

Lab 9: Sensor Fusion for Localization

This laboratory requires the following equipment:

- C development tools (gcc, make, etc.)
- Webots
- Matlab

The laboratory duration is approximately 3 hours. Although this laboratory is not graded, we encourage you to take your own personal notes as the homework and the final exam might leverage results acquired during this laboratory session. For any questions, please contact us at sis-ta@groupe.epfl.ch.

1.1 Information

In this assignment, you will find several exercises and questions.

- The notation **Q** means that the question can be answered theoretically or with simple commands in the Linux operating system, without implementing or running any code.
- The notation **S** means that the question can be solved only by compiling and running a piece of code or an additional simulation.
- The notation **I** means that the problem has to be solved by implementing, possibly compiling, and running a piece of code.
- The notation **B** means that the question is optional (bonus) and should be answered if you have enough time at your disposal.

1.2 Getting Started (Short reminder)

To start with this lab, you will need to download the material available on Moodle. Download *lab09.zip* and extract the lab archive.

1.3 General remarks and documentation

In this lab, you will continue working on odometry with the e-puck robot, extending the techniques introduced in Lab 8. Here you will understand the importance of sensor fusion to obtain an accurate interpretation of the data, and how noise on these data can be considered in this process. The first part demonstrates how the e-puck can navigate using 1D dead reckoning under non-deterministic uncertainties. The second part shows different techniques to fuse odometry with feature-based localization in Matlab. The last part demonstrates how to leverage a Kalman filter in Webots to fuse odometry with feature-based localization.

2 Odometry with non-deterministic uncertainties

As you saw in Lab 8, the e-puck can track its position very well with a properly calibrated dead reckoning system and perfect sensors (noise-free). However, non-deterministic sensor noise present on real robots will add additional difficulties to estimate the robot position, especially when information is only coming from one type of sensor. In these cases, the fusion of information from different sensors will help solve the issue. Open the world *featurenavigation.wbt*, clean and compile the *e-puck_feature* controller and run the simulation. The goal of this controller is to ensure the robot safely navigates in the corridor and reaches the goal position (front wall).

1. **(Q):** Open the robot's controller *e-puck_feature.c* and try to understand the code. Explain in your own words what happens. Try to come up with possible ways to improve the accuracy of the motion under non-deterministic sensor noise conditions (do not implement them). What are their downsides? *Note: run the simulation for about 3h of simulation time to fully see the degradation in performance of odometry.*
2. **(I):** One possibility to correct the accumulating errors is feature-based navigation. We now want to use the front wall as a feature to correct the estimation of robot position. We will do this by developing a function that measures the distance to the wall and use it as the feature to adjust the robot's position. Open the file *e-puck_feature.c* and implement the above-mentioned strategy, by supposing that the front wall is at the position (1, 0) and the robot start at position (0,0) in the function *feature_update*.

3 Sensor Fusion with 1D Kalman Filter

The feature-based correction of the 1D odometry implemented in Part 2 is a very basic sensor fusion. Indeed, we used the information of both the wheel encoders as well as a distance sensor, effectively resulting in a fusion of the information. This implementation, however, is a relatively crude one, since it assumes that the detection of the front wall is free of non-deterministic noise, which is not realistic. In this section we will gradually study a more general approach to sensor fusion leveraging a well-known estimation technique called Kalman filter. For this, open the file *Kalman1d.m* in Matlab.

3. **(Q)** Try to understand what this script does. The goal here is simply to familiarize oneself with the code.
Note: This lab is not very guided on purpose. Take the time to reflect on each step and refer back to lecture slides to understand what you are implementing and why.
4. **(Q)** Using linear propagation of covariance (c.f. slides Week 10), calculate the noise on the position estimate (*est_odo_x*) after 5 timesteps (position noise at $T=1$ is equal to 0). What is the noise after 200 steps? Compute this on paper.
Hint: if $Q = a + b$ then the associated standard deviation is $\delta Q = \sqrt{\delta a^2 + \delta b^2}$
Hint: you can refer to this link <https://ch.mathworks.com/help/matlab/math/random-numbers-with-specific-mean-and-variance.html> for the probabilistic distribution of random noise.
5. **(S)** What happens if you change the noise level *localization_noise*? Is this factor taken into account in the position estimation (*est_odo_x*)?
6. **(I)** Since for most real applications the noise level is known, it is suboptimal to ignore this information. Applying the central limit theorem, a very simple option to take into account the

noise levels of two variables when combining them, has been proposed by Fraser and Potter in 1969 [1], [2]:

$$x_{fused} = \frac{(\sigma_1^{-2}x_1 + \sigma_2^{-2}x_2)}{\sigma_1^{-2} + \sigma_2^{-2}}$$

Implement the position estimate using the Fraser Potter equation (*est_fp_x* in the code).

Hint: replace the simple resetting of the position with this formula using the noise level calculated in Question 4 for the odometry estimate (200 steps correspond to the interval between features). Use $\sigma = \text{localization_noise}$ for the measurement noise

Run the script and compare the resulting trajectories. What do you observe?

7. **(Q)** What happens if the measurement noise is significantly higher than the odometry noise and vice-versa? What is the issue of this sensor fusion algorithm?
8. **(Q)** Another method to fuse measurements is the Kalman filter, which tracks the uncertainty level of the estimation at each step. Take a look at the formula (seen in class) in Figure . Try to identify what each line (step) of the algorithm does. The final position estimate will be stored in the variable *est_kal_x*.

Estimation Based on Motion Model	Kalman Filter for Sensor and Motion Model Fusion
$\hat{x}_{k'} = \hat{x}_k + u[t_{k+1} - t_k]$	$\hat{x}_{k+1} = \hat{x}_{k'} + K_{k+1}(z_{k+1} - \hat{x}_{k'})$
$\sigma_{k'}^2 = \sigma_k^2 + \sigma_w^2[t_{k+1} - t_k]$	$\sigma_{k+1}^2 = \sigma_{k'}^2 - K_{k+1} \sigma_{k'}^2$
	$K_{k+1} = \frac{\sigma_{k'}^2}{\sigma_{k'}^2 + \sigma_z^2}$

Figure 1 1D Kalman Filter

9. **(I)** Create the new state variable $\mu = [x]$. Implement the prediction step for μ such that the position value of μ increases by the odometry measure every timestep (i.e. every time the prediction is called). Run your code and verify (e.g., through printing) that μ increases as expected.
10. **(I)** Initialize the noise state variable $\sigma = [0]$. By initializing σ in this way, what are we assuming? Implement the noise prediction update step.
11. **(I)** Implement now the correction step. This step is called every time a new measure is acquired to correct or update the prediction. Set the measurement noise to the square of the *localization_noise* variable. Run your code to plot μ and compare the result with what you obtained before. You can run your code several times to see how each approach (odometry, Fraser-Potter and Kalman filter) performs on average, the random noise introduced in the system will give you slightly different results each time.
12. **(Q)** What are the differences between the Fraser Potter algorithm and the Kalman filter in the 1D case?

4 Sensor Fusion with 2D Kalman Filter

13. **(B)** We shall now consider the case where the robot speed v is considered as part of the state. For this, you will work in the file *Kalman2d.m* in Matlab. Take a look at the formula (seen in class) in Fig. 2. What are μ , Σ , u and z ? What is the purpose of K ?

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1:   Algorithm Kalman_filter( $\mu_{t-1}, \Sigma_{t-1}, u_t, z_t$ ):
2:        $\bar{\mu}_t = A_t \mu_{t-1} + B_t u_t$ 
3:        $\bar{\Sigma}_t = A_t \Sigma_{t-1} A_t^T + R_t$ 
4:        $K_t = \bar{\Sigma}_t C_t^T (C_t \bar{\Sigma}_t C_t^T + Q_t)^{-1}$ 
5:        $\mu_t = \bar{\mu}_t + K_t (z_t - C_t \bar{\mu}_t)$ 
6:        $\Sigma_t = (I - K_t C_t) \bar{\Sigma}_t$ 
7:       return  $\mu_t, \Sigma_t$ 

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Figure 2: Kalman filter. Note that steps 2 and 3 (called prediction) can be done independently from steps 4 to 6 (update step).

14. **(B)** Then create the new state variable $\mu = [x; v]$ with the noise state variable $\Sigma = [0, 0; 0, 0]$. The input is given by $u = [\text{odo}]$, where *odo* is the velocity obtained from the odometry. To implement the prediction step, define A and B such that the position value of μ increases by odometry every timestep. Adapt R for your code to execute and run your code and verify that μ and Σ fluctuate as expected.
Hint: Be careful about the dimensions of your matrices.
15. **(B)** Implement the 2D case of the Kalman filter by inspiring yourself from your previously implemented correction step and adapting it to the new two-variable state. Run your code and observe the result. Is there a major difference to the previous Kalman filter result? *Hint: the correction happens only on x , leaving the speed part of μ untouched. This means that both z and $C^* \mu$ are 1 dimensional, whereas K needs to be in such a way that $K^*[1 \text{ dim}]$ results in a vector of the same dimension as μ .*
16. **(B)** Based on the changes you did to include the robot speed v in the state space, what changes would be necessary to include x , y , v_x and v_y ?

5 References

- [1] D. Fraser and J. Potter, "The optimum linear smoother as a combination of two optimum linear filters," in IEEE Transactions on Automatic Control, vol. 14, no. 4, pp. 387-390, August 1969, doi: 10.1109/TAC.1969.1099196, url: <https://ieeexplore.ieee.org/document/1099196>
- [2] Maybeck, Peter S. "Stochastic models, estimation, and control". Academic press, 1982.