate state
 - task state
 - environment
- Solution:
 - heuristics: bid assuming optimal conditions
 - learning: use prior information about environment
 - solution repair: re-auction tasks to teammates

Examples of Market-Based Multi-Robot Coordination

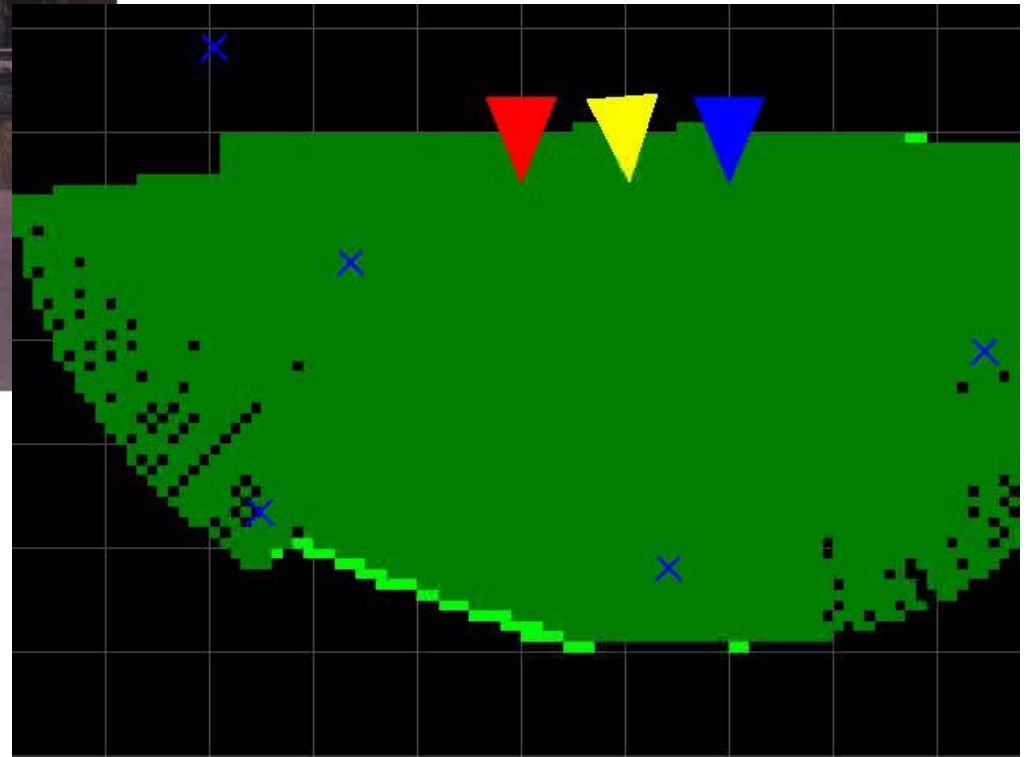
Market-Based Frameworks

- Murdoch [Gerkey, Mataric]
 - loosely coordinated tasks
 - demonstrated on box pushing
 - demonstrated robustness, fast auctioning
- TraderBots [Dias, Stentz, Zlot]
 - loosely coordinated tasks
 - demonstrated on exploration tasks
 - demonstrated robustness, scalability, auction types, task trees
- Hoplites [Kalra, Stentz]
 - tightly coordinated, spatial tasks
 - robots auction *plans* not tasks
 - demonstrated on perimeter sweeping, constrained exploration
 - relatively new

Murdoch



TraderBots



Dias, 2005

Zlot et. al., 2002

Hoplites

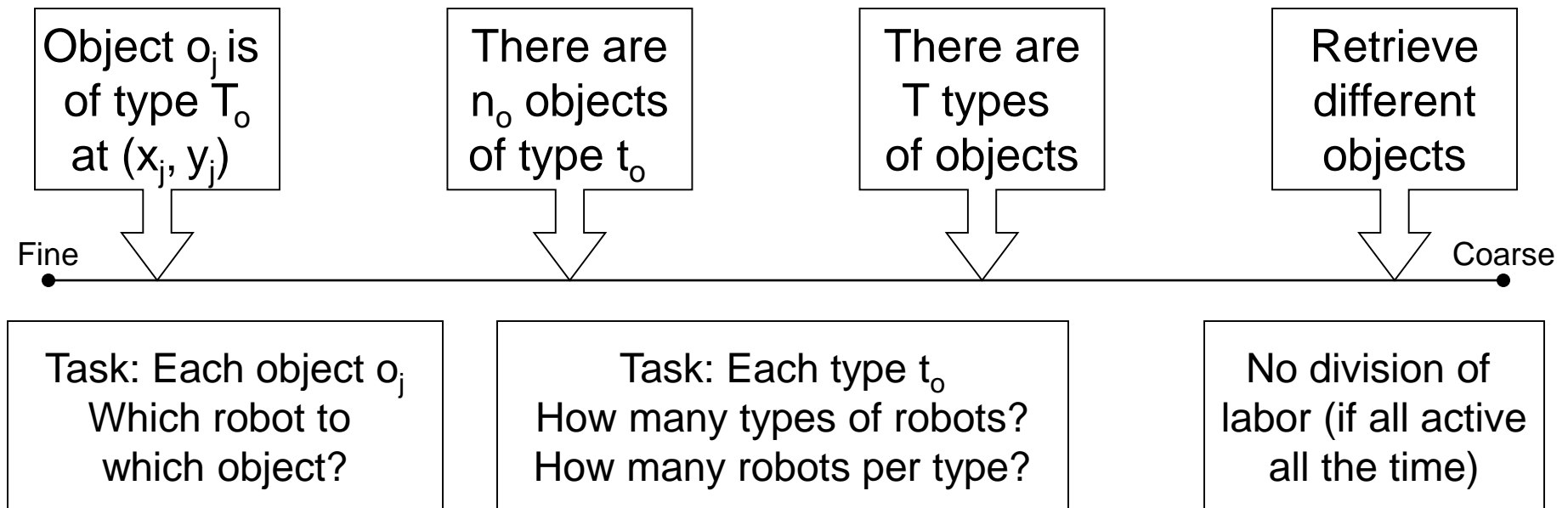


Comparing Threshold-Based and Market-Based Approaches for Multi-Robot Coordination

Division of Labor in Multi-Robot Systems

- Who does what? How many do what?
 - how many robots scan a room?
 - which robots track this object?
 - who plays goalie?
 - how many inspect each structure?
 - who is pulling first the stick?
- Who is actively engaged in a task and who is not?

Description of Tasks



Task allocation problem and solution depend heavily on task description

Differences

Threshold

Totally distributed

Simple

Non-directed communication

Little task knowledge

Use few resources

Tasks = Behaviors/Activities

Poorer solutions

Biologically Inspired

Market

Exploit centralization

Complex

Directed communication

Lots of task knowledge

Use lots of resources

Tasks = specific jobs

Better solutions

Economically inspired

What are costs and benefits of these fundamental features?

Event Handling Domain

- Random events handled by robots
 - Machine malfunctions,
 - toxic spills,
 - requests for delivery,
 - fires



$$E = \{(x_1, y_1) (x_2, y_2) \dots (x_n, y_n)\}$$

- Tasks:
 - One robot must handle each event
 - Instantaneous Assignment (no lookahead)

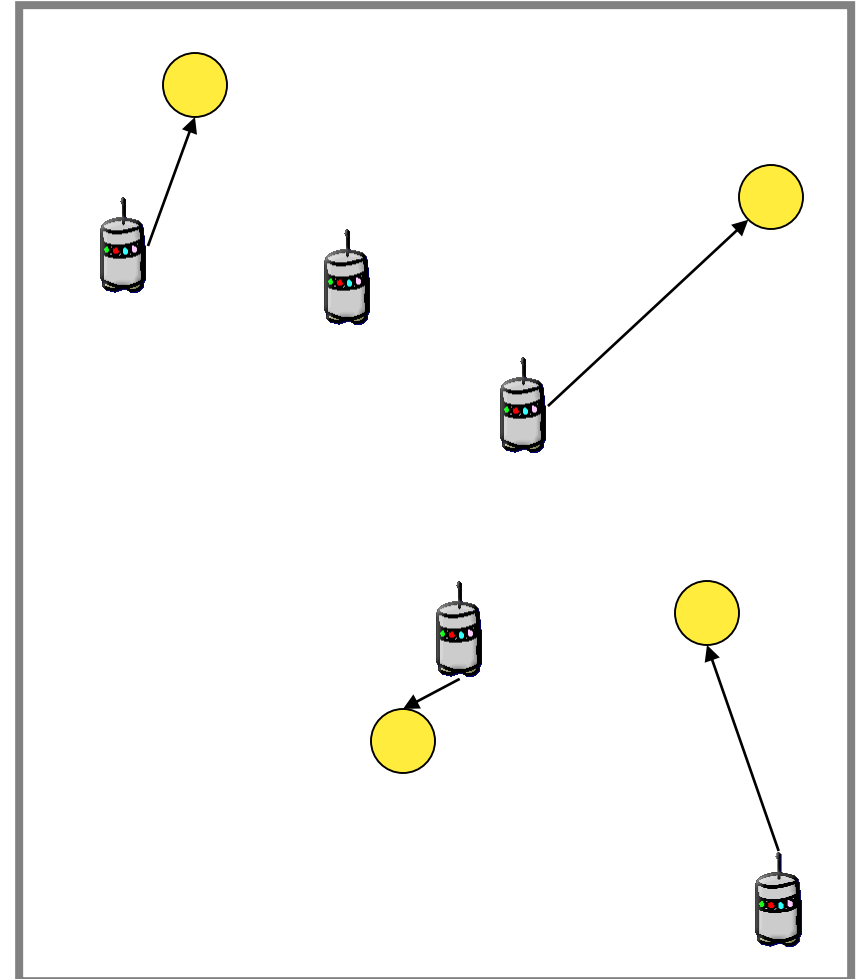


Metrics

- Cost of producing allocation:
 - Communication frequency
 - Computation
- Quality of allocation:
 - Number of events handled
 - Total distance traveled

The Market-Based approach

- Goal: assign event to nearest robot
- Allocation:
 - Sequential ordered auctions
 - Bid: distance to nearest event
 - Reserve price: $d_{expected}$



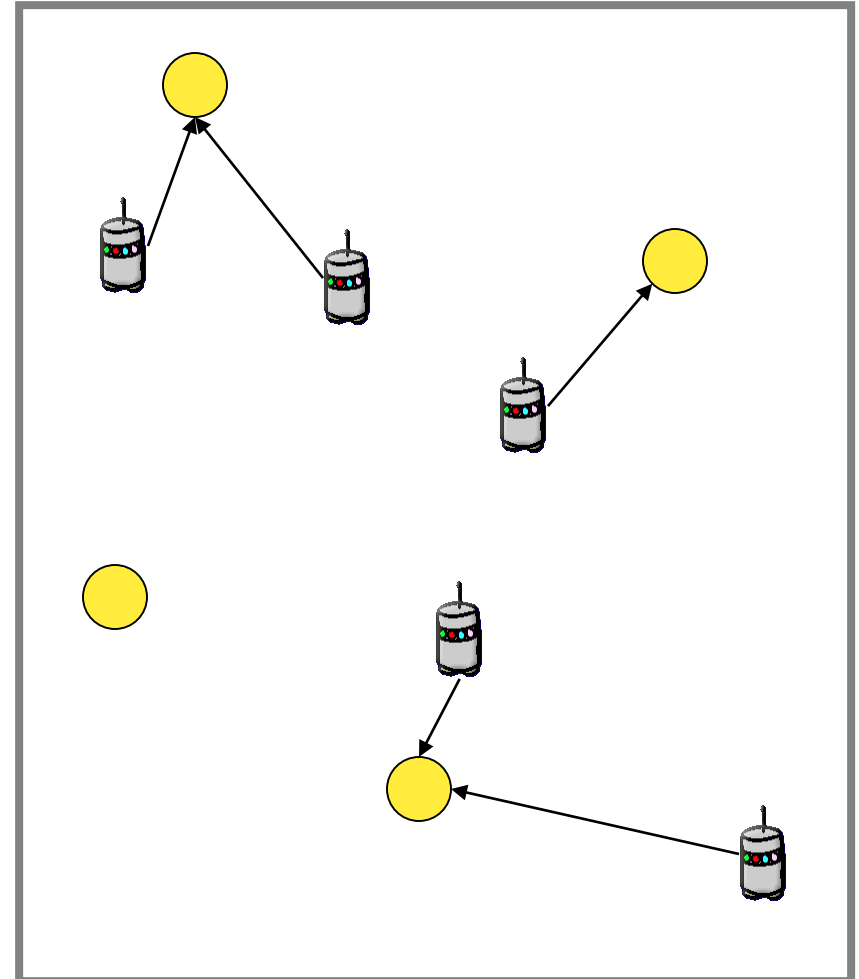
Threshold-Based Approach

- Goal: robot handles closest event w/o duplicating work

- $$\theta_e = \frac{1}{d_{expected}}$$

- $$\sigma_e = \frac{1}{d_e}$$

- Deterministic response
- Random walk



Axes

- Communication range (0 to full)
- Accuracy in state estimation (perfect to v. noisy)
- Accuracy in task estimation (perfect to v. noisy)
- Range in task estimation (0 to full)

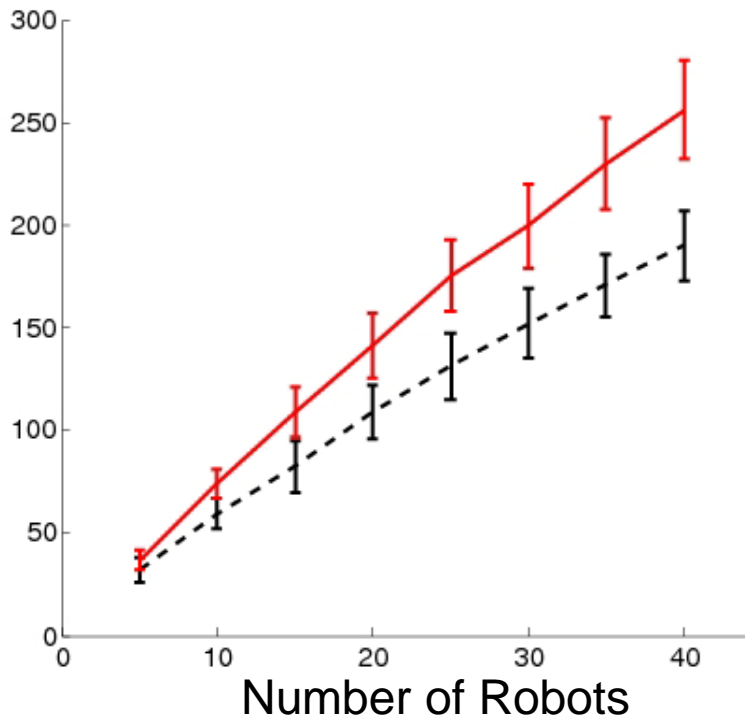
Experiments

- 10, 20, and 40 robots
- 100 x 100 environment
- 20 random events at all times
- 100 trials per experiment
- 100 steps per trial
- microscopic agent-based model (perfectly holonomic vehicles)

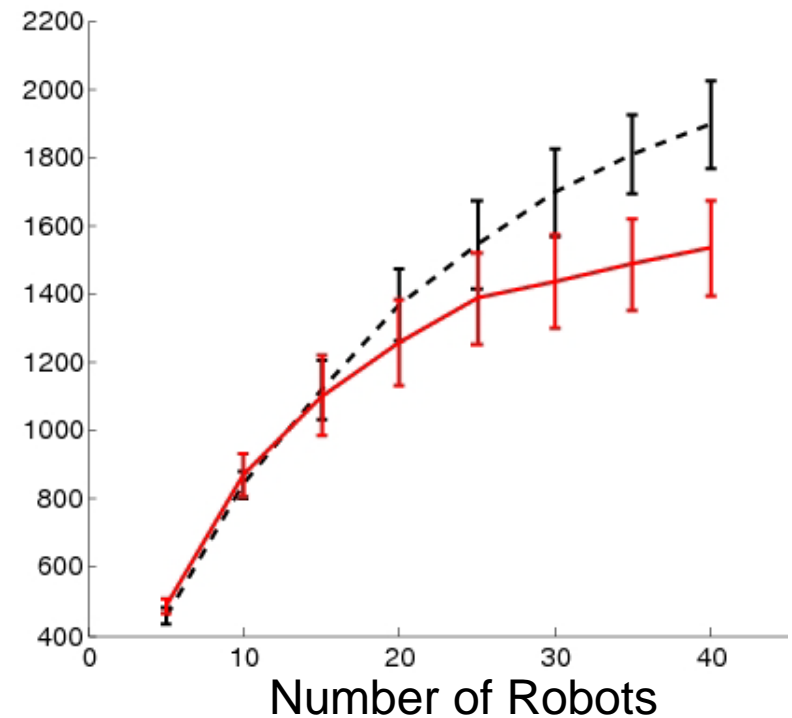
— Market
- - - Threshold

Quality: Baseline

Events Handled v Team Size



Distance v Team Size

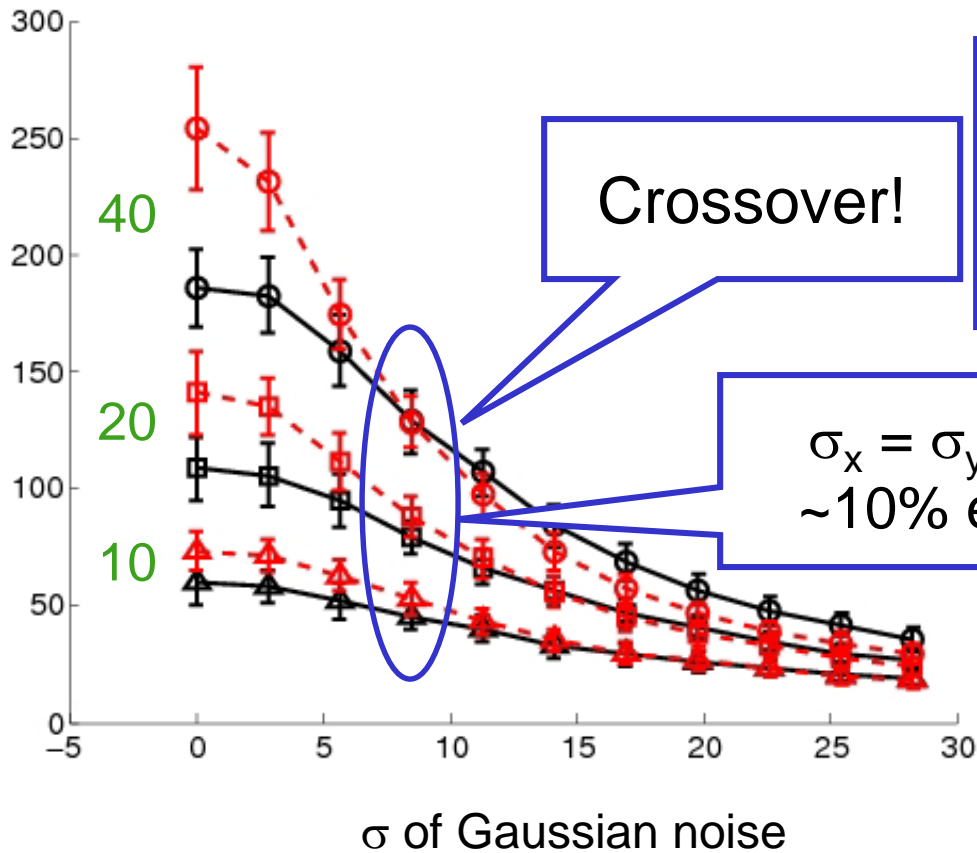


Perfect global com, no noise, parameters optimized for both

--- Market
 — Threshold
 10 Team Size

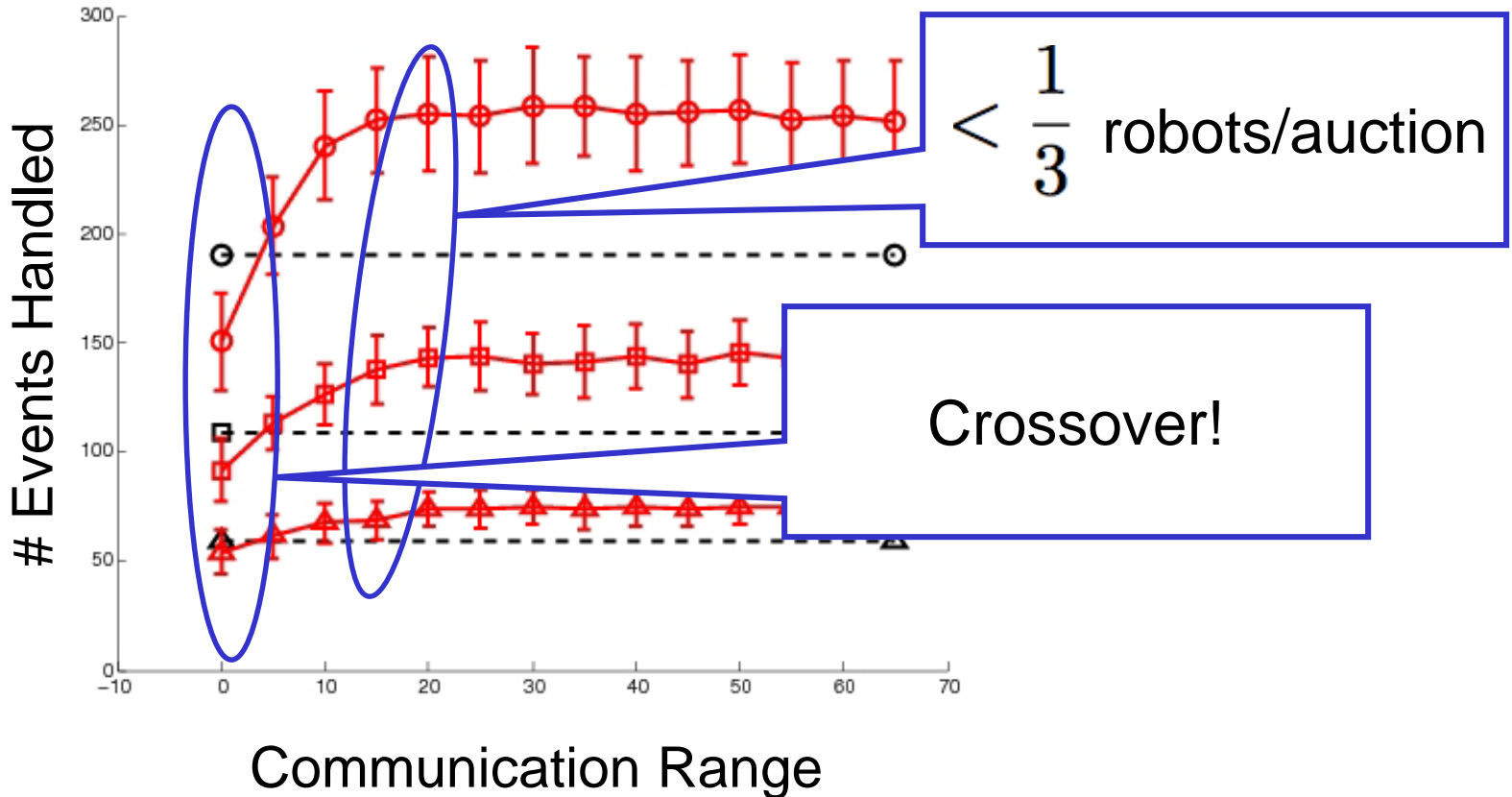
Noisy Task Estimation

Events Handled v Noise



— Market
- - - Threshold
10 Team Size

Poor communication range



Summary of Results

- Quality of information:
 - Markets “worth it” only with good information
 - Else, thresholds: same quality, less expensive
 - Markets worse in some respects!
- Robust to range of information
 - Markets successful with short comms
 - Both successful with short perception range

Conclusions

Take Home Messages

- Existence of several castes in natural societies (age-based, behavioral, morphological) are evident signs of division of labor
- From macroscopic models for natural systems to microscopic algorithms for artificial systems: non-trivial design choices must be carried out
- Two different classes of threshold-based algorithms: fixed and variable thresholds
- Market-based algorithms exploit points of centralization and networking for allocating more efficiently tasks
- Market-based algorithms can be more efficient than threshold-based ones at the price of more resources used; market-based allocation works efficiently only with accurate information but do not suffer too much about short communication ranges

Additional Literature – Week 6

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